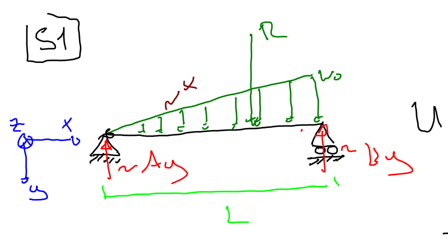
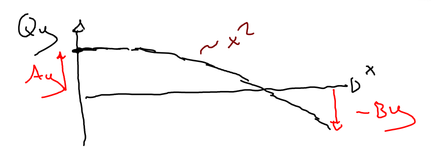
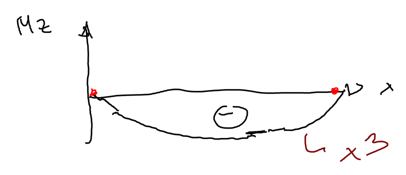
S1



$$U = \frac{1}{2}\left[\int_\ell \frac{Q_y^2}{G A_s}\,dx + \int_\ell \frac{M_z^2}{E I_z}\,dx\right]$$

→ Beanspruchung → Differentialbeziehungen





$\sum F_y$: $A_y + B_y = 1/2\, w_0 L$

$\sum M_z^{in A}$: $B_y \cancel{L} = \cancel{1/2}\, w_0 L\, \cancel{2}/3\, \cancel{L}$

$\hookrightarrow B_y = \frac{w_0 L}{3}$

$\hookrightarrow A_y = \frac{w_0 L}{6}$

$$Q_y = -\int q_y\,dx = -\int w(x)\,dx = -\int \frac{w_0}{L} x\,dx$$
$$= -\frac{w_0 x^2}{2L} + C_1$$
$$M_z = -\int Q_y\,dx = -\int -\frac{w_0 x^2}{2L} + C_1\,dx$$
$$= \frac{w_0 x^3}{6L} - C_1 x + C_2$$

RB: $M_z(x=0) = 0 \to C_2 = 0$

$M_z(x=L) = 0 \to \frac{w_0 L^{\cancel{3}}}{6\cancel{L}} - C_1 \cancel{L} = 0 \to C_1 = \frac{w_0 L}{6}$

$$M_z = \frac{w_0 x^3}{6L} - \frac{w_0 L}{6} x$$

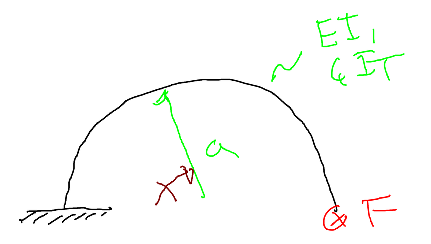
$$U = 1/2\left[\frac{1}{G A_s^y}\int_\ell \underbrace{\left(-\frac{w_0 x^2}{2L} + \frac{w_0 L}{6}\right)^2 dx}_{a} + \frac{1}{E I_z}\int_\ell \underbrace{\left(\frac{w_0 x^3}{6L} - \frac{w_0 L}{6} x\right)^2 dx}_{b}\right]$$

a: $$\int_0^L \frac{w_0^2 x^4}{4L^2} + \frac{w_0^2 L^2}{36} - \frac{\cancel{2} w_0^2 x^2 L}{\cancel{2}\cdot 6L}\,dx = \frac{w_0^2 L^{\cancel{5}\,3}}{20\cancel{L^2}} + \frac{w_0^2 L^3}{36} - \frac{w_0^2 L^3}{18}$$
$$= \frac{w_0^2 L^3}{\cancel{2}}\underbrace{\left[\frac{1}{10} + \frac{1}{18} - \frac{1}{9}\right]}_{\cancel{2}/45} = \frac{w_0^2 L^3}{45}$$

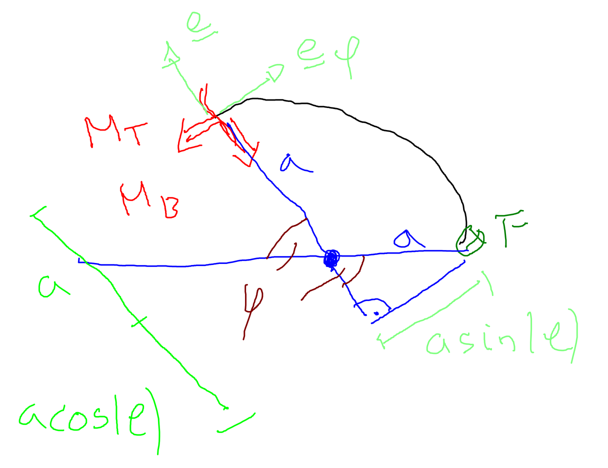
b: $$\int_0^L \frac{w_0^2 x^6}{36L^2} + \frac{w_0^2 L^2}{36} x^2 - \frac{2 w_0^2 x^4 L}{36 L}\,dx$$
$$= \frac{w_0^2 L^{\cancel{7}\,5}}{7\cdot 36\cdot \cancel{L^2}} + \frac{w_0^2 L^5}{3\cdot 36} - \frac{2 w_0^2 L^{\cancel{6}\,5}}{5\cdot 36 \cancel{L}} = \frac{w_0^2 L^5}{36}\underbrace{\left(\frac{1}{7} + \frac{1}{3} - \frac{2}{5}\right)}_{8/105}$$
$$= \frac{2 \cdot w_0^2 L^5}{945}$$

$$\boxed{U = \frac{w_0^2 L^3}{45}\left[\frac{1}{2 G A_s} + \frac{1}{21 E I_z}\right]}$$

