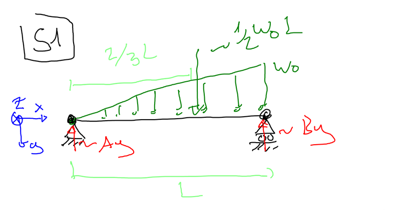
S1



1.) Beanspruchung: Differentialbeziehungen

ΣF_y: $A_y + B_y = \frac{1}{2} w_0 L$

$\Sigma M_z^{in A}$: $B_y \cancel{L} = \frac{1}{\cancel{2}} w_0 L \frac{\cancel{2}}{3} \cancel{L}$

$\hookrightarrow B_y = \frac{w_0 L}{3}$

$\hookrightarrow A_y = \frac{w_0 L}{6}$

$Q_y = -\int q_y \, dx = -\int \frac{w_0}{L} x \, dx$

$= -\frac{w_0 x^2}{2L} + C_1$

$M_z = -\int Q_y \, dx = -\int -\frac{w_0 x^2}{2L} + C_1 \, dx$

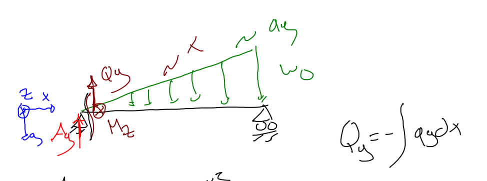
$= \frac{w_0 x^3}{6L} - C_1 x + C_2$

RB: $M_z(x=0) = 0 \to C_2 = 0$

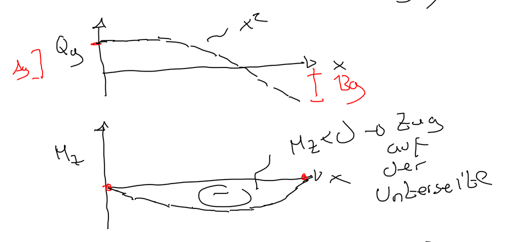
$M_z(x=L) = 0 \to \frac{w_0 L^{\cancel{3}}}{6 \cancel{L}} - C_1 \cancel{L} = 0$

$\hookrightarrow C_1 = \frac{w_0 L}{6}$

$\to M_z = \frac{w_0 x^3}{6L} - \frac{w_0 L}{6} x$



$Q_y = -\int q_y dx$



$U = \frac{1}{2}\left[\int_\ell \frac{Q_y^2}{GA_s} dx + \int_\ell \frac{M_z^2}{EI_z} dx\right]$

$= \frac{1}{2}\left[\underbrace{\int_\ell \left(-\frac{w_0 x^2}{2L} + \frac{w_0 L}{6}\right)^2 dx}_{a} + \frac{1}{EI_z} \underbrace{\int_\ell \left(\frac{w_0 x^3}{6L} - \frac{w_0 L}{6} x\right)^2 dx}_{b}\right]$

(a) $\int_0^L \frac{w_0^2 x^4}{4L^2} + \frac{w_0^2 L^2}{36} - \frac{\cancel{2} w_0^2 x^2 \cancel{L}}{\cancel{2} \cdot 6 \cancel{L}} \, dx = \frac{w_0^2 L^5}{20 L^2} + \frac{w_0^2 L^3}{36} - \frac{w_0^2 L^3}{18}$

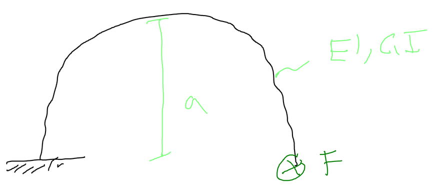
$= \frac{w_0^2 L^3}{\cancel{2}} \underbrace{\left[\frac{1}{10} + \frac{1}{18} - \frac{1}{9}\right]}_{\cancel{2}/45} = \frac{w_0^2 L^3}{45}$

