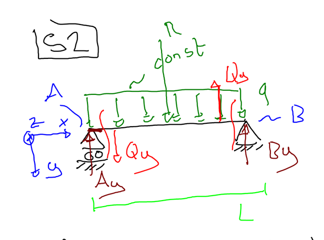
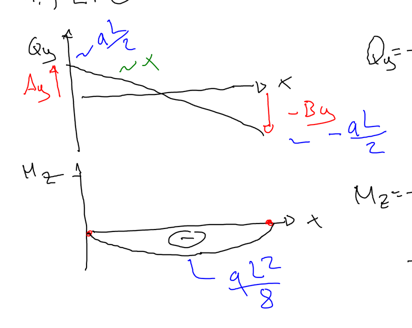
S2



$R = qL$

$\rightarrow A_y = B_y = \frac{qL}{2}$ wegen Symmetrie

1.) LF0 – reale System $\rightarrow Q_0(x), M_0(x)$



$Q_y = -\int q_y dx = -qx + C_1$

$Q(x=0) = C_1 = A_y = \frac{qL}{2}$

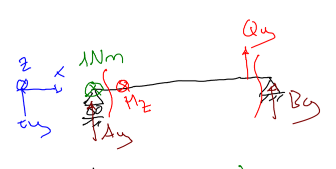
$M_z = -\int Q_y dx = -\int -qx + \frac{qL}{2} dx$

$= \frac{qx^2}{2} - \frac{qL}{2}x + C_2$

$M_z(x=0) = C_2 = 0$

$\hookrightarrow M_z(x = L/2) = \frac{qL^2}{8} - \frac{qL^2}{4} = -\frac{qL^2}{8}$

2.) LF1 – virtuelle System $\rightarrow Q_1(x), M_1(x)$

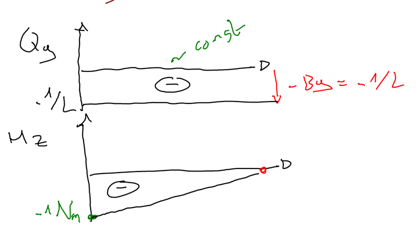


$\sum M_z^{in A}: \; 1 = B_y L \rightarrow B_y = 1/L$

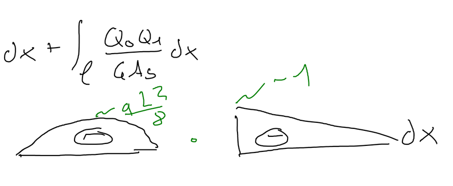
$\rightarrow Q_y = -\int q_y dx = 0 + C_1 = -1/L$

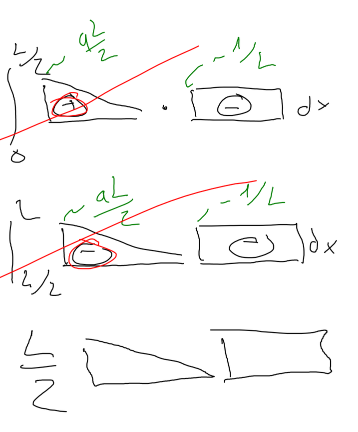
$\rightarrow M_z = -\int Q_y dx = -(-1/L)x + C_2$

$M_z(x=0) = C_2 = -1\,Nm$



3.) $\theta = \int_\ell \frac{M_0 M_1}{EI} dx + \int_\ell \frac{Q_0 Q_1}{GA_s} dx$

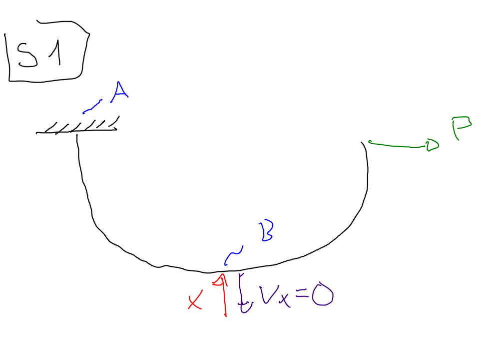
$\theta = \frac{1}{EI}\int_0^L$  $dx + \cancel{\frac{1}{GA_s}\int_0^{L/2} \ldots dx + \frac{1}{GA_s}\int_{L/2}^{L} \ldots dx}$



