Technical Note

Are Some Top-Heavy Structures More Stable?

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Abstract: This technical note investigates the dynamic response and stability of a rocking frame that consists of two identical free-standing slender columns capped with a freely supported rigid beam. Part of the motivation for this study is the emerging seismic design concept of allowing framing systems to uplift and rock along their plane in order to limit bending moments and shear forces— together with the need to stress that the rocking frame is more stable the more heavy is its cap-beam, a finding that may have significant implications in the prefabricated bridge technology. In this technical note, a direct approach is followed after taking dynamic force and moment equilibrium of the components of the rocking frame, and the remarkable results obtained in the past with a variational formulation (by the same authors) is confirmed—that the dynamics response of the rocking frame is identical to the rocking response of a solitary, free-standing column with the same slenderness, yet with larger size, which produces a more stable configuration. The motivation for reworking this problem by following a direct approach is to show, in the simplest possible way, that the heavier the freely supported cap beam, the more stable is the rocking frame, regardless of the rise of the center of gravity of the cap beam. The conclusion is that top-heavy rocking frames are more stable that when they are top-light. **DOI: 10.1061/(ASCE)ST.1943-541X.0000933.** *© 2014 American Society of Civil Engineers.*

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Introduction

It is common experience that a small, slender, free-standing, top-heavy object (such as a vase) may easily overturn due to a horizontal shaking, while a racing car with a low center of gravity remains stable even under the large horizontal forces that develop during a sharp turn. Whenever the stability of a free-standing object is an issue, the obvious, intuitive measure is to lower its center of gravity.

At the same time, ancient free-standing columns with aspect ratio as high as $6/1$ supporting heavy free-standing epistyles together with the even heavier frieze atop have survived the test of time and remain stable for several hundred years in areas with appreciable seismic hazard (Konstantinidis and Makris 2005).

The remarkable seismic stability of tall, free-standing solitary columns was understood some 50 years ago by Housner (1963), who uncovered a size-frequency scale effect that explained why (1) the larger of two geometrically similar blocks survives an excitation that will topple the smaller block, and (2) of two acceleration pulses with the same amplitude, the one with the longer duration is more capable of overturning. Following Housner's pioneering work, several studies from other investigators (Aslam et al. 1980; Yim et al. 1980; Spanos and Koh 1984; Tso and Wong 1989; Shenton 1996; Makris and Rousos 2000; Zhang and Makris 2001; Dimitrakopoulos and DeJong 2012) showed that the uplifting and rocking of solitary, tall, free-standing columns has beneficial effect to their seismic resistance, similar to the way that sliding reduces the base shears of heavy low-rise structures (Skinner and Robinson 1993; Kelly 1997; Konstantinidis and Makris 2009, 2010).

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Results on the dynamic response of two free-standing columns capped with a freely supported beam have been presented by Allen et al. (1986), who adopted a Lagrangian formulation. In the Allen et al. (1986) paper, it was assumed that the mass of each column, m_c, is much less than the mass of the freely supported beam, m_b, and therefore, the equation of motion derived was for $m_b/m_c \rightarrow \infty$. Furthermore, the results presented were obtained by solving the linearized equation of motion.

In this technical note, it is shown that the exact nonlinear equation of motion can be derived and solved without making any approximations and results are offered for any finite value of $\gamma = m_b/2m_c$. Furthermore, while the governing equation for the rocking frame appearing in the Allen et al. (1986) paper shows clearly that the response involves the slenderness, α, and size, R, of the columns of the rocking frame, the Allen et al. (1986) paper does not make any attempt to associate the dynamic response/stability of the rocking frame with that of the solitary rocking column.

Within the context of a planar rocking motion, this technical note shows that the dynamic response of the four-hinge free-standing rocking frame shown in Fig. 1 is more stable than the dynamic response of one of its columns when standing alone. Most importantly, this technical note shows that the heavier the freely supported beam, the more stable is the rocking frame, regardless of the rise of the center of gravity of the system. The conclusion is that rocking frames are more stable than when they are top-heavy than when they are top-light. Numerical studies with the discrete element method by Papaloizou and Komodromos (2009) are in agreement with our analytical result—that the planar response of free-standing columns supporting epistyles is more stable than the response of the solitary, free-standing column.

