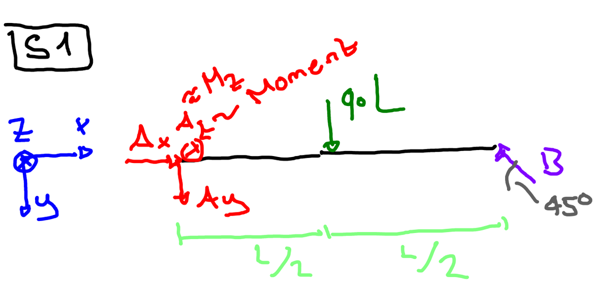
S1

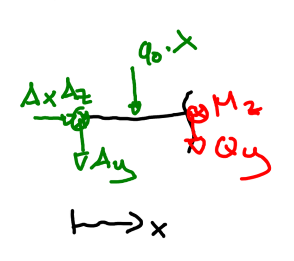


2.) $\sum F_x$: $A_x = \frac{\sqrt{2}}{2} B$

$\sum F_y$: $A_y = -q_0 L + \frac{\sqrt{2}}{2} B$

$\sum M_z^{\text{in } A}$: $A_z = \frac{\sqrt{2}}{2} BL - q_0 L \, L/2$

3.) M_z



$\sum M_z^{\text{im Schnittufer}}$:

$$M_z = A_z + A_y \cdot x + q_0 x \cdot \frac{x}{2}$$

$$= q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} BL + \frac{\sqrt{2}}{2} Bx - q_0 L x + \frac{q_0}{2} x^2$$

$$= \frac{q_0}{2} x^2 + x\left(\frac{\sqrt{2}}{2} B - q_0 L\right) + q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} BL$$

4.) $v_y(x) = \frac{1}{EI} \iint M_z(x)\, dx$

$$= \frac{1}{EI} \iint \frac{q_0}{2} x^2 + x\left(\frac{\sqrt{2}}{2} B - q_0 L\right) + q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} BL \, dx$$

$$= \frac{1}{EI}\left[\int \tfrac{1}{2}\tfrac{1}{3} q_0 x^3 + \tfrac{1}{2} x^2 \left(\frac{\sqrt{2}}{2} B - q_0 L\right) + x\left(q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} BL\right) + C_1 \, dx\right]$$

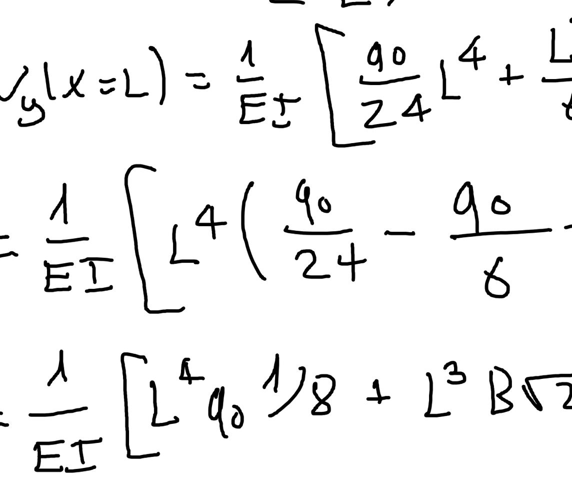
$$= \frac{1}{EI}\left[\frac{1}{24} q_0 x^4 + \frac{1}{6} x^3 \left(\frac{\sqrt{2}}{2} B - q_0 L\right) + \frac{x^2}{2}\left(q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} BL\right) + C_1 x + C_2\right]$$

5.) RB

$v_y(0) = 0 \to C_2 = 0$

$v_y'(0) = 0 \to C_1 = 0$

6.) Kinematische Bedingung



$|v_y(L)| = |u_x(L)|$

$$\Delta L = \varepsilon \cdot L_0 = \frac{\sigma}{E} L_0 = \frac{F_1 L_0}{EA}$$

F_1: Bx

$$u_x(L) = \frac{\sqrt{2}\, BL}{2\, EA}$$

$$v_y(x=L) = \frac{1}{EI}\left[\frac{q_0}{24} L^4 + \frac{L^3}{6}\left(\frac{\sqrt{2}}{2} B - q_0 L\right) + \frac{L^2}{2}\left(q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} BL\right)\right]$$

