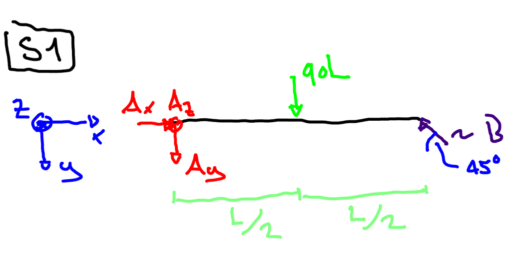
[S1]



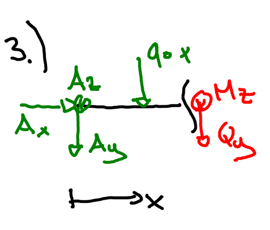
→ 2D-Problem

2.) $\Sigma F_x: \; A_x = \frac{\sqrt{2}}{2} B$

$\Sigma F_y: \; A_y = -q_0 L + \frac{\sqrt{2}}{2} B$

$\Sigma M_z^{\text{in } A}: \; \frac{\sqrt{2}}{2} B L - q_0 L \frac{L}{2} = A_z$

3.)



$\Sigma M_z^{\text{im Schnittufer}}$:

$M_z = -A_z + A_y x + q_0 x \frac{1}{2} x$

$\rightarrow M_z(x) = q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} B L + \frac{\sqrt{2}}{2} B x - q_0 L x + \frac{q_0}{2} x^2$

$M_z(x) = \frac{q_0}{2} x^2 + x\left(\frac{\sqrt{2}}{2} B - q_0 L\right) + q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} B L$

4.)

$v_y(x) = \frac{1}{EI} \iint M_z(x)\, dx$

$= \frac{1}{EI} \iint \frac{q_0}{2} x^2 + x\left(\frac{\sqrt{2}}{2} B - q_0 L\right) + q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} B L \, dx$

$= \frac{1}{EI} \left[\int \frac{1}{2}\frac{1}{3} q_0 x^3 + \frac{1}{2} x^2 \left(\frac{\sqrt{2}}{2} B - q_0 L\right) + x\left(q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} B L\right) + C_1 \, dx \right]$

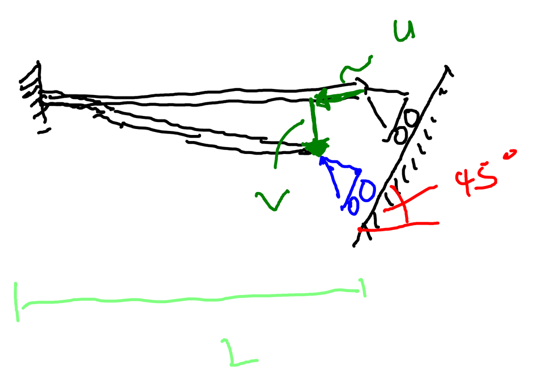
$= \frac{1}{EI} \left[\frac{1}{2}\frac{1}{3}\frac{1}{4} q_0 x^4 + \frac{1}{2}\frac{1}{3} x^3 \left(\frac{\sqrt{2}}{2} B - q_0 L\right) + \frac{x^2}{2}\left(q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} B L\right) + C_1 x + C_2 \right]$

5.) RB:

$v_y(0) = 0 \rightarrow C_2 = 0$

$v_y'(0) = 0 \rightarrow C_1 = 0$

6.) Kinematische Bedingung



$u_x(L) = v_y(L)$

$\Delta L = \varepsilon L_0 = \frac{\sigma}{E} L_0 = \frac{F_A L_0}{EA}$

$u_x(L) = \frac{\sqrt{2} B L}{2 E A} = v_y(L)$

$v_y(x=L) = \frac{1}{EI} \left[\frac{1}{24} q_0 L^4 + \frac{L^3}{6}\left(\frac{\sqrt{2}}{2} B - q_0 L\right) + \frac{L^2}{2}\left(q_0 \frac{L^2}{2} - \frac{\sqrt{2}}{2} B L\right) \right]$

$= \frac{1}{\cancel{EI}} \left[L^{\cancel{4}\,3} \left(\frac{q_0}{24} - \frac{q_0}{6} + \frac{q_0}{4}\right) + L^{\cancel{3}\,2} \left(\frac{\sqrt{2}}{12} B - \frac{\sqrt{2}}{4} B\right) \right] \overset{!}{=} \frac{\sqrt{2} B \cancel{L}\, I}{2 \cancel{E} A}$

