

Übung 11 - S1

$$U = \frac{1}{2} \left[\int_0^L \frac{N^2}{EA} dx + \int_0^L \frac{Q^2}{GA_S} dx + \int_0^L \frac{M^2}{EI} dx + \int_0^L \frac{T^2}{GJ_T} dx \right]$$

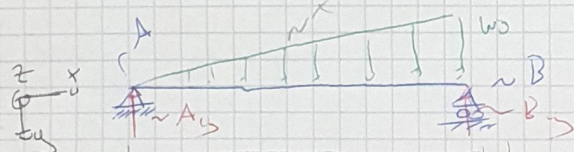
keine
Normalkräfte

keine
Torsion

→ nur Schub und Biegung → Q_y & M_z

$$w(x) = \frac{w_0}{L} x$$

→ Beanspruchung → Differentialbeziehungen

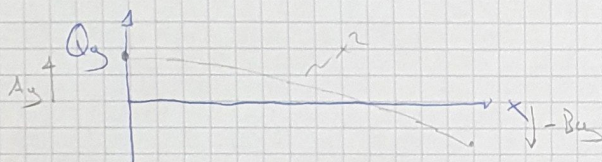


$$\sum F_y: A_y + B_y = \frac{1}{2} w_0 L$$

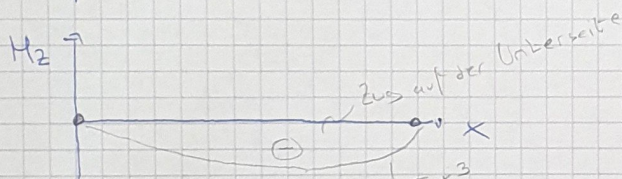
$$\sum M_z^A: B_y L = \frac{1}{2} w_0 L \cdot \frac{2}{3} L$$

$$L \cdot B_y = \frac{w_0 L^2}{3}$$

$$L \cdot A_y = \frac{w_0 L}{6}$$



$$Q_y = - \int q_y dx = - \int w(x) dx = - \int \frac{w_0}{L} x dx = - \frac{w_0 x^2}{2L} + C_1$$



$$M_z = - \int Q_y dx = - \int \left(- \frac{w_0 x^2}{2L} + C_1 \right) dx = \frac{w_0 x^3}{6L} - C_1 x + C_2$$

RB: $M_z(x=0) = 0 \rightarrow C_2 = 0$

$$M_z(x=L) = 0 \rightarrow \frac{w_0 L^3}{6L} - C_1 L = 0 \rightarrow C_1 = \frac{w_0 L}{6}$$

$$\rightarrow M_z = \frac{w_0 x^3}{6L} - \frac{w_0 L}{6} x$$

$$U = \frac{1}{2} \left[\frac{1}{GA_S} \int_0^L \left(- \frac{w_0 x^2}{2L} + \frac{w_0 L}{6} \right)^2 dx + \frac{1}{EI_z} \int_0^L \left(\frac{w_0 x^3}{6L} - \frac{w_0 L}{6} x \right)^2 dx \right]$$

①

$$\int_0^L \frac{w_0^2 x^4}{4L^2} + \frac{w_0^2 L^2}{36} - \frac{2w_0^2 x^2 L}{6L} dx = \frac{w_0^2 L^5}{20L^2} + \frac{w_0^2 L^3}{36} - \frac{w_0^2 L^3}{18}$$

$$= \frac{w_0^2 L^3}{2} \left[\frac{1}{10} + \frac{1}{18} - \frac{1}{9} \right] = \frac{w_0^2 L^3}{45}$$

$\frac{2}{45}$

$$(b) \int_0^L \frac{w_0^2 x^6}{36 L^2} + \frac{w_0^2 L^2}{36} x^2 - \frac{2 w_0^2 x^4 L}{36 L} dx$$

$$= \frac{w_0^2 L^7}{7 \cdot 36 L^2} + \frac{w_0^2 L^5}{3 \cdot 36} - \frac{2 w_0^2 L^5}{5 \cdot 36 L}$$

$$= \frac{w_0^2 L^5}{36} \left(\frac{1}{7} + \frac{1}{3} - \frac{2}{5} \right) = \frac{w_0^2 L^5 \cdot 2}{945}$$

$$\Rightarrow U = \frac{w_0^2 L^3}{45} \left[\frac{1}{2GA_s} + \frac{1}{EI \cdot 2.21} \right]$$