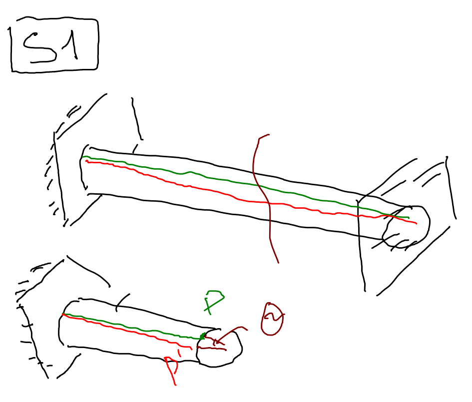
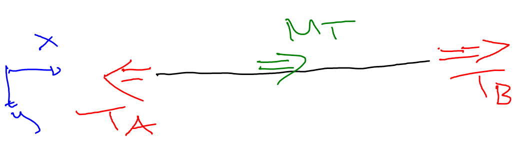
S1



$$\Theta(x_0) = \int_0^{x_0} \frac{T(x)}{G I_T(x)}\, dx$$

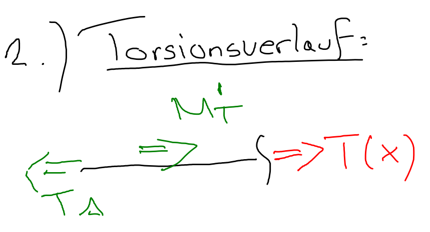
1.) Lagerkräften:



$$M_T = \int_0^{L_1+L_2} m_T \; x = m_T(L_1+L_2)$$

$$\Sigma M_x: \quad T_A = M_T + T_B$$
$$T_A = m_T(L_1+L_2) + T_B$$

2.) Torsionsverlauf:



$$M_T' = \int_0^x m_T\, dx = m_T x$$

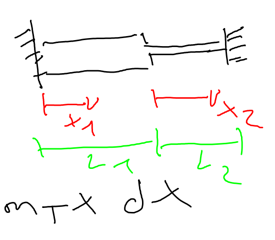
$$\Sigma M_x: \quad T(x) = T_A - M_T'$$
$$= T_A - m_T x$$
$$= m_T(L_1+L_2) + T_B - m_T x$$

3.) Stelle des Maximums:

$$\frac{d}{dx}\Theta(x) \overset{!}{=} 0 \Rightarrow \frac{d}{dx}\Theta(x) = \frac{d}{dx}\int_0^{x_0} \frac{T(x)}{G I_T(x)}\, dx = \frac{T(x)}{G I_T(x)} = 0$$

$$T(x) = 0 \Rightarrow T_A - m_T x = 0 \Rightarrow x = \frac{T_A}{m_T} = \frac{m_T(L_1+L_2)+T_B}{m_T}$$

4.) Verdrehungslinien $\Theta_1(x_1)$ & $\Theta_2(x_2)$:



$$\Theta_1(x_1) = \int_0^{x_1} \frac{T(x)}{G \cdot I_T(x)}\, dx = \frac{1}{G I_P}\int_0^{x_1} T(x)\, dx$$

$$= \frac{1}{G(\pi/2(r_1^4 - r_0^4))} \int_0^{x_1} m_T(L_1+L_2) + T_B - m_T x \; dx$$

$$= \frac{1}{G(\pi/2(r_1^4 - r_0^4))}\left([m_T(L_1+L_2) + T_B] x_1 - \frac{m_T x_1^2}{2} + C_1\right)$$

$$\Theta_2(x_2) = \frac{1}{G(\pi/2(r_2^4 - r_0^4))}\left([m_T(L_1+L_2) + T_B] x_2 - \frac{m_T x_2^2}{2} + C_2\right)$$

5.) RB:

→ Lagerbedingungen: $\Theta_1(x_1=0) = 0 \rightarrow C_1 = 0$

$\Theta_2(x_2=L_2) = 0 \rightarrow C_2 = \frac{m_T L_2^2}{2} - [m_T(L_1+L_2)+T_B] L_2$