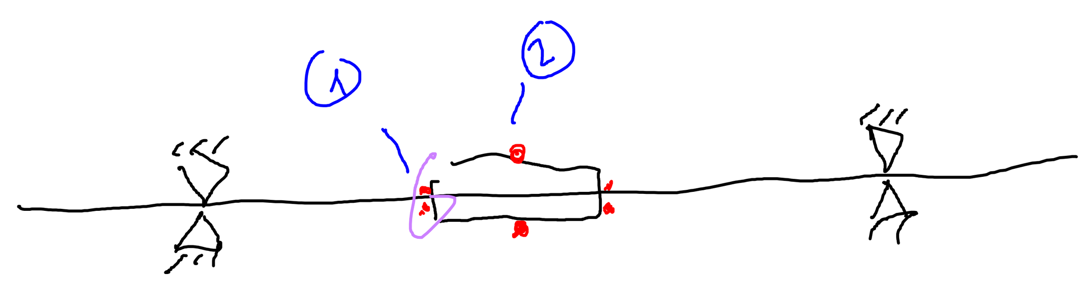
$\rightarrow \frac{L^3 q_0}{8} - L^2 B \frac{\sqrt{2}}{6} = \frac{\sqrt{2}}{2} \frac{I}{A} B$

$\rightarrow B\left(\frac{\sqrt{2}}{2}\frac{I}{A} + L^2 \frac{\sqrt{2}}{6}\right) = \frac{L^3 q_0}{8} \rightarrow B = \frac{\frac{L^3 q_0}{8}}{\frac{\sqrt{2}}{2}\frac{I}{A} + \frac{L^2 \sqrt{2}}{6}}$

$\rightarrow B = \frac{L^3 q_0 3 \cdot A}{\sqrt{2} \cdot 12 \cdot I + L^2 \sqrt{2} \cdot 4 \cdot A} = \boxed{\frac{\sqrt{2} \cdot 3 \cdot A \cdot L^3 \cdot q_0}{24 \cdot I + 8 \cdot A L^2}}$

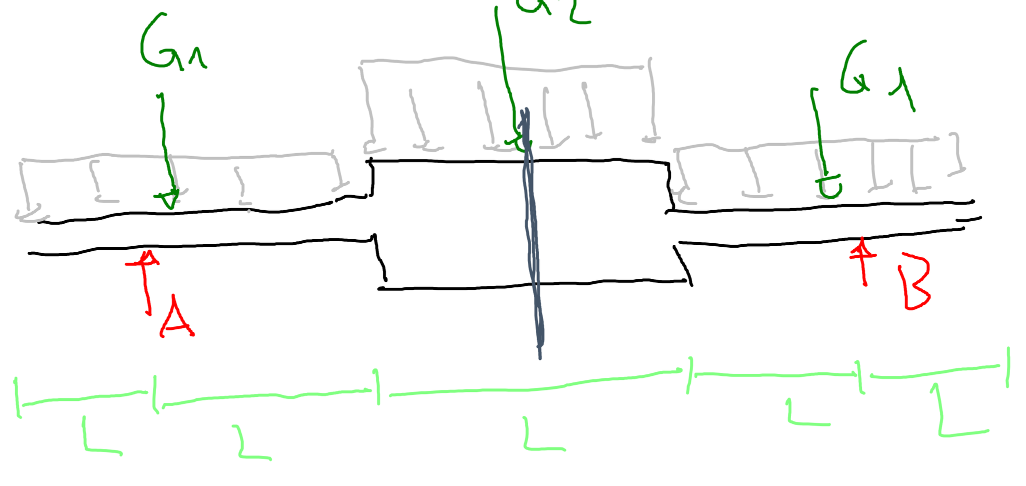
$\boxed{v_y = u_x = \frac{3 L^4 q_0}{8 E (A L^2 + 3 E I)}}$

[S2]



→ Ziel: $\sigma_B(x,y) = -\frac{M_b(x) \cdot y}{I_z}$

1.) Lagerkräfte



$[F_G] = \frac{N}{dm^3} = \frac{\text{Kraft}}{\text{Volumen}}$

$G_1 = F_G \cdot V_1 = 80 \frac{N}{dm^3} \cdot 20\, dm \cdot \pi \cdot R^2 = 1600 \cdot \pi \cdot R^2 \, N$

$G_2 = F_G \cdot V_2 = 80 \frac{N}{dm^3} \cdot 10\, dm \cdot \pi (\sqrt{2} R)^2 = 1600 \cdot \pi \cdot R^2 \, N$

