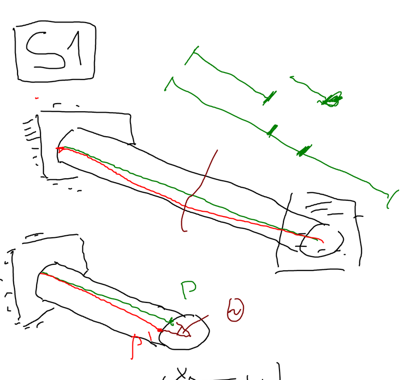
S1



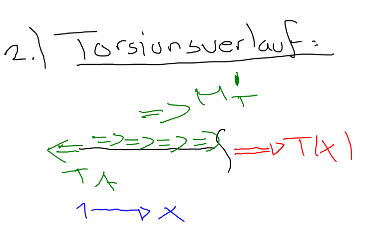
$$\Theta(x_0) = \int_0^{x_0} \frac{T(x)}{G \cdot I_T(x)}\,dx$$

1.) Lagerkräfte:

$$M_T = \int_0^{L_1+L_2} m_T\,dx = m_T(L_1+L_2)$$

$$\Sigma M_x: \quad T_A = M_T + T_B$$
$$T_A = m_T(L_1+L_2) + T_B$$

2.) Torsionsverlauf:



$$M_T' = \int_0^x m_T\,dx = m_T x$$

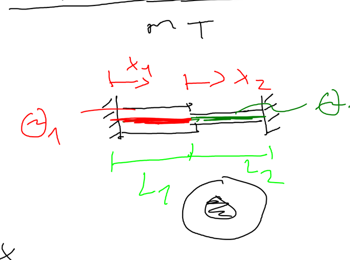
$$\Sigma M_x: \quad T(x) = T_A - M_T'$$
$$= T_A - m_T x$$
$$= m_T(L_1+L_2) + T_B - m_T x$$

3.) Stelle des Maximum:

$$\frac{d}{dx}\Theta(x) \overset{!}{=} 0 \implies \frac{d}{dx}\Theta(x) = \frac{d}{dx}\int_0^{x_0} \frac{T(x)}{G I_T(x)}\,dx = \frac{T(x)}{G I_T(x)} = 0$$

$$\implies T(x) = 0 \implies T_A - m_T x = 0 \implies x = \frac{T_A}{m_T} = \frac{m_T(L_1+L_2) + T_B}{m_T}$$

4.) Verdrehungslinie $\Theta_1(x)$ & $\Theta_2(x)$ bestimmen:



$$\Theta_1(x_1) = \int_0^{x_1} \frac{T(x)}{G \cdot I_T(x)}\,dx = \frac{1}{G I_p}\int_0^{x_1} T(x)\,dx$$

$$= \frac{1}{G\left(\frac{\pi}{2}(r_1^4 - r_0^4)\right)} \int_0^{x_1} m_T(L_1+L_2) + T_B - m_T x\,dx$$

$$= \frac{1}{G\left(\pi/2\,(r_1^4 - r_0^4)\right)} \left(\left[m_T(L_1+L_2) + T_B\right]x_1 - \frac{m_T x_1^2}{2} + C_1\right)$$

$$\Theta_2(x_2) = \frac{1}{G\left(\pi/2\,(r_2^4 - r_0^4)\right)} \left(\left[m_T(L_1+L_2) + T_B\right]x_2 - \frac{m_T x_2^2}{2} + C_2\right)$$

5.) RB:

→ Lagerbedingungen:

$$\Theta_1(x_1=0) \overset{!}{=} 0 \implies C_1 = 0$$
$$\Theta_2(x_2=L_2) \overset{!}{=} 0 \implies C_2 = \frac{m_T L_2^2}{2} - \left[m_T(L_1+L_2) + T_B\right]L_2$$

→ Stetigkeitsbedingung:

$$\Theta_1(x_1=L_1) = \Theta_2(x_2=0)$$

$$\frac{1}{\cancel{G}\left(\cancel{\pi/2}\,(r_1^4 - r_0^4)\right)}\left(\left[m_T(L_1+L_2) + T_B\right]L_1 - \frac{m_T L_1^2}{2}\right)$$

$$= \frac{1}{\cancel{G}\left(\cancel{\pi/2}\,(r_2^4 - r_0^4)\right)}\left(\frac{m_T L_2^2}{2} - \left[m_T(L_1+L_2) + T_B\right]L_2\right)$$

$$T_B = \ldots = -1519{,}91\ \text{kNmm}$$

6.) Maximum berechnen:

$$x = \frac{m_T(L_1+L_2) + T_B}{m_T} = 520\,\text{mm}$$

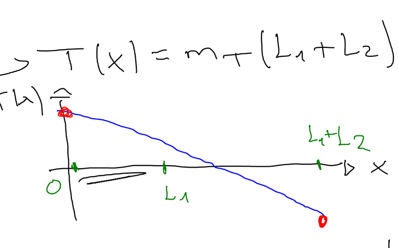
→ andere Stellen
→ Ränder: $\Theta_1(x_1=0) = \Theta_2(x_2=L_2) = 0$
→ Unstetigkeit: $x_1 = L_1$