→ Stetigkeitsbedingungen: $\Theta_1(x_1=L_1) = \Theta_2(x_2=0)$

$$\frac{1}{\cancel{G}(\pi/2(r_1^4 - r_0^4))}\left([m_T(L_1+L_2)+T_B]L_1 - \frac{m_T L_1^2}{2}\right) =$$

$$\frac{1}{\cancel{G}(\pi/2(r_2^4 - r_0^4))}\left(\frac{m_T L_2^2}{2} - [m_T(L_1+L_2)+T_B]L_2\right)$$

$$T_B = \ldots = -1519{,}91 \text{ kNmm}$$

6.) Maximum berechnen:

$$x = \frac{m_T(L_1+L_2)+T_B}{m_T} = 520 \text{ mm}$$

→ Ränder: $\Theta_1(x_1=0) = \Theta_2(x_2=L_2) = 0$

→ Unstetigkeit: $x_1 = L_1$

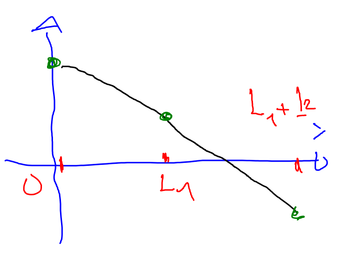
→ Stellen einsetzen: $\Theta_1(x_1=L_1) = 0{,}008426$ rad

$\Theta_2(x_2 = 520 - L_1) = \boxed{0{,}012675 \text{ rad}} = \Theta_{max}$

7.) τ_{max} bestimmen:

$$\tau_{max} = \frac{T_{max}}{W_T} = \frac{T_{max} \cdot 16 \cdot D}{\pi(D^4 - d^4)} = \frac{T_{max} \cdot \cancel{2^4} \cdot 2 \cdot R}{\pi(\cancel{2^4} R^4 - \cancel{2^4} r^4)} = \frac{2R \cdot T_{max}}{\pi(R^4 - r^4)}$$

$T(x) = m_T(L_1+L_2) + T_B - m_T x$ → lineare Fkt.



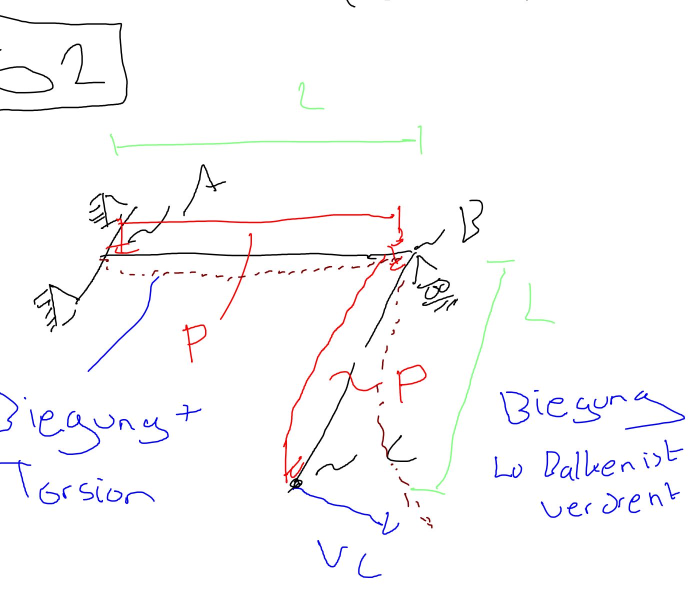
→ 1. Abschnitt: → $x_1 = 0$ → $T_{max} = m_T(L_1+L_2) + T_B$

$$\tau_{max} = \frac{2 r_1 [m_T(L_1+L_2)+T_B]}{\pi(r_1^4 - r_0^4)} = 94{,}7 \text{ MPa}$$