Dynamics of the Rocking Frame

With reference to Fig. 1, and assuming that the coefficient of friction is large enough that there is no sliding, the equations of motion of a free-standing column with size $R = \sqrt{b^2 + h^2}$ and slenderness $\alpha = \tan^{-1}(b/h)$, and subjected to a horizontal ground acceleration $\ddot{u}_g(t)$ when rocking around pivot points O and O',

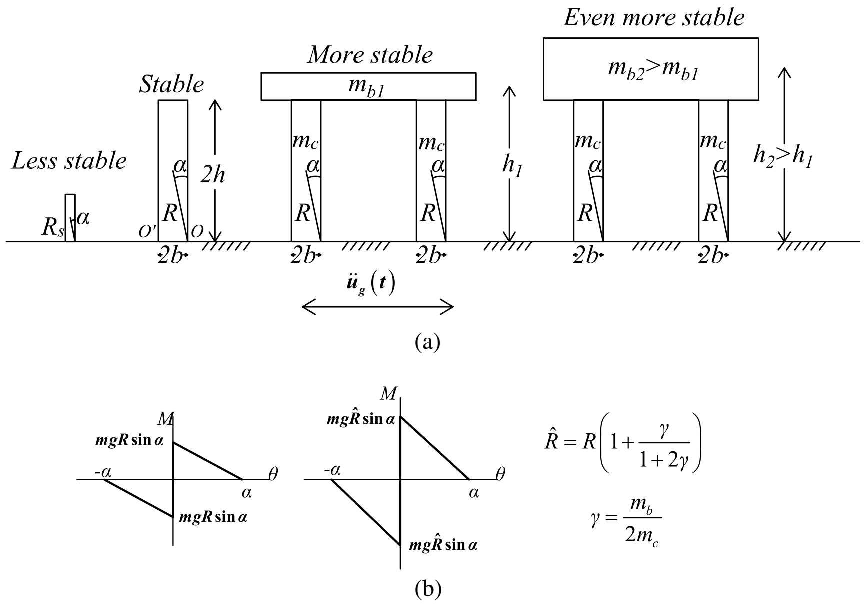

Fig. 1. A large free-standing column with size R and slenderness α is more stable than a geometrically similar smaller column shown at the far left of the figure; a free-standing rocking frame with columns having the same size R and same slenderness α is more stable than a solitary rocking column; a heavier freely supported cap-beam renders the rocking frame even more stable, regardless of the rise of the center of gravity of the system

respectively, is (Yim et al. 1980; Hogan 1989; Makris and Roussos 2000; Zhang and Makris 2001)

$$I_o\ddot{\theta}(t) = mgR\sin[\alpha + \theta(t)] - m\ddot{u}_g(t)R\cos[-\alpha - \theta(t)], \quad \theta(t) < 0 \tag{1}$$

$$I_o\ddot{\theta}(t) = -mgR\sin[\alpha - \theta(t)] - m\ddot{u}_g(t)R\cos[\alpha - \theta(t)], \quad \theta(t) > 0 \tag{2}$$

For a rocking motion to be initiated, $\ddot{u}_g(t) > g\tan\alpha$ at some time in its excitation history. The equations can be expressed in the compact form

$$\ddot{\theta}(t) = -p^2\left\{\sin[\alpha\text{sgn}(\theta(t)) - \theta(t)] + \frac{\ddot{u}_g}{g}\cos[\alpha\text{sgn}(\theta(t)) - \theta(t)]\right\} \tag{3}$$

where $p = \sqrt{mRg/I_o}$ The oscillation frequency of a rigid block under free vibration is not constant, because it strongly depends on the vibration amplitude Housner (1963). Nevertheless, the quantity p is a measure of the dynamic characteristics of the block. For rectangular blocks, $I_o = (4/3)$ mR^2 and the frequency parameter assumes the value $p = \sqrt{3g/4R}$

Fig. 1(b) shows the moment-rotation relationship during the rocking motion of a free-standing column. The rocking system has infinite stiffness until the external horizontal forces induce a moment as high as mgR $\sin\alpha$, and once the column is rocking, its restoring force decreases monotonically, reaching zero when $\theta = \alpha$.