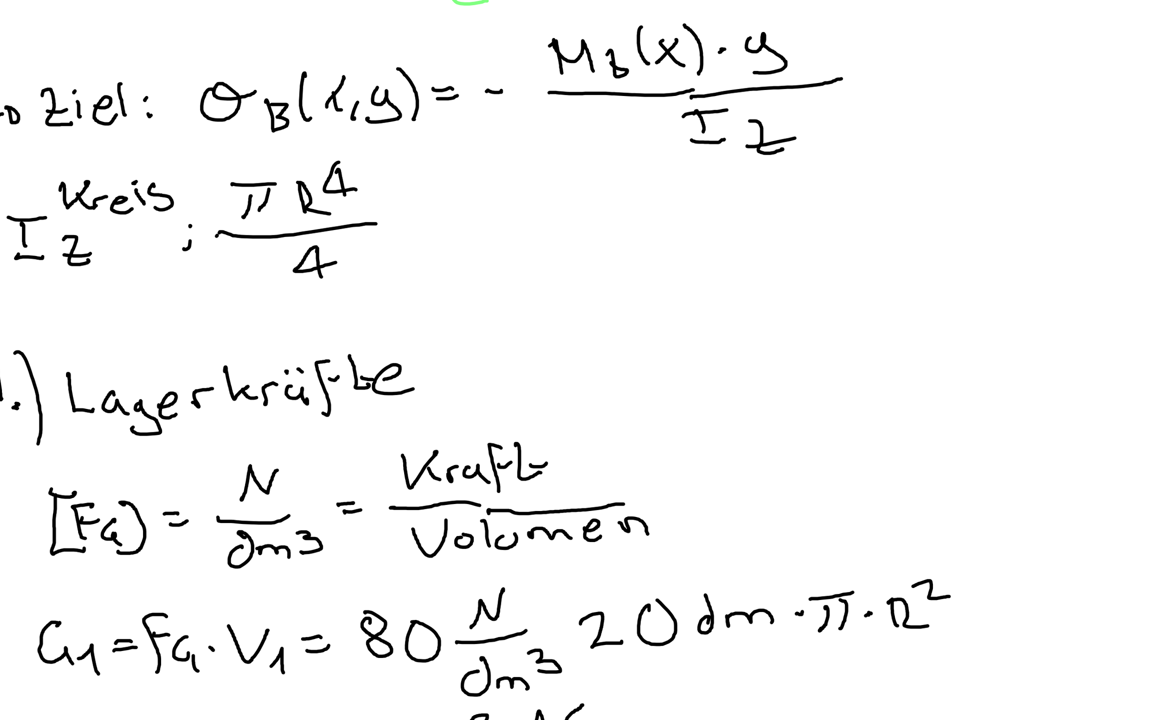
$$= \frac{1}{EI}\left[L^4\left(\frac{q_0}{24} - \frac{q_0}{6} + \frac{q_0}{4}\right) + L^3\left(\frac{\sqrt{2}}{12} B - \frac{\sqrt{2}}{4} B\right)\right]$$

$$= \frac{1}{EI}\left[L^4 q_0 \tfrac{1}{8} + L^3 B \sqrt{2} \left(-\tfrac{1}{6}\right)\right] \overset{!}{=} \frac{\sqrt{2}}{2} \frac{BL}{EA}$$

$$B = \frac{\sqrt{2} \cdot 3 \cdot A \cdot L^3 \cdot q_0}{24 I + 8 A L^2}$$

$$\boxed{v_y = u_x = \frac{\sqrt{2}}{2}\frac{BL}{EA} = \frac{3 L^4 q_0}{8E(AL^2 + 3I)}}$$

