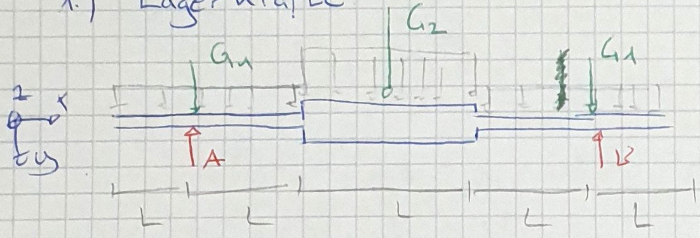
Übung 7: $\boxed{S2}$

→ Ziel: $\sigma_b(x,y) = -\frac{M_b(x)\cdot y}{I_z}$

Eigengewicht → linienverteilte Kraft ↳ was sind die kritischen Stellen?

1.) Lagerkräfte



→ Symmetrie

$[F_G] = \frac{N}{dm^3} = \frac{\text{Kraft}}{\text{Volumen}}$

$G_i = F_G \cdot V$

$G_1 = F_G \cdot V_1 = 80\,\frac{N}{dm^3}\,\underbrace{20\,dm}_{\text{Länge}} \cdot \pi \cdot R^2 = \overset{1600}{\cancel{800}}\,\pi R^2\ N$

$G_2 = F_G \cdot V_2 = 80\,\frac{N}{dm^3}\,10\,dm \cdot \pi\left(\sqrt{2}R\right)^2 = 1600\,\pi R^2\ N$

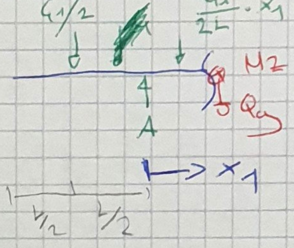
$A = B$ wegen Symmetrie

$\sum F_y:\ 2G_1 + G_2 = 2A \rightarrow A = \frac{\overbrace{2G_1 + G_2}^{3G_1}}{2} \left(= \frac{4800\pi R^2}{2} N \right)$

$\left(A = B = 2400\,\pi R^2\,N\right)$

$\boxed{2.)\ \text{Biegemoment}}$ unnötig

- Abschnitt $0 \le x \le L$: irrelevant bzgl. max. Spannung
- Abschnitt $L \le x \le 2L$:

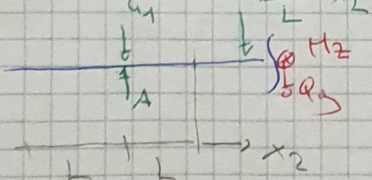


$\sum M_z^{\text{im Schnittufer}}:\ M_z = -Ax_1 + \frac{G_1}{2}\left(\frac{L}{2} + x_1\right) + \frac{G_1}{2L}x_1 \cdot \frac{x_1}{2}$

$\rightarrow M_z = -\frac{3G_1}{\underset{2}{\cancel{4}}L}x_1 + \frac{G_1 L}{4} + \frac{G_1}{2}x_1 + \frac{G_1}{4L}x_1^2$

$\rightarrow M_z = \frac{G_1}{4L}x_1^2 + x_1\left(\frac{G_1}{2} - \frac{3G_1}{2L}\right) + \frac{G_1 L}{4}$

- Abschnitt $2L \le x \le 3L$:



$\sum M_z^{\text{im Schnittufer}}$:

$M_z = -A(L + x_2) + G_1(L + x_2) + \frac{G_1}{L}x_2\,\frac{1}{2}x_2$

$= -\frac{3}{2}G_1 L - \frac{3}{2}G_1 x_2 + G_1 L + G_1 x_2 + \frac{G_1}{2L}x_2^2$

2.2) Kritische Stellen $\qquad I_z^{Kreis} = \frac{\pi R^4}{4}$

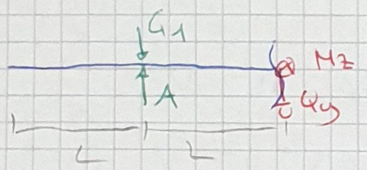
Stelle 1: $x_1 = 2L \quad y = R$

$$\sigma(x_1 = 2L, y = R) = -\frac{M_z(x_1 = 2L)\cdot y}{I_z} = -\frac{-\frac{G_1}{2}L\cdot R}{\frac{\pi R^{3}}{2}} = \frac{G_1 L\cdot 2}{\pi R^3}$$

→ $M_z(x = 2L)$:

$\sum M_z^{\text{im Schnittufer}}$: $\quad M_z = G_1\cdot L - A L$

$$= G_1\cdot L - \frac{3G_1}{2}L = -\frac{G_1}{2}L$$



Stelle 2: $x = 2.5\,L \quad y = \sqrt{2}R$

$$\sigma(x = \tfrac{5}{2}L, y\sqrt{2}R) = -\frac{M_z(x = \frac{5}{2}L)\cdot y}{I_z} = -\frac{G_1\cdot L(-1/2)\sqrt{2}R}{\pi R^3}$$

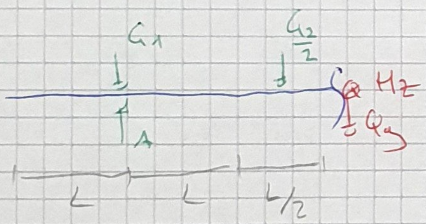
→ $M_z(x = \frac{5}{2}L)$

$\sum M_z^{\text{im Schnittufer}}$:

$$M_z = G_1\cdot\frac{3}{2}L - A\frac{3}{2}L + \frac{G_2}{2}\frac{L}{2}$$

$$= G_1\cdot\frac{3}{2}L - \frac{3}{2}\frac{3}{2}G_1 L + \frac{G_1}{4}L$$

$$= G_1\cdot L\,(-1/2)$$



→ Welcher Wert ist grösser?

$$\frac{2G_1L}{\pi R^3} \longleftrightarrow \frac{\sqrt{2}\,G_1L}{2\pi R^3} \quad \to \text{Stelle } 1$$

→ 3.) Zulässigen Radius berechnen

$$\sigma(x = 2L, y = R) = \frac{2G_1L}{\pi R^3} = \sigma_{zul} = 20\,\frac{kN}{cm^2} = 20'000\,\frac{N}{cm^2}$$

$$= 2'000'000\,\frac{N}{dm^2} = \frac{2L}{\pi R^3}\,1600\,\pi\,\frac{N}{dm^2}$$

$$R = 2L\cdot 1600\,\frac{1}{2'000'000}\,dm$$

$$R = 0.016\,dm = 1{,}6\,mm$$