Equations of Motion of the Rocking Frame

The free-standing rocking frame shown in Fig. 1 is a single DOF structure with size $R = \sqrt{b^2 + h^2}$ and slenderness $\alpha = \tan^{-1}(b/h)$. The only additional parameter that influences the dynamics of the rocking frame is the ratio of the mass of the cap beam, m_b, to the mass of its two columns, m_c, $\gamma = m_b/2m_c$. As in the case of the single rocking column, the coefficient of friction is large enough that sliding does not occur at the pivot point at the base and at the cap-beam. Accordingly, the horizontal translation displacement $u(t)$ and vertical lift $v(t)$ of the cap beam are functions of the single DOF $\theta(t)$. For a positive horizontal ground acceleration (the ground is accelerating to the right), the rocking frame will initially rock to the left [$\theta(t) < 0$], as shown in Fig. 2. Assuming that the rocking frame will not topple, it will recenter, impacts will happen at the pivot points (at the base and the top of the columns with the cap-beam), and subsequently it will rock to the right [$\theta(t) > 0$]. During rocking, the dependent variables $u(t)$, $v(t)$ and their time derivatives are given for $\theta(t) < 0$ and $\theta(t) > 0$ by the following expressions:

$$u = \mp 2R(\sin\alpha - \sin[\alpha \pm \theta]) \tag{4}$$

$$\dot{u} = 2R\cos(\alpha \pm \theta)\dot{\theta} \tag{5}$$

$$\ddot{u} = 2R(\mp\sin[\alpha \pm \theta][\dot{\theta}]^2 + \cos[\alpha \pm \theta]\ddot{\theta}) \tag{6}$$

and

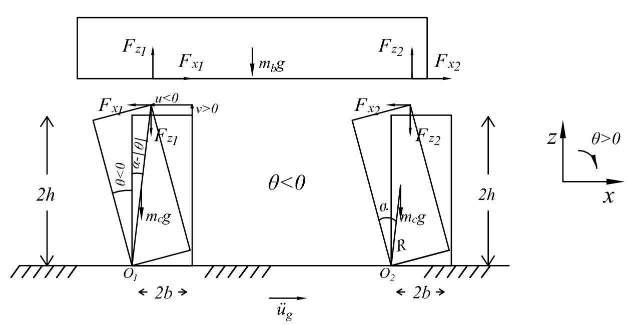

Fig. 2. Rocking frame with negative rotation ($\theta < 0$) together with the free-body diagram of the cap-beam

$$v = 2R(\cos[\alpha \pm \theta] - \cos\alpha) \tag{7}$$

$$\dot{v} = \mp 2R\sin(\alpha \pm \theta)\dot{\theta} \tag{8}$$

$$\ddot{v} = -2R(\cos[\alpha \pm \theta][\dot{\theta}]^2 + \sin[\alpha \pm \theta]\ddot{\theta}) \tag{9}$$

In the Eqs. (4)–(9), whenever there is a double sign (say $\pm$) the top sign is for $\theta(t) < 0$ and the bottom sign is for $\theta(t) > 0$.

With reference to Fig. 2, moment equilibrium of each column about their pivot point O_1 and O_2 at the ground gives

$$I_o\ddot{\theta} + m_c\ddot{u}_g R\cos(\alpha+\theta) = m_c gR\sin(\alpha+\theta) - F_{x1}2R\cos(\alpha+\theta) + F_{z1}2R\sin(\alpha+\theta) \tag{10}$$

$$I_o\ddot{\theta} + m_c\ddot{u}_g R\cos(\alpha+\theta) = m_c gR\sin(\alpha+\theta) - F_{x2}2R\cos(\alpha+\theta) + F_{z2}2R\sin(\alpha+\theta) \tag{11}$$