→ 2. Abschnitt: → $x_1 = L_1+L_2$ → $T_{max} = m_T(\cancel{L_1+L_2}) + T_B - m_T(\cancel{L_1+L_2})$

$$\tau_{max} = \frac{2 r_2 T_B}{\pi(r_2^4 - r_0^4)} = \boxed{-146.8 \text{ MPa}}$$

S2



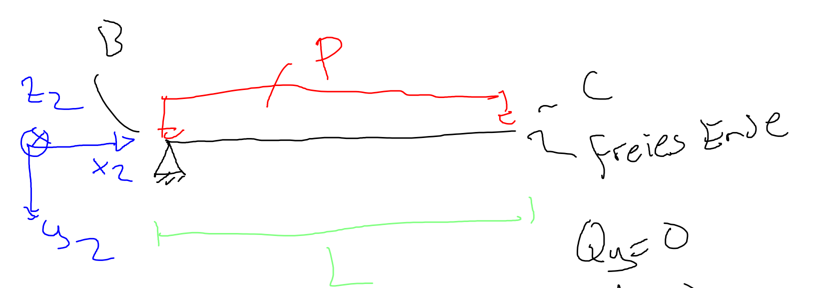
• Annahmen:

→ Biegung in A-B bewirkt keine Verschiebung in C

→ Schub wird vernachlässigt

→ Ziel: Biegelinie in B-C berechnen mit RB aus Torsion in A-B

1.) Biegelinie B-C:



→ Diff'beziehungen:

$$Q_y = -\int q_y\, dx = -px + C_1$$

$$Q_y(x_2 = L) = -pL + C_1 \overset{!}{=} 0 \rightarrow C_1 = pL$$

$$M_z = -\int Q_y\, dx = -\int -px + pL\, dx$$

$$= +\frac{px^2}{2} - pLx + C_2$$

$$M_z(x_2 = L) = \frac{pL^2}{2} - pL^2 + C_2 \overset{!}{=} 0$$
$$\Rightarrow C_2 = \frac{pL^2}{2}$$

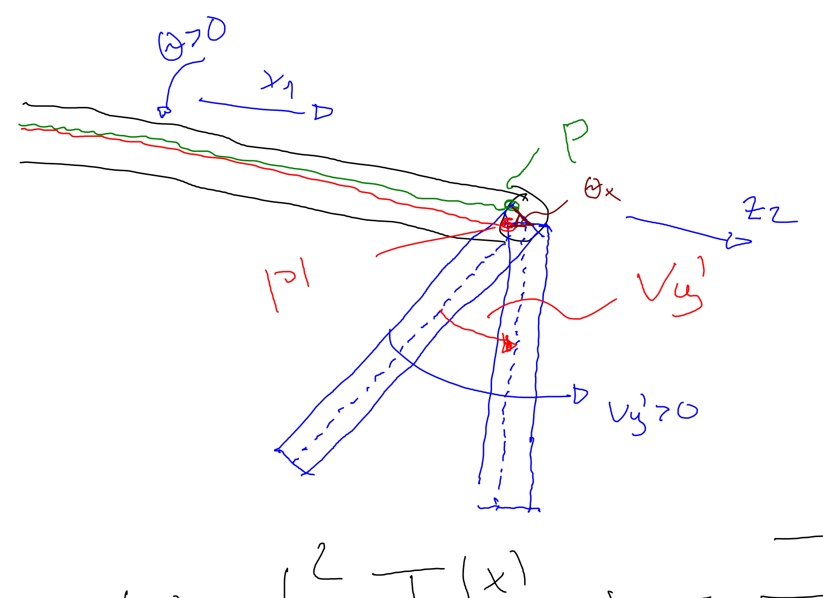
$$v_y(x_2) = \frac{1}{EI_z}\iint M_z(x)\, dx = \frac{1}{EI_z}\iint \frac{px^2}{2} - pLx + \frac{pL^2}{2}\, dx$$