→ Stellen einsetzen: $\Theta_1(x_1=L_1) = 0{,}008426$ rad
$\Theta_2(x_2=520-L_1) = \boxed{0.012675 \text{ rad}}$
$= \Theta_{max}$

7.) τ_{max} bestimmen:

$$|\tau_{max}| = \frac{|T_{max}|}{W_T} = \frac{T_{max}\cdot 16 \cdot D}{\pi(D^4 - d^4)} = \frac{T_{max}\cdot 2^4 \cdot 2R}{\pi\,(2^4R^4 - 2^4 r^4)} = \frac{2R \cdot T_{max}}{\pi(R^4 - r^4)}$$

$T_{max} \implies T(x) = m_T(L_1+L_2) + T_B - m_T x$ ~ lineare Fkt.



→ 1. Abschnitt: $T_{max} = m_T(L_1+L_2) + T_B$

$$\tau_{max} = \frac{2r_1\left[m_T(L_1+L_2) + T_B\right]}{\pi(r_1^4 - r_0^4)} = 94{,}7\ \text{MPa}$$

→ 2. Abschnitt $T_{max} = m_T(\cancel{L_1}+L_2) + T_B - m_T(\cancel{L_1}+L_2)$

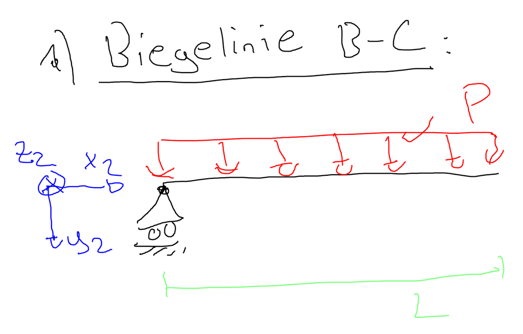
$$\tau_{max} = \frac{2r_2 T_B}{\pi(r_2^4 - r_0^4)} = \boxed{-146.8\ \text{MPa}}$$

S2

Annahmen:

→ Biegung in A-B bewirkt keine Verschiebung in C
→ Schub wird vernachlässigt
→ Ziel: Biegelinie in B-C berechnen
→ RB: mit Torsion in A-B

1) Biegelinie B-C:



→ Differentialbeziehungen

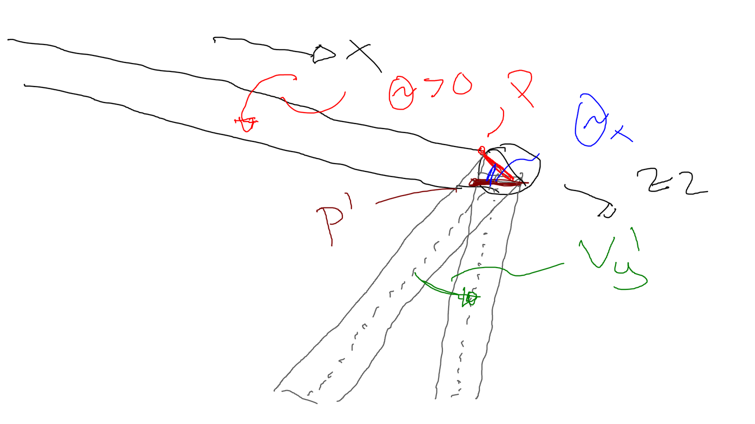
$$Q_y = -\int q_y\,dx = -px + C_1$$
$$Q_y(x_2=L) = -pL + C_1 = 0 \rightarrow C_1 = pL$$