Given that Eqs. (10) and (11) are for $\theta < 0$, the argument $\alpha - |\theta|$ has been replaced with $\alpha + \theta$. Subtraction of Eq. (11) from Eq. (10) gives

$$F_{x2} - F_{x1} = -(F_{z1} - F_{z2})\tan(\alpha+\theta) \tag{12}$$

Force equilibrium of the cap-beam along the horizontal direction gives

$$m_b(\ddot{u}_g + \ddot{u}) = F_{x1} + F_{x2} \tag{13}$$

and subtraction of Eq. (12) from Eq. (13) gives

$$-2F_{x1} = -(F_{z1} - F_{z2})\tan(\alpha+\theta) - m_b(\ddot{u}_g + \ddot{u}) \tag{14}$$

Substitution of the expression of $-2F_{x1}$ given by Eq. (14) into Eq. (10) gives

$$\frac{I_o\ddot{\theta}}{R} + m_c\ddot{u}_g\cos(\alpha+\theta) = m_c g\sin(\alpha+\theta) + (F_{z1}+F_{z2})\sin(\alpha+\theta) - m_b(\ddot{u}_g + \ddot{u})\cos(\alpha+\theta) \tag{15}$$

Eq. (15) is a statement of the dynamic equilibrium of the rocking frame; nevertheless, the vertical reactions, F_{z1} and F_{z2} are functions of $\ddot{\theta}(t)$, which needs to appear explicitly in the equation of motion.

Force equilibrium of the cap-beam along the vertical direction gives

$$m_b(g + \ddot{v}) = F_{z1} + F_{z2} \tag{16}$$

Substitution of Eq. (16) into (15) and after rearranging terms gives

$$\frac{I_o\ddot{\theta}}{R} = -(m_c+m_b)\ddot{u}_g\cos(\alpha+\theta) + (m_c+m_b)g\sin(\alpha+\theta) + m_b[\ddot{v}\sin(\alpha+\theta) - \ddot{u}\cos(\alpha+\theta)] \tag{17}$$

Replacing the relative horizontal and vertical accelerations $\ddot{u}$ and $\ddot{v}$ with the expressions given by Eqs. (6) and (9), and after cancelling the quadratic angular velocity terms, Eq. (17) simplifies to

$$\left(\frac{I_o}{R} + 2m_bR\right)\frac{\ddot{\theta}}{g} = (m_c+m_b)\sin(a+\theta) - (m_c+m_b)\frac{\ddot{u}_g}{g}\cos(\alpha+\theta) \tag{18}$$

Using that for rectangular columns, $I_o = 4/3\ m_cR^2$, and if $\gamma = m_b/2m_c$, Eq. (18) assumes the form

$$\ddot{\theta}(t) = -\frac{1+2\gamma}{1+3\gamma}\frac{3g}{4R}\left\{\sin[a+\theta(t)] - \frac{\ddot{u}_g(t)}{g}\cos[a+\theta(t)]\right\},\quad \theta(t) < 0 \tag{19}$$

When the rotation of the rocking frame is positive $[\theta(t) > 0]$, moment equilibrium of each column about their pivot point O_1 and O_2 at the ground gives,

$$I_o\ddot{\theta} + m_c\ddot{u}_g R\cos(\alpha-\theta) = -m_c gR\sin(\alpha-\theta) + F_{x1}2R\cos(\alpha-\theta) - F_{z1}2R\sin(\alpha-\theta) \tag{20}$$

$$I_o\ddot{\theta} + m_c\ddot{u}_g R\cos(\alpha-\theta) = -m_c gR\sin(\alpha-\theta) + F_{x2}2R\cos(\alpha-\theta) - F_{z2}2R\sin(\alpha-\theta) \tag{21}$$

By following an equivalent approach to the case for $\theta(t) < 0$, the equation of motion of the rocking frame for $\theta(t) > 0$ is

$$\ddot{\theta}(t) = -\frac{1+2\gamma}{1+3\gamma}\frac{3g}{4R}\left\{-\sin[a-\theta(t)] - \frac{\ddot{u}_g(t)}{g}\cos[a-\theta(t)]\right\},\quad \theta(t) > 0 \tag{22}$$