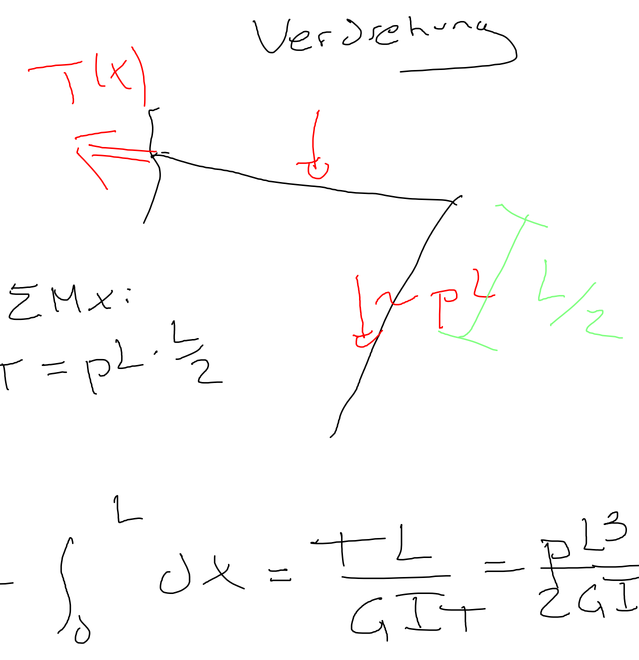
$$= \frac{1}{EI_z}\int \frac{px^3}{6} - \frac{pLx^2}{2} + \frac{pL^2 x}{2} + C_3\, dx$$

$$= \frac{1}{EI_z}\left(\frac{px^4}{24} - \frac{pLx^3}{6} + \frac{pL^2x^2}{4} + C_3 x + C_4\right)$$

RB: $v_y(x_2 = 0) \overset{!}{=} 0 \rightarrow C_4 = 0$

$v_y'(x_2 = 0) = \Theta_x(x_1 = L)$ — Neigung der Mittellinie = Verdrehung





$\Sigma M_x:$
$T = pL \cdot \frac{L}{2}$

$$\Theta_x(L) = \int_0^L \frac{T(x)}{G I_T(x)}\, dx = \frac{T}{G I_T}\int_0^L dx = \frac{TL}{G I_T} = \frac{pL^3}{2 G I_T}$$

$$\frac{C_3}{EI_z} = \frac{pL^3}{2 G I_T} \rightarrow C_3 = \frac{E I_z p L^3}{2 G I_T}$$

v_C berechnen:

$$v_y(x_2 = L) = \frac{1}{EI_z}\left[\frac{pL^4}{24} - \frac{pL^4}{6} + \frac{pL^4}{4} + \frac{E I_z p L^4}{2 G I_T}\right]$$

$$= \frac{1}{EI_z}\left[pL^4\left(\frac{1}{8} + \frac{E I_z}{2 G I_T}\right)\right]$$

$$I_z = \frac{\pi}{4}\left((R+t)^4 - R^4\right) \overset{t \ll R}{=} \pi R^3 t$$

(Serie 9: S1)

$$v_y(x_2 = L) = v_C = \frac{pL^4}{2 E I_z}\left[\frac{1}{4} + \frac{E I_z}{G I_T}\right]$$

Design II: $I_T = I_p = 2 \times I_z = 2\pi R^3 t$

Design I: $I_T = 1/3\, 2\pi R t^3$

$$v_C^{II} = \frac{pL^4}{4 E \pi R^3 t}\left[\frac{E}{G} + 1/2\right]$$

$$v_C^{I} = \frac{pL^4}{4 E \pi R^3 t}\left[\frac{3 E R^2}{G t^2} + 1/2\right]$$

$$\Delta v_C = |v_C^I - v_C^{II}| = \left|\frac{pL^4}{4 \cancel{E} \pi R^3 t}\left[\frac{3 \cancel{E} R^2}{G t^2} + \cancel{1/2} - \frac{\cancel{E}}{G} - \cancel{1/2}\right)\right|$$

$$= \left|\frac{pL^4}{4 G \pi R^3 t}\left[\frac{3R^2}{t^2} - 1\right]\right|$$