S2



→ Ziel: $\sigma_B(x,y) = -\frac{M_z(x) \cdot y}{I_z}$

I_z^{Kreis}; $\frac{\pi R^4}{4}$

1.) Lagerkräfte

$[F_G] = \frac{N}{dm^3} = \frac{\text{Kraft}}{\text{Volumen}}$

$G_1 = F_G \cdot V_1 = 80 \frac{N}{dm^3}\, 20\, dm \cdot \pi \cdot R^2$

$= 1600 \cdot \pi \cdot R^2\, N$

$G_2 = F_G \cdot V_2 = 80\, N/dm^3 \cdot 10\, dm \cdot \pi (\sqrt{2} R)^2$

$= 1600 \cdot \pi \cdot R^2\, N$ → $G_1 = G_2$

$A = B$ → wegen Symmetrie

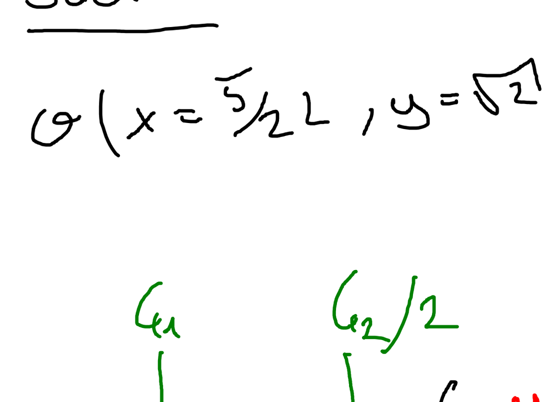
$\sum F_y$: $2G_1 + G_2 = 2A \to A = \frac{2G_1 + G_2}{2}$

$A = \frac{3 G_1}{2}$

2.) Kritische Stellen:

Stelle 1: $x = 2L$, $y = R$

$$\sigma(x=2L, y=R) = -\frac{M_z(x=2L) \cdot \cancel{R}}{\frac{\pi R^{\cancel{4}\,3}}{4}} = \frac{G_1 \cdot L \cdot 2}{\pi R^3}$$



$\sum M_z^{\text{im Schnittufer}}$:

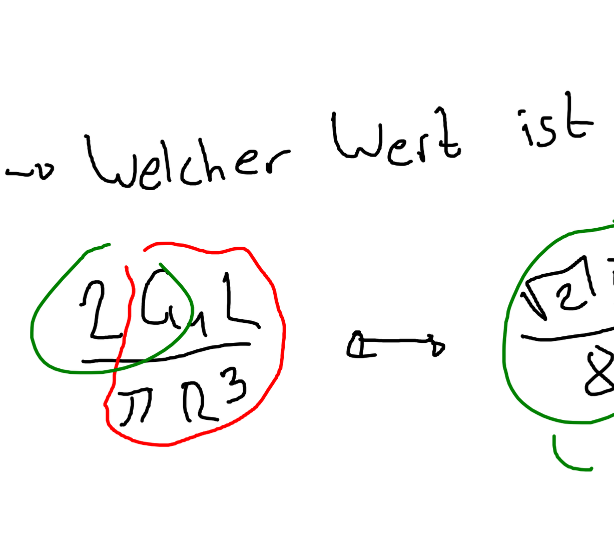
$M_z = G_1 \cdot L - A \cdot L$

$= G_1 \cdot L - \frac{3G_1}{2} L$

$= -\frac{G_1}{2} L$

Stelle 2: $x = 2.5L$ $y = \sqrt{2} R$

$$\sigma(x = 5/2 L, y = \sqrt{2} R) = -\frac{M_z(x = 5/2 L) \cdot \sqrt{2} R}{\pi R^4} = \frac{\sqrt{2}\, 5\, G_1 \cdot L}{8 \cdot \pi \cdot R^3}$$



$\sum M_z^{\text{im Schnittufer}}$:

$M_z = G_1 \cdot 3/2 L - A\, 3/2 L + \frac{G_2}{2}\frac{L}{4}$

$= G_1 \cdot \frac{3}{2} L - \frac{3}{2}\frac{3}{2} G_1 L + \frac{G_1}{8} L$

$= -\frac{5}{8} G_1 \cdot L$

→ Welcher Wert ist grösser?