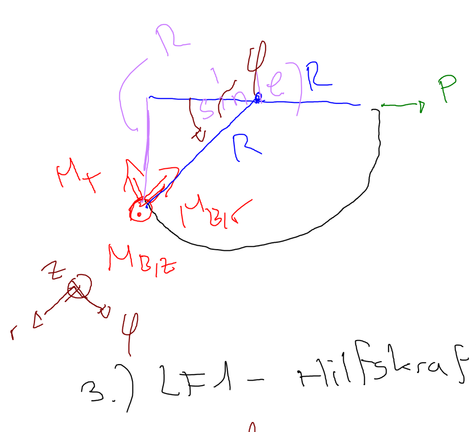
$= \frac{1}{EI}\frac{1}{3}\frac{qL^2}{8}(-1)L$

$\boxed{= \frac{1}{EI_z}\frac{1}{24}qL^3}$

S1



2.) LF0 – realen Lasten $\rightarrow M_0, T_0$

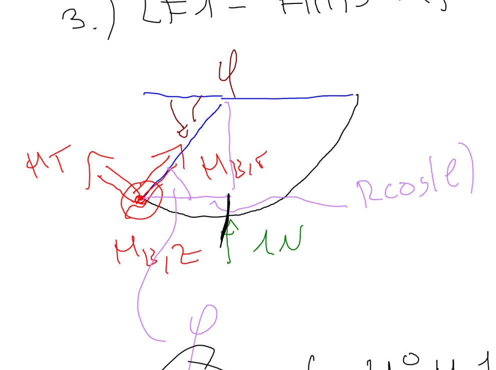


$\sum M_r^{\text{im Schnittufer}}: \; M_{B,r}(\varphi) = 0$

$\sum M_\varphi^{\text{im Schnittufer}}: \; M_T(\varphi) = 0$

$\sum M_z^{\text{im Schnittufer}}: \; M^0_{B,z}(\varphi) = R\sin(\varphi)\cdot P$

3.) LF1 – Hilfskraft am ersetzten Lager mit 1N $\rightarrow M_1, T_1$



$M_{B,r} = M_T = 0$

$\sum M_z^{\text{im Schnittufer}}: \; M^1_{B,z}(\varphi) = -R\cos(\varphi)$

4.) $\delta_{10} = \int_\ell \frac{M_z^0 M_z^1}{EI_z} dx = \frac{1}{EI_z}\int_0^{\pi} M^0_{B,z}(\varphi)\cdot M^1_{B,z}(\varphi)\, s\, d\varphi$

$= -\frac{1}{EI_z}\int_0^{\pi/2} R\sin(\varphi) P R\cos(\varphi) R\, d\varphi$

$= -\frac{PR^3}{EI_z}\int_0^{\pi/2} \cancel{\sin(\varphi)\cos(\varphi)}^{1/2} d\varphi = -\frac{PR^3}{2EI_z}$

$\delta_{11} = \int_\ell \frac{M_z^1 M_z^1}{EI_z} dx = \frac{1}{EI_z}\int_0^{\pi/2} [M^1_{B,z}(\varphi)]^2 s\, d\varphi$

