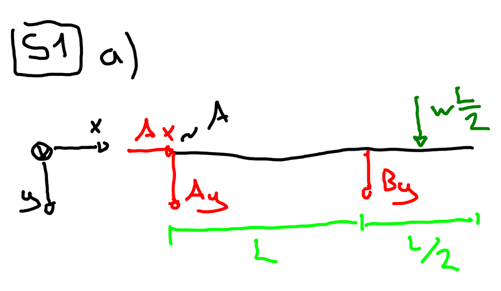
S1 a)



1.) Lagerkräfte berechnen:

$\sum F_x: \; A_x = 0$

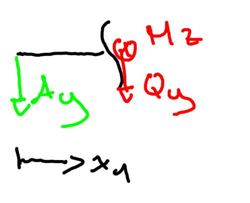
$\sum F_y: \; A_y + B_y = -w\frac{L}{2}$

$\sum M_z^{in\,A}: \; B_y \cdot L = -w\frac{L}{2}\frac{5}{4}L$

$\to B_y = -wL\frac{5}{8}$

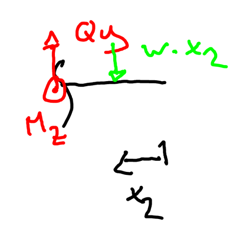
$\to A_y = -wL\left(\frac{1}{2} - \frac{5}{8}\right) = \frac{wL}{8}$

2.) Biegemoment:



$\sum M_z^{\text{im Schnittufer}}$:

$M_z(x_1) = A_y \cdot x_1 = \frac{wL}{8} \cdot x_1$



$\sum M_z^{\text{im Schnittufer}}$:

$M_z = w \cdot x_2 \frac{x_2}{2}$

$M_z(x_2) = \frac{w}{2}x_2^2$

4.) DGL

$v_y^{①}(x_1) = \frac{1}{E}\iint \frac{M_z(x_1)}{I}\, dx_1 = \frac{1}{EI}\iint \frac{wL}{8}x_1\, dx_1$

$= \frac{wL}{8EI}\left[\int x_1^2 \cdot 1/2 + C_1 \, dx_1\right]$

$= \frac{wL}{8EI}\left[\frac{1}{3}\frac{1}{2}x_1^3 + C_1 \cdot x_1 + C_2\right]$

$v_y^{②}(x_2) = \frac{1}{E}\iint \frac{M_z(x_2)}{I}\, dx_2 = \frac{1}{EI}\iint \frac{w}{2}x_2^2\, dx_2$

$= \frac{w}{2EI}\left[\int \frac{1}{3}x_2^3 + C_3 \, dx_2\right]$

$= \frac{w}{2EI}\left[\frac{1}{4}\frac{1}{3}x_2^4 + C_3 \cdot x_2 + C_4\right]$

5.) RB

Lagerbedingungen

$v_y^{①}(x_1=0) = 0 \to C_2 = 0$

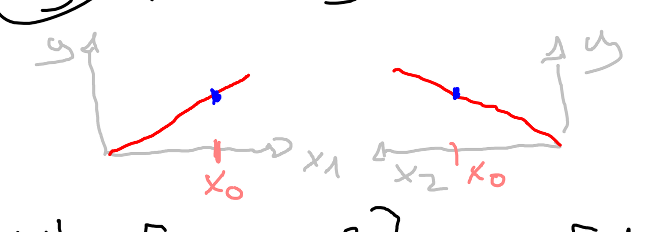
$v_y^{①}(x_1=L) = 0 \to \cancel{\frac{wL}{8EI}}\left[\frac{1}{6}L^3 + C_1 L\right] = 0$

$\to C_1 = -\frac{L^2}{6}$

$v_y^{②}(x_2=\frac{L}{2}) = 0 \to \cancel{\frac{w}{2EI}}\left[\frac{1}{12}\frac{L^4}{16} + C_3\frac{L}{2} + C_4\right] = 0$

→ Stetigkeitsbedingung:

$v_y'^{①}(x_1=L) = v_y'^{②}(x_2=L/2)\;(-1)$



$\frac{wL}{8EI}\left[\frac{1}{2}L^2 - \frac{L^2}{6}\right] = \frac{w}{2EI}\left[\frac{1}{3}\frac{L^3}{8} + C_3\right](-1)$