(b) $\int_0^L \frac{w_0^2 x^6}{36 L^2} + \frac{w_0^2 L^2}{36} x^2 - \frac{2 w_0^2 x^4 L}{36 L} \, dx = \frac{w_0^2 L^7}{7 \cdot 36 L^2} + \frac{w_0^2 L^5}{3 \cdot 36} - \frac{2 w_0^2 L^{\cancel{6}\,5}}{5 \cdot 36 \cancel{L}}$

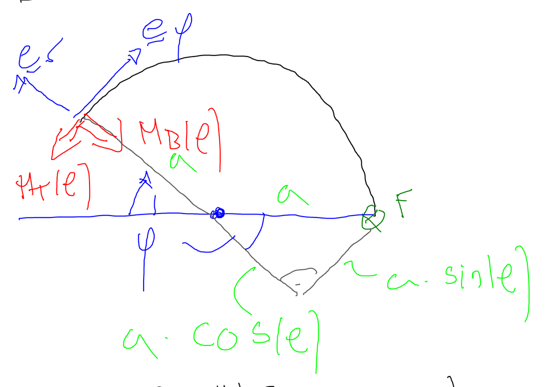
$= \frac{w_0^2 L^5}{36} \underbrace{\left(\frac{1}{7} + \frac{1}{3} - \frac{2}{5}\right)}_{8/105} = \frac{2 w_0^2 L^5}{945}$

$$\boxed{U = \frac{w_0^2 L^3}{45}\left[\frac{1}{2GA_s} + \frac{L^2}{EI_z \cdot 21}\right]}$$

S2



1.) Beanspruchung: $M_B(\varphi)$, $M_T(\varphi)$



$\Sigma M_r^{\text{im Schnittufer}}$: $M_B(\varphi) = F a \sin(\varphi)$

$\Sigma M_\varphi^{\text{im Schnittufer}}$: $M_T(\varphi) = F(a + a\cos(\varphi))$

$= Fa(\cos(\varphi) + 1)$

2.) U berechnen:

$U = \frac{1}{2}\left[\underbrace{\frac{1}{EI}\int_\ell M_B(\varphi)^2 a \, d\varphi}_{a} + \underbrace{\frac{1}{GI_T}\int_\ell M_T(\varphi)^2 a \, d\varphi}_{b}\right]$

Integraltabelle

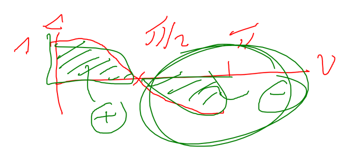
(a) $\int_0^\pi F^2 a^2 \sin^2(\varphi) a \, d\varphi = F^2 a^3 \int_0^\pi \underbrace{\sin^2(\varphi) \, d\varphi}_{\pi/2} = F^2 a^3 \pi/2$

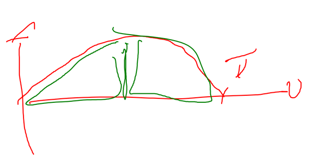
(b) $\int_0^\pi F^2 a^2 (\cos(\varphi) + 1)^2 a \, d\varphi = F^2 a^3 \int_0^\pi \cos^2(\varphi) + 2\cos(\varphi) + 1 \, d\varphi$

$= F^2 a^3 \left[\underbrace{\int_0^\pi \cos^2(\varphi) \, d\varphi}_{\pi/2} + 2 \underbrace{\int_0^\pi \cos(\varphi) \, d\varphi}_{0} + \underbrace{\int_0^\pi 1 \, d\varphi}_{\pi}\right] = F^2 a^3 \left(\frac{\pi}{2} + 0 + \pi\right)$