As in the case of the solitary free-standing rocking column Eqs. (19) and (22) can be expressed in the compact form

$$\ddot{\theta} = -\frac{1+2\gamma}{1+3\gamma}p^2\left\{\sin[a\,\mathrm{sgn}(\theta(t)) - \theta(t)] + \frac{\ddot{u}_g(t)}{g}\cos[a\,\mathrm{sgn}(\theta(t)) - \theta(t)]\right\} \tag{23}$$

where $p = \sqrt{3g/4R}$, is the frequency parameter of the solitary rocking column of the rocking frame.

Eq. (23), which describes the planar motion of the free-standing rocking frame, is precisely the same as Eq. (3), which describes the planar rocking motion of a solitary free-standing rigid column with the same slenderness α, except that in the rocking frame, the term p^2 is multiplied by the factor $(1+2\gamma)/(1+3\gamma)$. Accordingly, the frequency parameter of the rocking frame is, $\hat{p} = \sqrt{1+2\gamma/1+3\gamma}p$, in which $\gamma = m_b/2m_c$.

Eq. (23) indicates that the rocking response and stability analysis of the free- standing rocking frame with a cap-beam supported on columns having slenderness α and size R is that of a solitary column with the same slenderness α and a larger size

$$\hat{R} = \frac{1+3\gamma}{1+2\gamma}R = \left(1 + \frac{\gamma}{1+2\gamma}\right)R \tag{24}$$

During rocking motion of a free-standing frame, the moment-rotation curve follows the curve shown in Fig. 1(b) without enclosing any area. Energy is lost during impact when the angle of rotation reverses. At this instant, it is assumed that the rotation continues smoothly and that the impact force is concentrated at the new pivot points. Application of the angular momentum-impulsive theorem in association with the change of the linear momentum of the cap-beam (Makris and Vassiliou 2013) offers the ratio of the kinetic energy of the rocking frame after and before impact

$$r = \left(\frac{\dot{\theta}_2}{\dot{\theta}_1}\right)^2 = \left(\frac{1 - \frac{3}{2}\sin^2\alpha + 3\gamma\cos 2a}{1 + 3\gamma}\right)^2 \tag{25}$$

Eq. (25) indicates that the maximum coefficient of restitution, $\sqrt{r}$, of the rocking frame that is needed to engage into rocking motion is always smaller (therefore, more energy is dissipated) than the maximum coefficient of restitution of the solitary column = $1–(3/2)\sin^2\alpha$ (Housner 1963), which is recovered when $\gamma = m_b/2m_c = 0$

Experimental studies on the dynamic response of the rocking frame (trilith) have been presented by Peña et al. (2008). In that study, the slenderness of the columns is $\tan\alpha = 2b/2\ h = 0.22/0.8 = 0.275(\alpha = 15.38^\circ)$, the size of the column is $R = \sqrt{(0.4\ \text{m})^2 + (0.11\ \text{m})^2} = 0.415$ m, the frequency parameter of the columns, $p = 4.21$ rad/s and $\gamma = 265\ \text{kg}/265\ \text{kg} = 0.434$. With these values, the frequency parameter of the rocking frame is $\hat{p} = \sqrt{1 + 2\gamma/1 + 3\gamma}p = 0.9p = 3.79$ rad/s. Fig. 11 of the Peña et al. (2008) paper plots response histories of the columns and cap-beam of the trilith when subjected to a constant sine excitation with amplitude $u_o = 5$ mm $= 0.005$ m and frequency $f = 3.3$ Hz. With these values, the amplitude of the base excitation is $\ddot{u}_{go} = u_o(2\pi f)^2 = 2.15\ \text{m/s}^2 = 0.219$ g. Accordingly, the peak base excitation $\ddot{u}_{go} = 0.219$ g is smaller than $g\tan\alpha = 0.275$ g which is the minimum acceleration that is needed for uplifting of the rocking frame [trilith, see Eq. (23) and Makris and Vassiliou (2013)].