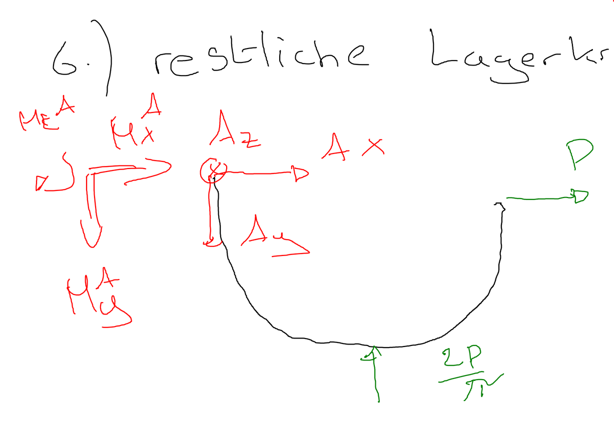
$= \frac{1}{EI_z}\int_0^{\pi/2} R^2\cos^2(\varphi) R\, d\varphi = \frac{R^3}{EI_z}\int_0^{\pi/2} \cancel{\cos^2(\varphi)}^{\pi/4} d\varphi = \frac{\pi}{4}\frac{R^3}{EI_z}$

5.) Lagerbedingung

$X = -\frac{\delta_{10}}{\delta_{11}}$

$X = + \frac{P\cancel{R^3}}{\cancel{2}\cancel{EI_z}} \frac{\cancel{4}^{2}\,\cancel{EI_z}}{\pi\,\cancel{R^3}} = \boxed{\frac{2P}{\pi} = B_y}$

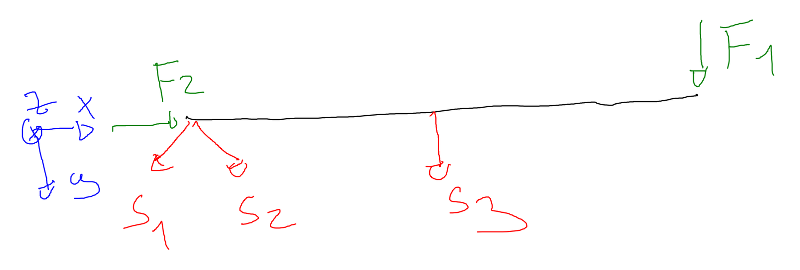
6.) restliche Lagerkräfte:



$\sum F_x: \; A_x = -P$
$\sum F_y: \; A_y = \frac{2P}{\pi}$
$\sum F_z: \; A_z = 0$
$\sum M_x^{in A}: \; M_x^A = 0$
$\sum M_y^{in A}: \; M_y^A = 0$
$\sum M_z^{in A}: \; M_z^A = R\frac{2P}{\pi}$