$\to C_3 = -\frac{1}{8}L^3$

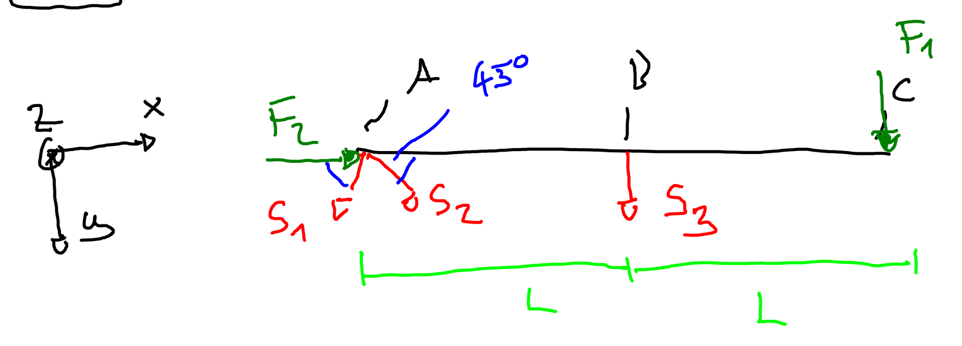
$C_4 = -\frac{L^4}{12 \cdot 16} + \frac{L}{2}\frac{L^3}{8} = \frac{11}{192}L^4$

$$\boxed{v_y^{①}(x_1) = \frac{wL}{8EI}\left[\frac{1}{6}x_1^3 - \frac{L^2}{6}x_1\right] \quad \text{für } 0 \le x_1 \le L}$$

$$\boxed{v_y^{②}(x_2) = \frac{w}{2EI}\left[\frac{1}{12}x_2^4 - \frac{L^3}{8}x_2 + \frac{11}{192}L^4\right]}$$

b) $v_y^{②}(x_2=0) = \frac{wL^4}{2EI}\frac{11}{192} = \boxed{\frac{11}{384}\frac{wL^4}{EI}}$

S2



$\sum M_z^{in\,A}: \; S_3 \cdot L = F_1 \cdot 2L(-1) \to S_3 = -2F_1$

$\sum F_x: \; -\frac{\sqrt{2}}{2}S_1 + \frac{\sqrt{2}}{2}S_2 + F_2 = 0$

$\sum F_y: \; \frac{\sqrt{2}}{2}S_1 + \frac{\sqrt{2}}{2}S_2 + S_3 + F_1 = 0$

$S_1 = S_2 + \sqrt{2}F_2$

$\frac{\sqrt{2}}{2}\left(S_2 + \sqrt{2}F_2\right) + \frac{\sqrt{2}}{2}S_2 - 2F_1 + F_1 = 0$

$S_2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = F_1 - F_2 \to S_2 = \frac{\sqrt{2}}{2}(F_1 - F_2)$

$S_1 = \frac{\sqrt{2}}{2}(F_1 - F_2) + \sqrt{2}F_2 = \frac{\sqrt{2}}{2}(F_2 + F_1)$

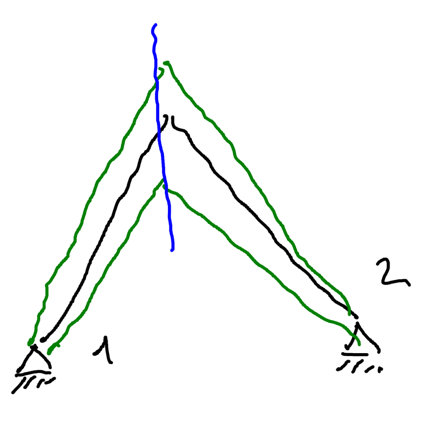
→ Verlängerung der Stäbe:

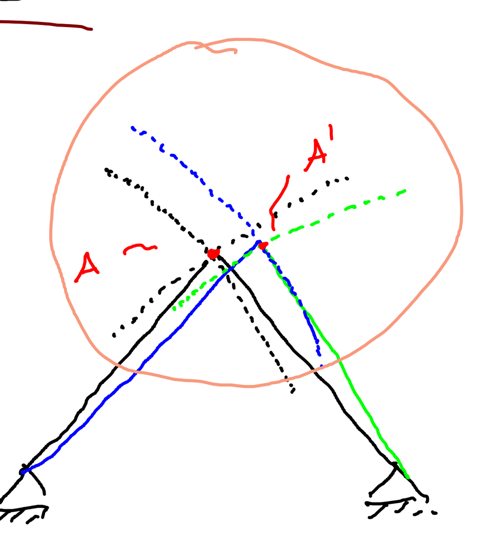
$\Delta L = \varepsilon \cdot L_0 = \frac{\sigma}{E}L_0 = \frac{F \cdot L_0}{EA}$

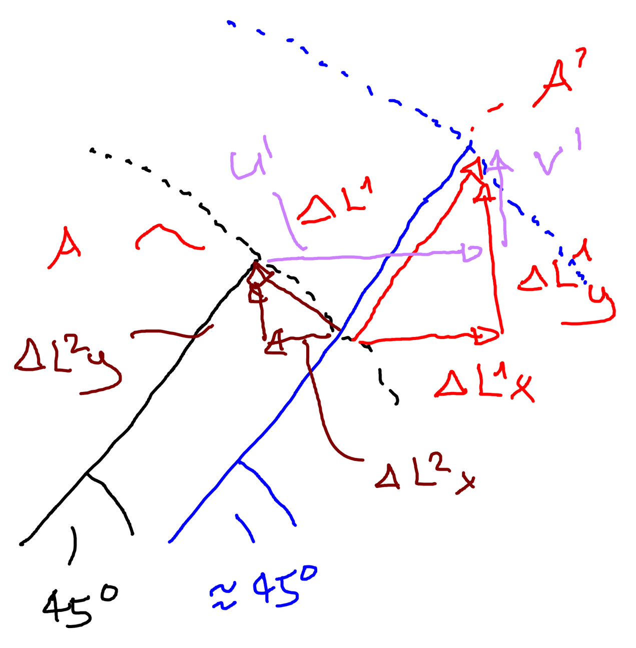
$\Delta L_1 = \frac{S_1 L_0^1}{EA} = \frac{\sqrt{2}}{2}(F_2 + F_1)\sqrt{2}\frac{L}{EA} = (F_1 + F_2)\frac{L}{EA}$

$\Delta L_2 = \frac{S_2 L_0^2}{EA} = \frac{\sqrt{2}}{2}(F_1 - F_2)\sqrt{2}\frac{L}{EA} = (F_1 - F_2)\frac{L}{EA}$