S2



1.) Beanspruchung:



$\sum M_r$ im Schnittufer: $M_B(\varphi) = F a \sin(\varphi)$

$\sum M_\varphi$ im Schnittufer: $M_T(\varphi) = F(a\cos(\varphi) + a)$
$= Fa(\cos(\varphi) + 1)$

2.) Berechnung von U:

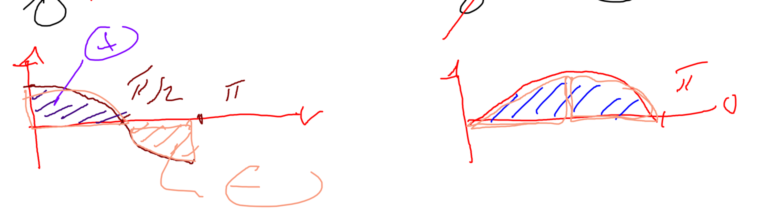
$$U = 1/2\left[\frac{1}{EI}\int_\ell \underbrace{M_B(\varphi)^2 a\,d\varphi}_{a} + \frac{1}{G I_T}\int_\ell \underbrace{M_T(\varphi)^2 a\,d\varphi}_{b}\right]$$

a: $$\int_0^\pi F^2 a^2 \sin^2(\varphi)\, a\,d\varphi = F^2 a^3 \int_0^\pi \sin^2(\varphi)\,d\varphi = F^2 a^3\, \pi/2$$

Integraltabelle: $\pi/2$

b: $$\int_0^\pi F^2 a^2 (\cos(\varphi) + 1)^2 a\,d\varphi = F^2 a^3 \int_0^\pi \cos^2(\varphi) + 2\cos(\varphi) + 1\,d\varphi$$
$$= F^2 a^3\left[\underbrace{\int_0^\pi \cos^2(\varphi)\,d\varphi}_{\pi/2} + 2\underbrace{\int_0^\pi \cos(\varphi)\,d\varphi}_{0} + \underbrace{\int_0^\pi 1\,d\varphi}_{\pi}\right] = F^2 a^3\left(\frac{\pi}{2} + \pi\right)$$
$$= F^2 a^3 \frac{3\pi}{2}$$

$$= \int_0^\pi \sin^2(\varphi)\,d\varphi$$



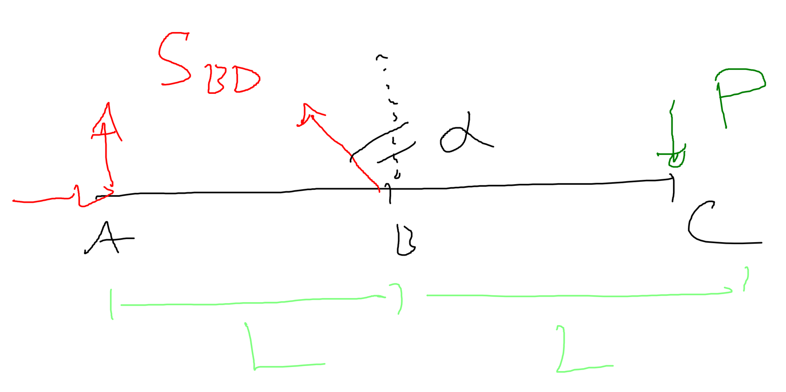
3.) Definition:

$$v_F = \frac{2U}{F} = \frac{\cancel{2}}{\cancel{F}}\frac{1}{\cancel{2}}\left[\frac{1}{E I_z} F^{\cancel{2}} a^3 \pi/2 + \frac{1}{G I_T} F^{\cancel{2}} a^3\, 3\pi/2\right]$$

$$\boxed{= \frac{\pi F a^3}{2}\left[\frac{1}{E I_z} + \frac{3}{G I_T}\right]}$$

Serie 6 H1

1.) Beanspruchung:



$\sum M_z^{in A}$: $L\cos(\alpha)\, S_{BD} = 2LP$

$\hookrightarrow S_{BD} = \frac{2P}{\cos(\alpha)} = 2.5P$

2.) U berechnen:

$$U = 1/2\left[\int_\ell \frac{N^2}{EA}\,dx\right] = \frac{1}{2EA}\int_{BD} (2.5P)^2\,dx = \frac{1}{2EA}\, 6.25 P^2 \cdot L_{BD}$$

$$L_{BD} = \sqrt{0.3^2 + 1.2^2}$$

3.) Arbeitssatz:

$$v_C = v_F = \frac{2U}{P} = \frac{1}{EA}\, 6.25 P^{\cancel{2}} \cdot L_{BD} = 5.98\ mm$$

$$\boxed{v_D = \frac{v_C}{2} = 2.98\ mm}$$