Serie 6 – S2

1.) LF0

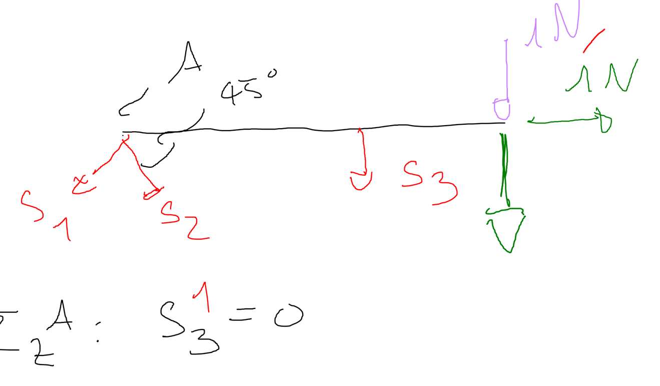


$S_1^0 = \frac{\sqrt{2}}{2}(F_1 + F_2)$

$S_2^0 = \frac{\sqrt{2}}{2}(F_1 - F_2)$

$S_3^0 = -2F_1$

2.) LF1 $\rightarrow u$



$\sum_z^A: \; S_3^1 = 0$

$\sum F_y: \; S_1 = -S_2$

$\sum F_x: \; -\frac{\sqrt{2}}{2}S_1 + \frac{\sqrt{2}}{2}S_2 + 1 = 0$

$\hookrightarrow \frac{\sqrt{2}}{2}S_2 + \frac{\sqrt{2}}{2}S_2 + 1 = 0$

$S_2^1 = -\frac{\sqrt{2}}{2}$

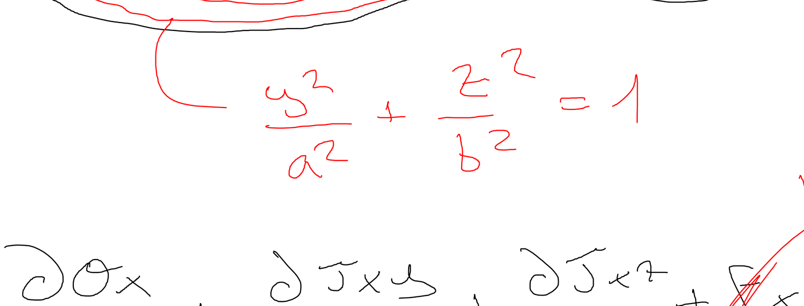
$\hookrightarrow S_1^1 = \sqrt{2}/2$

3.) $u = \int_{\ell_1} \frac{N_0 N_1}{EA} dx + \int_{\ell_2} \frac{N_0 N_1}{EA} dx + \int_{\ell_3} \frac{N_0 N_1}{EA} dx$

$= \frac{1}{EA}\Big[\frac{\cancel{\sqrt{2}}}{2}(F_1 + F_2)\cancel{\frac{\sqrt{2}}{2}}\sqrt{2}L + \cancel{\sqrt{2}L}\,\cancel{\frac{\sqrt{2}}{2}}(F_1 - F_2)\left(-\frac{\sqrt{2}}{2}\right) + \cancel{L(-2F_1 \cdot 0)}$

$= \frac{L}{EA}\Big[\frac{\sqrt{2}}{2}(\cancel{F_1} + F_2) - \frac{\sqrt{2}}{2}(\cancel{F_1} - F_2)\Big] = \boxed{\frac{L}{EA}\sqrt{2}F_2}$ u

Serie 10 H12



$\frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$

keine Gravitation

$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \cancel{f_x} = 0$

$\frac{\partial \tau_{xy}}{\partial x} + \cancel{\frac{\partial \sigma_y}{\partial y}} + \frac{\partial \tau_{yz}}{\partial z} + \cancel{f_y} = 0 \quad \cdot dz$

$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \cancel{\frac{\partial \sigma_z}{\partial z}} + \cancel{f_z} = 0 \quad \cdot dy$

$\sigma_y = \sigma_z = 0$

$\frac{\partial \tau_{xy}}{\partial x}\cdot dz + \tau_{yz} = 0$

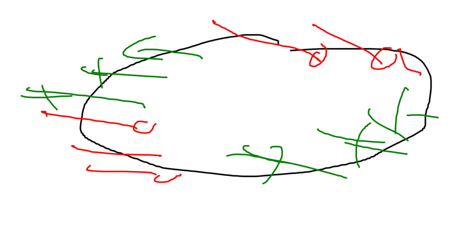
$\frac{\partial \tau_{xz}}{\partial x}\cdot dy + \tau_{yz} = 0$

$-$

$\frac{\partial \tau_{xy}}{\cancel{\partial x}} dz - \frac{\partial \tau_{xz}}{\cancel{\partial x}} dy = 0 \quad \Big| \int dx$

$\tau_{xy}\, dz = \tau_{xz}\, dy$

$\frac{\tau_{xy}}{\tau_{xz}} = \frac{dy}{dz}$



$\vdots$

$\underbrace{\frac{\tau_{xy}}{a^2 z}}_{\text{nur } z \text{ abhängig}} = \underbrace{\frac{\tau_{xz}}{b^2 y}}_{\text{nur } y \text{ abhängig}} = C \quad \rightarrow \tau_{xy} = \pm C a^2 z$