Given that $\ddot{u}_{go} = 0.219\ \text{g} < g\tan\alpha = 0.275$ g, the trilith tested by Peña et al. (2008) apparently did not experience a pure rocking motion as depicted schematically in Fig. 2; but rather it experienced an inferior vibration mode, due to possible minor anomalies of the contact surfaces. It is possible that these anomalies are responsible for the highly three-dimensional behavior of the rocking frame that was recorded by Peña et al. (2008). Given that such anomalies may be present in future implementations of the concept of the rocking frame, the potential sliding during impact can be prevented with the creation of a recess at the base of the columns and at the cap-beam.

Emerging Concept of Rocking Isolation for Bridges

The concept of allowing the piers of tall bridges to rock is not new. For instance, the beneficial effects that derive from uplifting and rocking have been implemented since the early 1970s in the South Rangitikei Bridge in New Zealand (Beck and Skinner 1974).

Nevertheless, despite the successful design of the South Rangitikei Bridge and the ample dynamic stability of the rocking frame as documented in Makris and Vassiliou (2013) and further confirmed in this work, most modern tall bridges (with tall slender piers) are protected from seismic action via base (shear) isolation of the deck, rather than from (the most natural) rocking isolation. Part of the motivation of this work is to show in the simplest possible way that in the event that a rocking system is selected, a heavy deck atop tall slender columns not only does not harm the stability of the columns, but enhances the stability of the entire system, as shown by Eq. (23).

Our work comes to support the emerging design concept (mainly advanced by the prefabricated bridge technology) of concentrating the inelastic deformations of bridge frame at the locations where the bridge piers meet the foundation and the deck (Mander and Cheng 1997; Sakai and Mahin 2004; Wacker et al. 2005; Cheng 2008; Cohagen et al. 2008). It shall, however, be stressed that in prefabricated bridge technologies, the bridge piers and the deck are not free-standing; therefore, the structural system is essentially a hybrid system in between the rocking frame examined in this work and a traditional ductile moment-resisting frame.

At present, the equivalent static lateral force procedure is deeply rooted in the design philosophy of the structural engineering community, which is primarily preoccupied with how to improve the ductility and performance of the seismic connections, while the ample dynamic rocking stability that derives from the beneficial coexistence of negative stiffness and gravity as described by Eq. (23) is ignored. At the same time, it should be recognized that during the last decade there have been several publications which have voiced the need to go beyond the elastic response spectrum and the associated equivalent static lateral force procedure (Makris and Konstantinidis 2003; Apostolou et al. 2007; Resemini et al. 2008; Acikgoz and DeJong 2012). The time is therefore ripe for the development of new, physically motivated response/design curves which are relevant (in a technically sound way) to the response/design of large, slender structures. Part of the motivation for this paper is to bring forward the ample seismic stability associated with the free rocking of large, slender structures and the corresponding rocking frame.

Conclusions

In this technical note, we investigated the dynamic response and stability analysis of the rocking frame that consists of two identical free-standing, rigid columns capped with a freely supported rigid beam. Following a direct formulation after taking force and moment equilibrium of the components of the rocking frame, the technical note confirms the remarkable result obtained in the past by the same investigators with a variational formulation (Makris and Vassiliou 2013), namely, that the dynamic response of the rocking frame is identical to the rocking response of a solitary free-standing column with the same slenderness as the columns of the frame, yet with larger size, which is a more stable configuration. Consequently, the presence of the freely supported cap-beam atop the columns renders the rocking frame a more stable structure, despite the rise of the center of gravity. Most importantly, the study shows, via the derivation of Eq. (23), that the heavier the freely supported cap beam is, the more stable the rocking frame, implying that top-heavy rocking frames are more stable that when they are top-light.

The ultimate goal of this technical note is to accept and establish the rocking frame and its inherent hinging mechanism not just as a limit-state mechanism, but as an operational state (seismic protection) mechanism for large slender structures, as was accepted more than 2.5 millennia ago by the builders of archaic and classical temples.

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