$\Delta L_3 = \frac{S_3 L_0^3}{EA} = -2F_1\frac{L}{EA}$

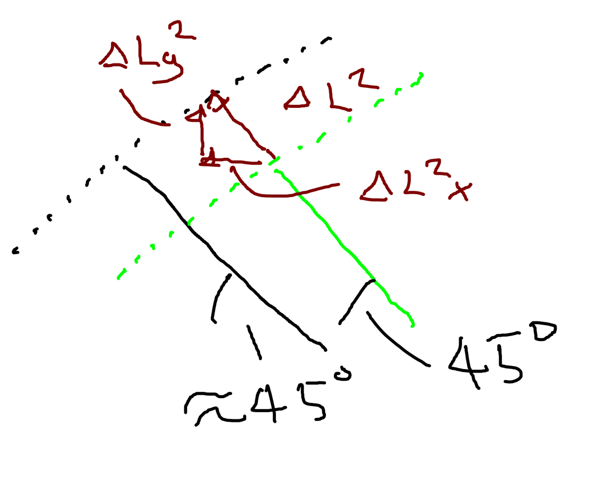






$u' = \Delta L^1_x + (-\Delta L^2_x)$

$v' = \Delta L^1_y - (-\Delta L^2_y)$



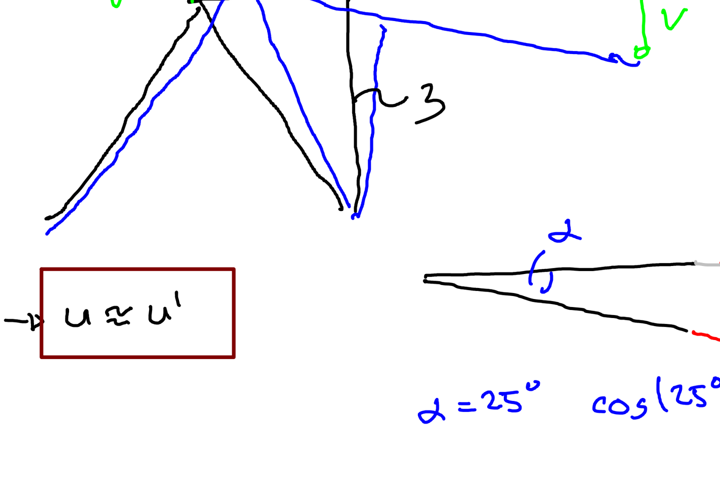
$\Delta L^1_x = \frac{\sqrt{2}}{2}\Delta L^1 = \frac{\sqrt{2}}{2}(F_1 + F_2)\frac{L}{EA}$

$\Delta L^1_y = \frac{\sqrt{2}}{2}\Delta L^1 = \frac{\sqrt{2}}{2}(F_1 + F_2)\frac{L}{EA}$

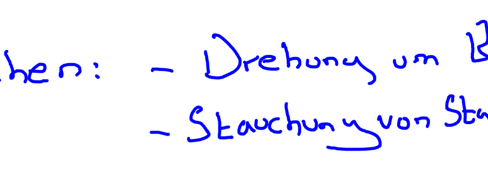
$\Delta L^2_x = \frac{\sqrt{2}}{2}\Delta L^2 = \frac{\sqrt{2}}{2}(F_1 - F_2)\frac{L}{EA}$

$\Delta L^2_y = \frac{\sqrt{2}}{2}\Delta L^2 = \frac{\sqrt{2}}{2}(F_1 - F_2)\frac{L}{EA}$

$u' = \sqrt{2}F_2\frac{L}{EA}$ $\qquad$ $v' = \sqrt{2}F_1\frac{L}{EA}$



$\to \boxed{u \approx u'}$

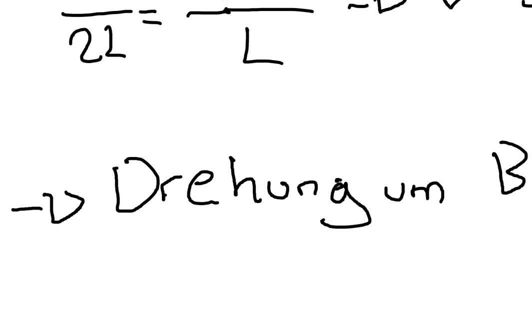


$\alpha = 25°$ $\quad \cos(25°) \approx 0{,}9$

→ v: 2 Ursachen: – Drehung um B
– Stauchung von Stab 3

→ Superpositionsprinzip

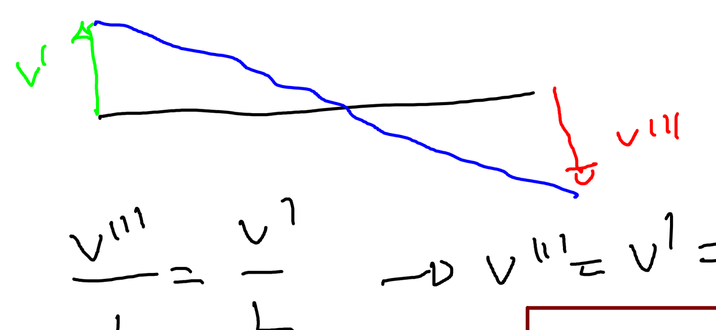
→ Stauchung von 3 Falls 1 & 2 starr



→ Strahlensatz:

$\frac{v''}{2L} = \frac{-\Delta L_3}{L} \to v'' = 2\Delta L_3 = 4F_1\frac{L}{EA}$

→ Drehung um B Falls 3 starr:



$\frac{v'''}{L} = \frac{v'}{L} \to v''' = v' = \sqrt{2}F_1\frac{L}{EA}$

$\to v = v'' + v''' = \boxed{(4 + \sqrt{2})F_1\frac{L}{EA}}$