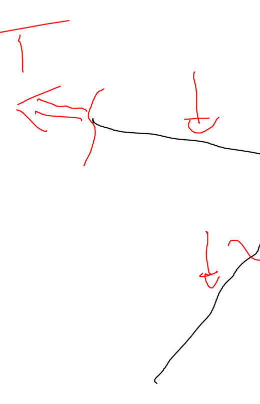
$$M_z = -\int Q_y\,dx = -\int -px + pL\,dx$$
$$= +\frac{px^2}{2} - pLx + C_2$$
$$M_z(x_2=L) = +\frac{pL^2}{2} - pL^2 + C_2 = 0$$
$$\rightarrow C_2 = +\frac{pL^2}{2}$$

$$v_y(x_2) = \frac{1}{EI_z}\iint M_z(x)\,dx = \frac{1}{EI_z}\iint + \frac{px^2}{2} - pLx + \frac{pL^2}{2}\,dx$$
$$= \frac{1}{EI_z}\int \frac{px^3}{6} - \frac{pLx^2}{2} + \frac{pLx}{2} + C_3\,dx$$
$$= \frac{1}{EI_z}\left(+\frac{px^4}{24} - \frac{pLx^3}{6} + \frac{pL^2x^2}{4} + C_3 x + C_4\right)$$

RB:

$v_y(x_2=0) \overset{!}{=} 0 \rightarrow C_4 = 0$
$|v_y'(x_2=0)| = |\Theta_x(x_1=L)|$ → Neigung der Mittellinie $\hat{=}$ Verdrehung





ΣM_x:
$$T = pL\frac{L}{2} = \frac{pL^2}{2}$$

$$\Theta_x(L) = \int_0^L \frac{T(x)}{G I_T(x)}\,dx = \frac{T}{G I_p}\int_0^L dx = \frac{TL}{G I_p} = \frac{pL^3}{2 G I_p}$$

$$|v_y'(x_2=0)| = |\Theta_x(x_1=L)|$$

$$\frac{C_3}{EI_z} = \frac{pL^3}{2GI_p} \rightarrow C_3 = \frac{EI_z\,pL^3}{2GI_p}$$

v_c berechnen:

$$v_y(x_2=L) = \frac{1}{EI_z}\left[+\frac{pL^4}{24} - \frac{pL^4}{6} - \frac{pL^4}{4} + \frac{EI_z\,pL^4}{2GI_p}\right]$$
$$= \frac{1}{EI_z}\left[pL^4\left(+\frac{1}{8} - \frac{EI_z}{2GI_p}\right)\right]$$

$$I_z = \frac{\pi}{4}\left((R+t)^4 - R^4\right) \overset{t \ll R}{=} \pi R^3 t$$ → gleich Design I & II

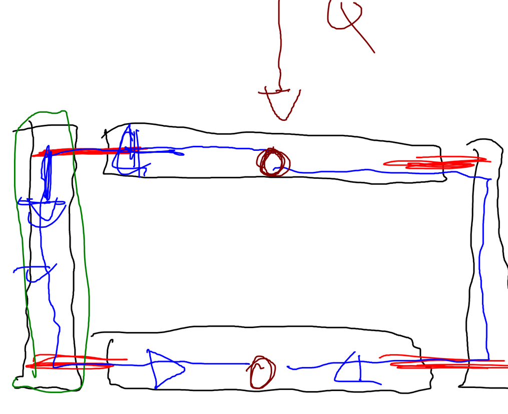
$$v_y(x_2=L) = v_c = \frac{pL^4}{2EI_z}\left[\frac{EI_z}{GI_p} - \frac{1}{4}\right]$$

Design II: $I_T = I_p = 2\pi R^3 t$
Design I: $I_T = 1/3\, s_i e_i^3 = 1/3\, 2\pi R t^3$

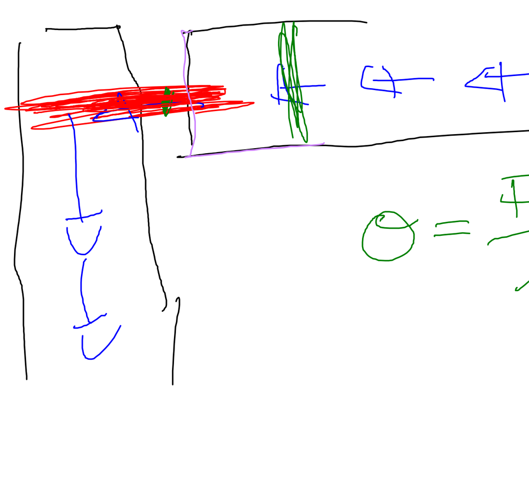
$v_c^{II} = \ldots$
$v_c^{I} = \ldots$

$$\Delta = |v_c^I - v_c^{II}| = \frac{pL^4}{4G\pi R^3 t}\left[\frac{3R^2}{t^2} - 1\right]$$

W9



→ Normalspannung
→ Schubspannung



$$\sigma = \frac{F}{A}$$