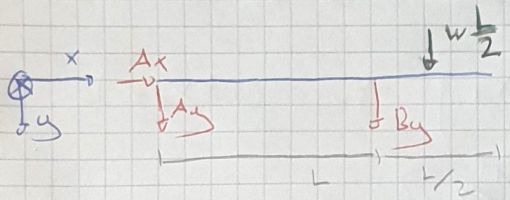
Übung 6 · S 1

a) 1.) Lagerkräfte bestimmen → KOS



$\sum F_x = 0 \rightarrow A_x = 0$

$\sum F_y: \; A_y + B_y = -w\frac{L}{2}$

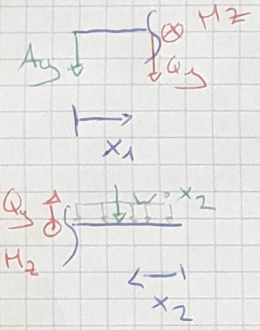
$\sum M_z^{in A}: \; B_y \cdot L = -w\frac{L}{2}\frac{5}{4}L$

$\rightarrow B_y = -\frac{5}{8}wL$

$\rightarrow A_y = -wL\left(\frac{1}{2} - \frac{5}{8}\right) = \frac{wL}{8}$

2.) Beanspruchung berechnen

→ M_z



$\sum M_z^{im\ Schnittufer}: \; M_z = A_y \cdot x_1 \rightarrow M_z(x_1) = \frac{wL}{8}x_1$

$\sum M_z^{im\ Schnittufer}: \; M_z = w \cdot x_2 \cdot x_2 \frac{1}{2} \rightarrow M_z(x_2) = \frac{w}{2}x_2^2$

3.) I bestimmen: I ist bereits gegeben

4.) DGL → Biegelinie

$$v_y^{(1)}(x_1) = \frac{1}{E}\iint \frac{M_z(x_1)}{I}\,dx_1 = \frac{1}{E}\iint \frac{wL}{8}\frac{x_1}{I}\,dx_1 = \frac{wL}{8EI}\left[\int x_1^2 \tfrac{1}{2}\,dx_1 + C_1\,dx_1\right]$$

$$= \frac{wL}{8EI}\left[\frac{1}{2}\frac{1}{3}x_1^3 + C_1 x_1 + C_2\right]$$

$$v_y^{(2)}(x_2) = \frac{1}{E}\iint \frac{M_z(x_2)}{I}\,dx_2 = \frac{1}{E}\iint \frac{w}{2}x_2^2\,dx_2 = \frac{w}{2EI}\left[\int \frac{1}{3}x_2^3 + C_3\,dx_2\right]$$

$$= \frac{w}{2EI}\left[\frac{1}{3}\frac{1}{4}x_2^4 + C_3 x_2 + C_4\right]$$

5.) RB: Lager

$v_y^{(1)}(x_1 = 0) = 0 \rightarrow C_2 = 0$

$v_y^{(1)}(x_1 = L) = 0 \rightarrow \frac{wL}{8EI}\left[\frac{1}{6}L^3 + C_1 L\right] = 0 \rightarrow C_1 = -\frac{L^2}{6}$

$v_y^{(2)}\left(x_2 = \frac{L}{2}\right) = 0 \rightarrow \frac{w}{2EI}\left[\frac{1}{12}\frac{L^4}{16} + C_3\frac{L}{2} + C_4\right] = 0$

Stetigkeit: $v_y'^{(1)}(x_1 = L) = v_y'^{(2)}\left(x_2 = \frac{L}{2}\right) \cdot (-1)$ ~ Laufvariablen haben unterschiedliche Vorzeichen

$$\frac{wL}{8EI}\left[\frac{1}{2}L^2 - \frac{L^2}{6}\right] = \frac{w}{2EI}\left[\frac{1}{3}\frac{L^3}{8} + C_3\right] \cdot (-1)$$

$$C_3 = \overset{(-1)}{\downarrow} \frac{L \cdot \frac{L}{2}}{8} \left[L^2 \cdot \frac{2}{3} \right] - \frac{L^3}{24} = L^3 \left(\frac{1}{12} - \frac{1}{24} \right) = \frac{1}{8} L^3$$

$$C_4 = -\frac{L^4}{12 \cdot 16} + \frac{L}{2} \frac{L^3}{8} = L^4 \left(\frac{1}{192} \right) \frac{11}{192}$$

$$v_S^{(1)}(x_1) = \frac{wL}{8EI} \left[\frac{1}{6} x_1^3 - \frac{L^2}{6} x_1 \right] \quad \text{für } 0 \le x_1 \le L$$

$$v_S^{(2)}(x_2) = \frac{w}{2EI} \left[\frac{1}{12} x_2^4 - \frac{L^3}{8} x_2 + \frac{11}{192} L^4 \right] \quad \text{für } 0 \le x_2 \le L/2$$

b) $$v_L = v_S^{(2)}(x_2 = 0) = \frac{wL^4}{2EI} \cdot \frac{11}{192} = \frac{11}{384} \frac{wL^4}{EI}$$