$\frac{2 G_1 L}{\pi R^3} \longleftrightarrow \frac{\sqrt{2}\, 5}{8} \frac{G_1 \cdot L}{\pi R^3}$

$\approx 0{,}883$

→ Zulässiger Radius:

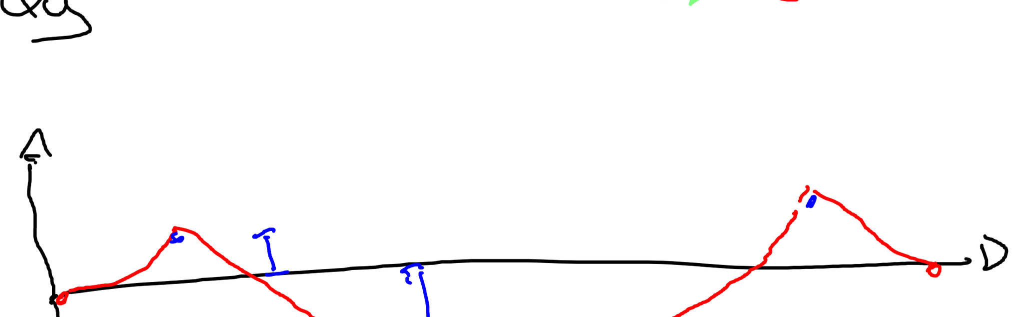
$\frac{2 G_1 L}{\pi R^3} = \sigma_{zul} = 20 \frac{kN}{cm^2} = 20'000 \frac{N}{cm^2}$

$= 2'000'000 \frac{N}{dm^2} = \frac{2L}{\pi R^3}\, 1'600 \cdot \pi \cdot R^2 \frac{N}{dm^2}$

$R = 2L \cdot 1'600 \frac{1}{2'000'000}\, dm$

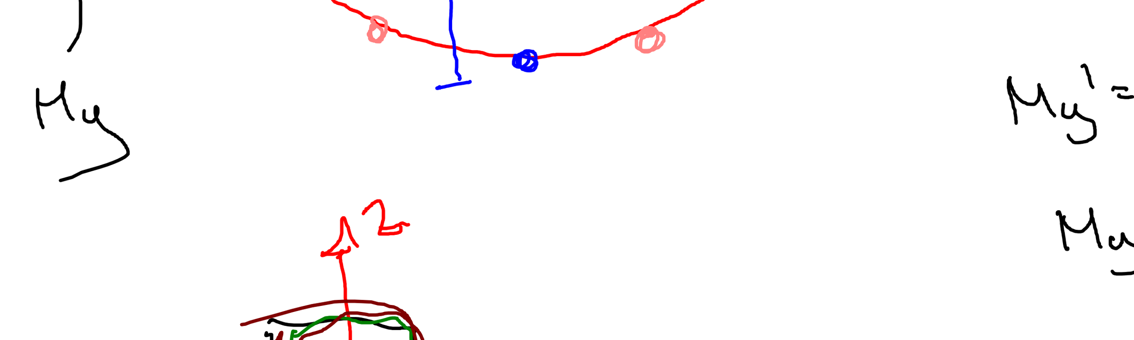
$$\boxed{R = 0.016\, dm = 1{,}6\, mm}$$

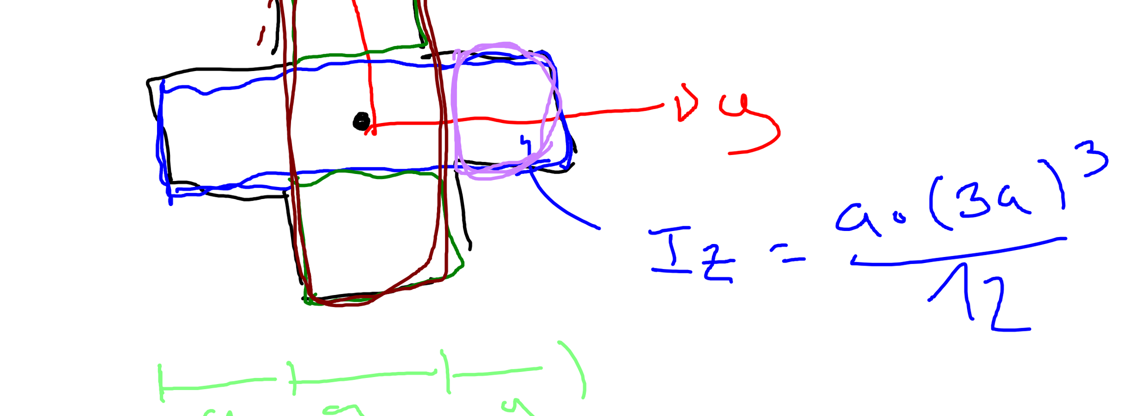
→ Beanspruchungsverlauf:



$-Q_y' = q_y$

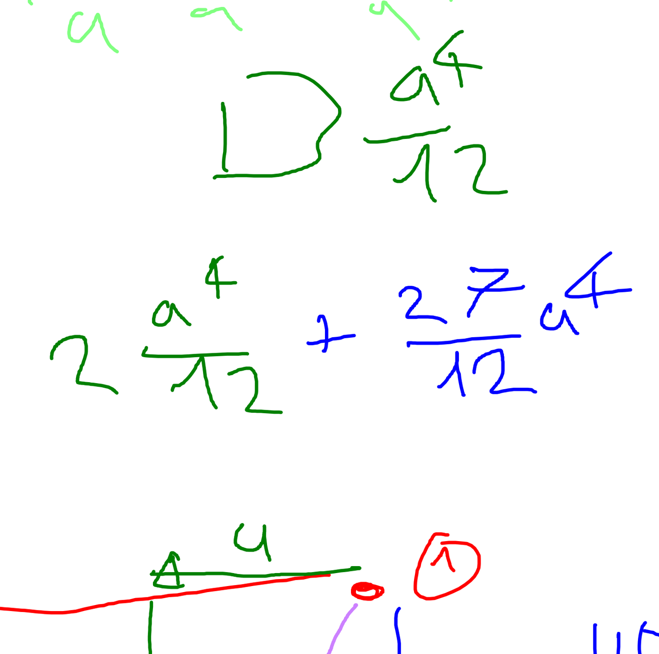
$Q_y' = -\int q_y\, dx$





$M_y' = -Q_y$

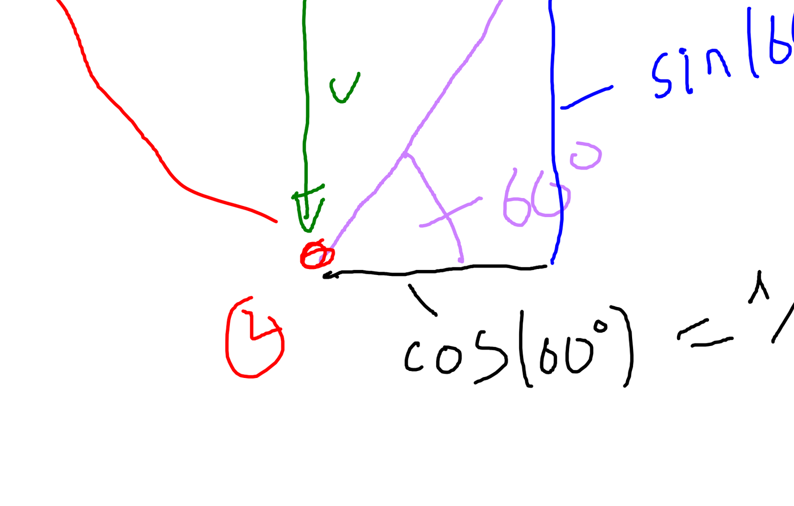
$M_y = -\int Q_y\, dx$



$I_z = \frac{a \cdot (3a)^3}{12}$

$\frac{a^4}{12}$

$2 \frac{a^4}{12} + \frac{27}{12} a^4 = \frac{29}{12} a^4$



$\sin(60°) = \sqrt{3}/2$

$\cos(60°) = 1/2$

$\frac{u}{v} = \frac{1/\cancel{2}}{\sqrt{3}/\cancel{2}} = \frac{1}{\sqrt{3}}$

$$\boxed{u = \frac{v}{\sqrt{3}}}$$