$G_1 = G_2$

→ wegen Symmetrie $A = B$

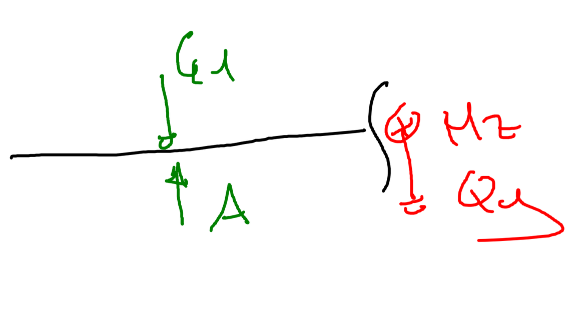
$\Sigma F_y: \; 2 G_1 + \cancel{G_2}^{\,G_1} = 2A \rightarrow A = \frac{3 G_1}{2}$

2.) Kritische Stellen:

→ Stelle 1: $x = 2L \quad y = R$

$\sigma(x=2L, y=R) = -\frac{M_z(x=2L) \cdot y}{I_z} = \cancel{-}\frac{-\frac{G_1}{2} L \cdot R}{\frac{\pi R^4}{2}} = \frac{G_1 \cdot L \cdot 2}{\pi R^3}$

→ $M_z(x=2L)$:



$\Sigma M_z^{\text{im Schnittufer}}$:

$M_z = G_1 \cdot L - A L$

$= G_1 L - \frac{3 G_1}{2} L$

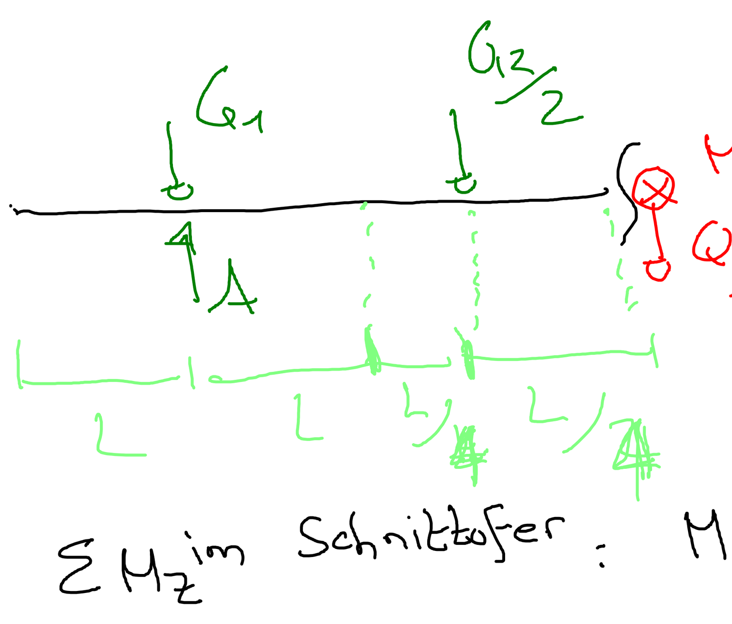
$= -\frac{G_1}{2} L$

→ Stelle 2: $x = 2.5L \quad y = \sqrt{2} R$

$\sigma(x = 5/2 L, y = \sqrt{2} R) = -\frac{M_z(x = 5/2 L) \cdot y}{I_z}$

→ $M_z(x = 5/2 L)$:

$= \cancel{-} \frac{\cancel{1/2}\, G_1 L \sqrt{2}\, \cancel{R}\; -5/8}{\pi R^{\cancel{4}\,3}}$



$\Sigma M_z^{\text{im Schnittufer}}$: $M_z = G_1 \cdot 3/2 L - A\, 3/2 L + \frac{G_2}{2} \frac{L}{4}$

$= G_1 3/2 L - 3/2\, 3/2\, G_1 L + \frac{G_1}{8} L$

$= G_1 \cdot L (\cancel{-1/2})\; -5/8$

→ Vergleich der kritischen Stellen:

Stelle 1:

$\frac{2\, G_1 \cdot L}{\pi R^3}$

stelle 2

$\approx 0{,}883$

$\frac{-5\sqrt{2}}{8} \frac{\cancel{\sqrt{2}}\, G_1 L}{\cancel{2}\, \pi R^3}$

$\longleftrightarrow$

→ kritische Stelle ist bei $\boxed{1}$

3.) Zulässigen Radius berechnen

$\frac{2 G_1 L}{\pi R^3} \overset{!}{=} \sigma_{zul} = 20 \frac{kN}{cm^2} = 20'000 \frac{N}{cm^2}$

$= 2'000'000 \frac{N}{dm^2} = \frac{2L}{\pi R^3} 1600 \pi R^2 \frac{N}{dm^2}$

$R = 2L \cdot 1'600 \frac{1}{2'000'000} dm$

$\boxed{R = 0.016\, dm = 1.6\, mm}$