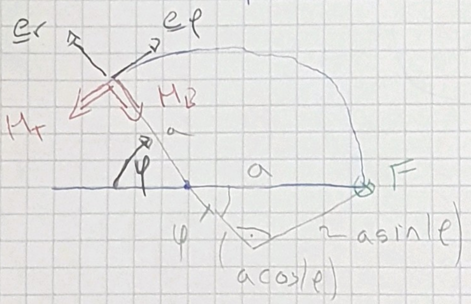
Übung 11 - $\boxed{S2}$

→ Direkte Anwendung des Arbeitssatzes:

$$W_{ext} = U = \tfrac{1}{2} F v_F \rightarrow v_F = \frac{2U}{F}$$

1.) Beanspruchung:



$$\sum M_r^{\text{im Schnittufer}}: \quad M_B(\varphi) = F a \sin(\varphi)$$

$$\sum M_\varphi^{\text{im Schnittufer}}: \quad M_T(\varphi) = F\left(a\cos(\varphi) + a\right) = Fa\left(\cos(\varphi)+1\right)$$

2.) Berechnung von U:

$$U = \frac{1}{2}\left[\underbrace{\frac{1}{EI}\int_\ell M_B(\varphi)^2\, a\, d\varphi}_{(a)} + \underbrace{\frac{1}{GI_T}\int_\ell M_T(\varphi)^2\, a\, d\varphi}_{(b)}\right]$$

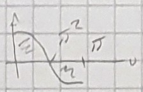
Integraltabelle

$$(a): \int_0^\pi F^2 a^2 \sin^2(\varphi)\, a\, d\varphi = F^2 a^3 \underbrace{\int_0^\pi \sin^2(\varphi)\, d\varphi}_{\pi/2} = F^2 a^3 \frac{\pi}{2}$$

$$(b): \int_0^\pi F^2 a^2 (\cos(\varphi)+1)^2\, a\, d\varphi = F^2 a^3 \int_0^\pi \cos(\varphi)^2 + 2\cos(\varphi) + 1 \, d\varphi$$

$$= F^2 a^3 \left[\underbrace{\int_0^\pi \cos(\varphi)^2\, d\varphi}_{\pi/2} + 2\underbrace{\int_0^\pi \cos(\varphi)\, d\varphi}_{0} + \underbrace{\int_0^\pi d\varphi}_{\pi}\right] = F^2 a^3\left(\frac{\pi}{2} + \pi\right)$$

$$= \int_0^\pi \sin(\varphi)^2\, d\varphi \qquad\qquad = F^2 a^3 \frac{3\pi}{2}$$



3.) Definition anwenden:

$$v_F = \frac{2U}{F} = \frac{\cancel{2}}{F}\,\frac{1}{\cancel{2}}\left[\frac{1}{EI} F^{\cancel{2}} a^3 \frac{\pi}{2} + \frac{1}{GI_T} F^{\cancel{2}} a^3 \frac{3\pi}{2}\right]$$

$$= \frac{\pi F a^3}{2}\left[\frac{1}{EI} + \frac{3}{GI_T}\right]$$