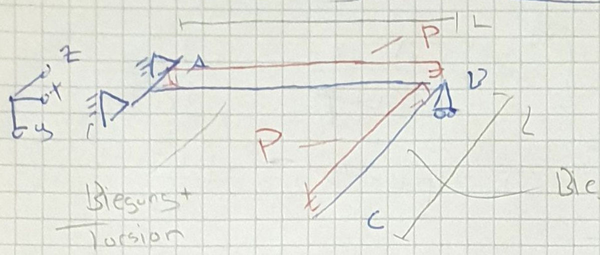
Übung 10 - S2



Annahmen:

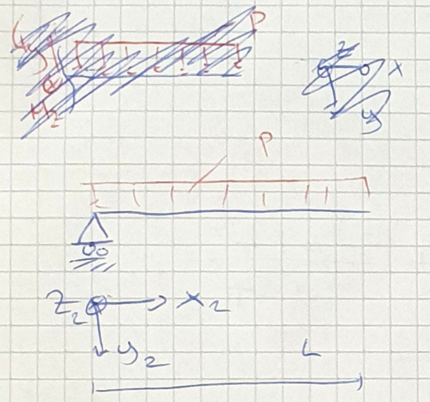
→ Biegung in A-B bewirkt keine Verschiebung in C

Biegung → Schub wird vernachlässigt

→ Ziel: Biegelinie in B-C berechnen

→ RB mit Torsion in A-B

· Biegelinie B-C:



→ Differentialbeziehungen:

$$Q_y = -\int q_y\,dx = -px + C_1$$

$$Q_y(x_2 = L) = -pL + C_1 \overset{!}{=} 0$$

$$\hookrightarrow C_1 = pL$$

$$M_z = -\int Q_y\,dx = -\int -px + pL\,dx$$

$$= -\frac{px^2}{2} + pLx + C_2$$

$$M_z(x_2 = L) = -\frac{pL^2}{2} + pL^2 + C_2 = 0$$

$$\hookrightarrow C_2 = -\frac{pL^2}{2}$$

$$v_y(x_2) = \frac{1}{EI_z}\iint M_z(x)\,dx$$

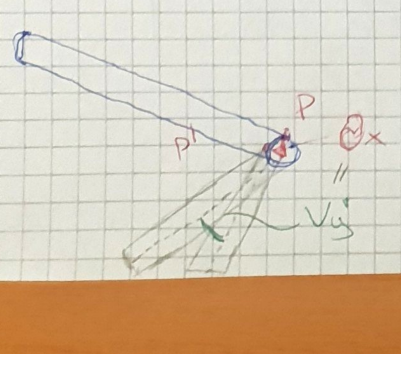
$$= \frac{1}{EI_z}\iint \left\{-\frac{px^2}{2} + pLx - \frac{pL^2}{2}\right\} dx = \frac{1}{EI_z}\int -\frac{px^3}{6} + \frac{pLx^2}{2} - \frac{pL^2x}{2} + C_3\,dx$$

$$= \frac{1}{EI_z}\left(-\frac{px^4}{24} + \frac{pLx^3}{6} - \frac{pL^2x^2}{4} + C_3x + C_4\right)$$

RB:

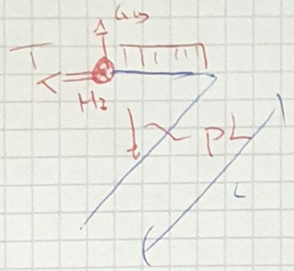
$v_y(x_2 = 0) \overset{!}{=} 0 \rightarrow C_4 = 0$

$v_y'(x_2 = 0) = \theta_x(x_1 = L)$ Neigung der Mittellinie = Verdrehung



· Verdrehung in B

$$\Theta_x(L) = \int_0^L \frac{T(x)}{G I_T(x)}\,dx = \frac{T}{G I_p}\int_0^L dx = \frac{T\,L}{G I_p} = \frac{pL^3}{2 G I_p}$$



$$\sum M_x^{\text{im Schnittufer}}: \quad T = pL \cdot L/2 = \frac{pL^2}{2}$$

→ RB: $v_y'(x_2 = 0) = \Theta_x(x_1 = L)$

$$\frac{C_3}{EI_z} = \frac{pL^3}{2GI_p} \to C_3 = \frac{EI_z\, pL^3}{2GI_p}$$

v_C berechnen:

$$v_y(x_2 = L) = \frac{1}{EI_z}\left[-\frac{pL^4}{24} + \frac{pL^4}{6} - \frac{pL^4}{4} + \frac{EI_z\, pL^4}{2GI_p}\right]$$

$$= \frac{1}{EI_z}\left[pL^4\left(-\frac{1}{8} + \frac{EI_z}{2GI_p}\right)\right]$$

Serie 9: S.1

$$I_z = \frac{\pi}{4}\left((R+t)^4 - R^4\right) \overset{t \ll R}{=} \pi R^3 t$$ gleich für Design I & II

$$v_y(x_2 = L) = v_C = \frac{pL^4}{2EI_z}\left[\frac{EI_z}{GI_p} - \frac{1}{4}\right]$$

Design II: $I_T = I_p = 2\pi R^3 t$

Design I: $I_T = \frac{1}{3}\sum_i s_i e_i^3 = \frac{1}{3} 2\pi R t^3$

$$v_C^{II} = \frac{pL^4}{2E\pi R^3 t}\left[\frac{E\pi R^3 t}{G 2\pi R^3 t} - \frac{1}{4}\right] = \frac{pL^4}{4E\pi R^3 t}\left[\frac{E}{G} - \frac{1}{2}\right]$$

$$v_C^{I} = \frac{pL^4}{2E\pi R^3 t}\left[\frac{E\pi R^3 t}{G \frac{1}{3} 2\pi R t^3} - \frac{1}{4}\right] = \frac{pL^4}{4E\pi R^3 t}\left[\frac{3ER^2}{Gt^2} - \frac{1}{2}\right]$$

$$\Delta C = \left|v_C^I - v_C^{II}\right| = \left|\frac{pL^4}{4E\pi R^3 t}\left[\frac{3ER^2}{Gt^2} - \cancel{\frac{1}{2}} - \frac{E}{G} + \cancel{\frac{1}{2}}\right]\right|$$

$$\boxed{= \left|\frac{pL^4}{4G\pi R^3 t}\left[\frac{3R^2}{t^2} - 1\right]\right|}$$