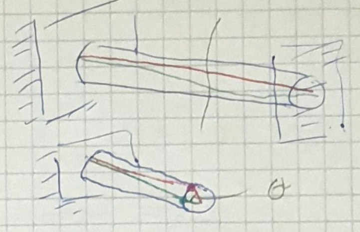
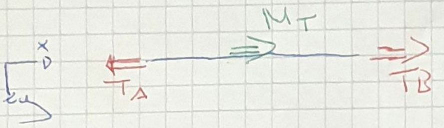
Serie 10 – $\boxed{S_1}$

$$\Theta(x_0) = \int_0^{x_0} \frac{T(x)}{G \cdot I_T(x)}\, dx$$



1.) Lagerkräfte

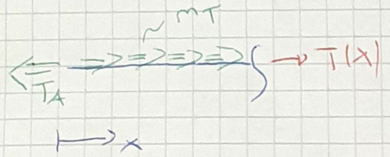


$$M_T = \int_0^{L_1+L_2} m_T\, dx = m_T (L_1 + L_2)$$

$$\sum M_x: \; T_A = M_T + T_B$$

$$T_A = m_T (L_1 + L_2) + T_B$$

2.) Torsionsverlauf



$$M_T' = \int_0^x m_T\, dx = m_T x$$

$$\sum M_x: \; T(x) = T_A - \cancel{}\, M_T' = T_A - m_T x = m_T(L_1+L_2) + T_B - m_T x$$

3.) ~~Maximum~~ Stelle des Maximum bestimmen:

$$\frac{d}{dx}\Theta(x) \overset{!}{=} 0 \Rightarrow \frac{d}{dx}\Theta(x) = \frac{d}{dx}\int_0^{x_0} \frac{T(x)}{G \cdot I_T(x)}\, dx = \frac{T(x)}{G I_T(x)} = 0$$

$$\Rightarrow T(x) = 0 \Rightarrow T_A - m_T x = 0 \Rightarrow \cancel{x_A}\; x = \frac{T_A}{m_T} = \frac{m_T(L_1+L_2) + T_B}{m_T}$$

→ T_B ist noch unbekannt

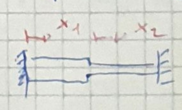
→ Verdrehungslinie als ~~~~ ~~~~ zusätzliche Gleichung verwenden
↳ analog zur Biegelinie bei statisch überbestimmten Systemen

4.) Verdrehungslinien $\Theta_1(x)$ & $\Theta_2(x)$ bestimmen

$$\Theta_1(x_1) = \int_0^{x_1} \frac{T(x)}{G \cdot I_T(x)}\, dx = \frac{1}{G \underbrace{I_p}_{I_z + I_y}} \int_0^{x_1} T(x)\, dx$$

$$= \frac{1}{G\left(\frac{\pi}{2}(r_1^4 - r_0^4)\right)} \int_0^{x_1} m_T(L_1+L_2) + T_B - m_T x\, dx$$

$$= \frac{1}{G\left(\frac{\pi}{2}(r_1^4 - r_0^4)\right)} \left(\left[m_T(L_1+L_2)\, \cancel{x_0} + T_B \right] x_1 - \frac{m_T x_1^2}{2} + C_1 \right)$$



→ analog für $\Theta_2(x)$

$$\Theta_2(x_2) = \frac{1}{G\left(\frac{\pi}{2}(r_2^4 - r_0^4)\right)} \left(\left[m_T(L_1+L_2) + T_B \right] x_2 - \frac{m_T x_2^2}{2} + C_2 \right)$$

5.) RB

Lager-bedingung $\begin{cases} \Theta_1(x_1=0) \overset{!}{=} 0 \Rightarrow C_1 = 0 \\ \Theta_2(x_2=L_2) \overset{!}{=} 0 \Rightarrow C_2 = \frac{m_T L_2^2}{2} - [m_T(L_1+L_2)+T_B] L_2 \end{cases}$

Stetigkeitsbedingung

$$\Theta_1(x_1=L_1) = \Theta_2(x_2=0)$$

$$\frac{1}{G(\frac{\pi}{2}(r_1^4-r_0^4))}\left([m_T(L_1+L_2)+T_B]L_1 - \frac{m_T L_1^2}{2}\right)$$

$$= \frac{1}{G(\frac{\pi}{2}(r_2^4-r_0^4))}\left(\frac{m_T L_2^2}{2} - [m_T(L_1+L_2)+T_B]L_2\right)$$

$T_B = \dots = -1519.91$ kNmm

6.) Maximum berechnen:

$$x = \frac{m_T(L_1+L_2)+T_B}{m_T} = 520 \text{ mm}$$

→ Ist das die einzige mögliche Stelle

→ Ränder: $\Theta_1(x_1=L_1) = \Theta_2(x_2=L_2) = 0$

→ Unstetigkeit: $x_1 = L_1$

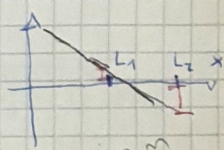
→ Stellen einsetzen: $\Theta_1(x_1=L_1) = 0,008426$ rad

$\Theta_2(x_2 = 520\text{mm} - L_1) = \boxed{|0.012675 \text{ rad}|} = \theta_{max}$

7.) τ_{max} bestimmen ~ betragsmässig

$$\tau_{max} = \frac{T_{max}}{W_T} = \frac{T_{max}\cdot 16\cdot D}{\pi(D^4-d^4)} = \frac{T_{max}\cdot 2^4\cdot 2R}{\pi(2^4R^4-2^4r^4)} = \frac{2R\cdot T_{max}}{\pi(R^4-r^4)}$$

$T_{max} \Rightarrow T(x) = m_T(L_1+L_2)+T_B - m_T x$ ~ lineare Funktion



→ Maximum an den Rändern

→ 1. Abschnitt: $T_{max} = m_T(L_1+L_2)+T_B$ → $x_1 = 0$

$$\tau_{max} = \frac{2r_1[m_T(L_1+L_2)+T_B]}{\pi(r_1^4-r_0^4)} = \boxed{94,7 \text{ MPa}}$$

→ 2. Abschnitt $\tau_{max} = m_T(L_1+L_2)+T_B - m_T(L_1+L_2)$ → $x_1 = L_1+L_2$

$$\tau_{max} = \frac{2r_2 T_B}{\pi(r_2^4-r_0^4)} = -146.8 \text{ MPa}$$