$= \int_0^\pi \sin^2(\varphi) \, d\varphi$

$= F^2 a^3 \frac{3\pi}{2}$



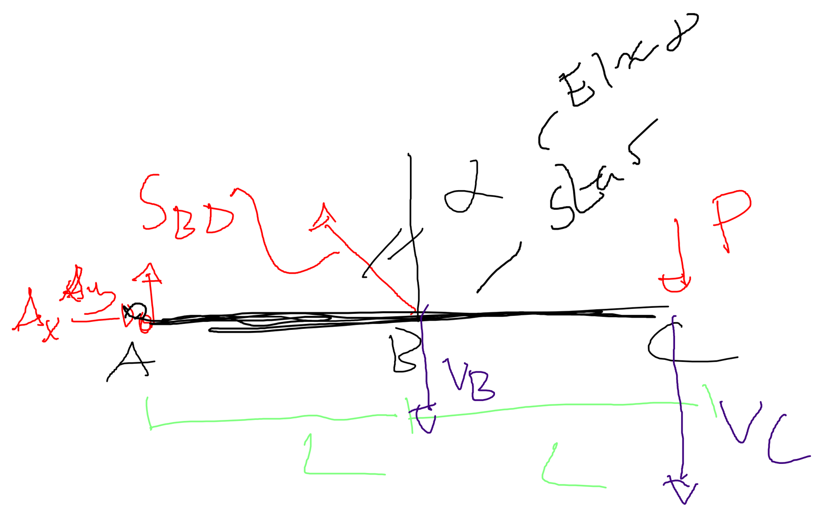


3.) Definition:

$v_F = \frac{2U}{F} = \frac{\cancel{2}}{\cancel{F}} \frac{1}{\cancel{2}}\left[\frac{1}{EI} F^{\cancel{2}} a^3 \pi/2 + \frac{1}{GI_T} F^{\cancel{2}} a^3 \frac{3\pi}{2}\right]$

$$\boxed{= \frac{\pi F a^3}{2}\left[\frac{1}{EI} + \frac{3}{GI_T}\right]}$$

Serie 6: HM ~ mit Energiemethode



1.) Beanspruchung

$\Sigma M_z^{in A}$: $\cancel{L}\cos(\alpha) S_{BD} = 2\cancel{L} P$

$\hookrightarrow S_{BD} = \frac{2P}{\cos(\alpha)} = 2.5P$

2.) Berechnung von U:

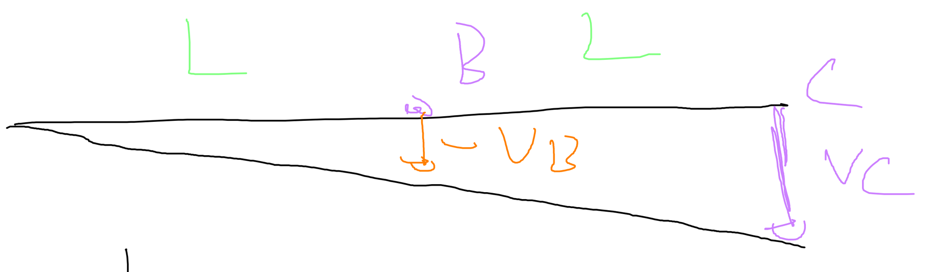
$\sqrt{0{,}3^2 + 1{,}2^2}$

$U = \frac{1}{2}\int_\ell \frac{N^2}{EA} dx = \frac{1}{2EA}\int_{BD} (2.5P)^2 dx = \frac{1}{\cancel{2}EA} 6.25 P^{\cancel{2}} L_{BD}$

3. Definition:

$v_F = \frac{\cancel{2}U}{\cancel{P}} = \frac{1}{EA} 6.25 P \cdot L_{BD} = 5.98\,mm$

$$\boxed{v_B = \frac{v_C}{2} = 2.98\,mm}$$



$\frac{2L}{v_C} = \frac{L}{v_B} \to \frac{v_C}{2L} = \frac{v_B}{L}$

$\to v_B = \frac{v_C}{2}$

$\to EA, EI, GI_T \to \infty$

$\hookrightarrow$ Balken ist starr