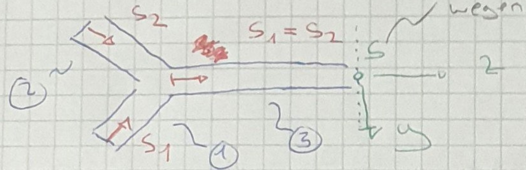
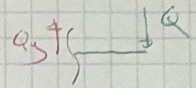
Übung 8 : $\boxed{S2}$

→ System ist symmetrisch → Dünnwandiger Querschnitt

1.) Laufvariablen einführen





→ $Q_y = Q = \text{konst}$

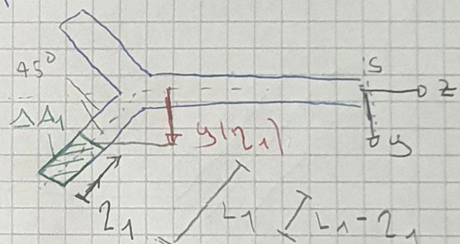
2.) Schubfluss im Abschnitt ①:

$$\tau_{xs_1}(\cancel{x}, s_1) = -\frac{Q_y(x)}{I_z(x)} \cdot \frac{H_z(s_1)}{e(s_1)}$$

→ Q_y ist konstant im ganzen Balken

→ A ist konst ⇒ I_z = konst

→ gemäß Musterlösung: definiere $\frac{Q_y}{I_z} = q$:)



$$H_z(s_1) = \int_0^{s_1} y(\eta_1)\, e(\eta_1)\, d\eta_1$$

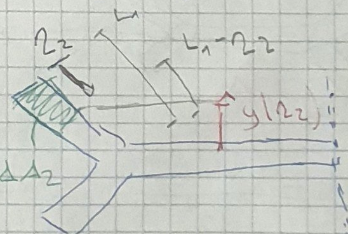
$$= \int_0^{s_1} (L_1 - \eta_1) \sin(\alpha)\, e\, d\eta_1$$

$$= \underbrace{\sin(\overset{=45°}{\alpha})}_{\frac{\sqrt{2}}{2}} e \left[L_1 \eta_1 - \frac{\eta_1^2}{2}\right]_{\eta_1=0}^{\eta_1=s_1} = \frac{\sqrt{2}}{2} e \left[L_1 s_1 - \frac{s_1^2}{2}\right]$$

$$\tau_{xs_1}(s_1) = -q \frac{\sqrt{2}}{2} \frac{\cancel{e}}{\cancel{e}} \left(L_1 s_1 - \frac{s_1^2}{2}\right)$$

3.) Schubfluss im Abschnitt ②:

$$\tau_{xs_2}(\cancel{x}, s_2) = -\underbrace{\frac{Q_y(x)}{I_z(\cancel{x})}}_{q} \cdot \frac{H_z(s_1)}{\cancel{e(s_2)}\; e} \rightarrow \tau_{xs_2}(s_2) = -q \frac{H_z(s_2)}{e}$$



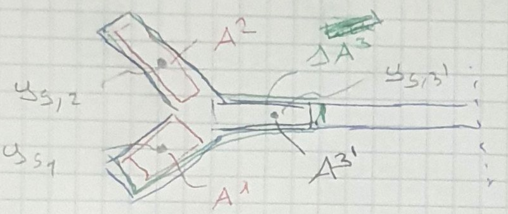
$$H_z(s_2) = \int_0^{s_2} y(\eta_2)\, e(\eta_2)\, d\eta_2 = \int_0^{s_2} (L_1 - \eta_2) \sin(\alpha)\, e\, d\eta_2$$

$$= -\sin(\alpha)\, e \left[L_1 \eta_2 - \frac{\eta_2^2}{2}\right]_{\eta_2=0}^{\eta_2=s_2}$$

$$= -\frac{\sqrt{2}}{2} e \left[L_1 s_2 - \frac{s_2^2}{2}\right]$$

$$\rightarrow \tau_{xs_2}(s_2) = q \frac{\sqrt{2}}{2} \left(L_1 s_2 - \frac{s_2^2}{2}\right) \mathrel{\hat{=}} -\tau_{xs_1}$$

4.) Schubspannungen im Abschnitt 3



→ Superpositionsprinzip

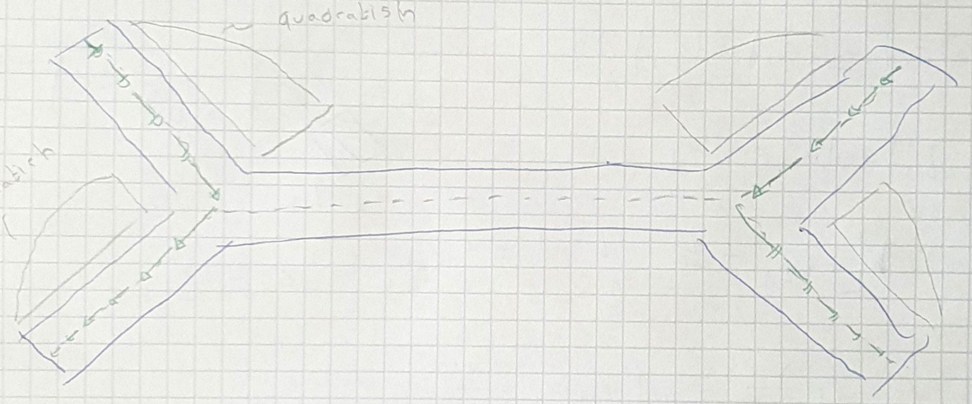
$$\Delta A_1 = \Delta A_2$$

$$H_z^3 = H_z^1 + H_z^2 + H_z^{3'} = y_{S,1} \cdot \Delta A_1 + y_{S,2} \cdot \Delta A_2 + \cancel{y_{S,3'}} \cdot \Delta A_{3'}$$

$$= \Delta A_1 \left(y_{S,1} + y_{S,2} \right) = 0 \qquad \rightarrow \tau_{xs}(s_1 > L_1) = 0$$

$$y_{S,1} = -y_{S,2}$$

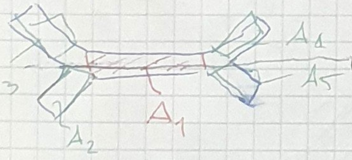
5.) Schubfluss



→ ohnmwardig : entlang der Mittellinie

$$I_z = I_{z''} + A \cdot d^2$$
$$= \frac{eL_1^3}{24} + eL_1 \left(\frac{L_1 \sqrt{2}}{2 \cdot 2} \right)^2$$
$$= eL_1^3 \left(\frac{1}{24} + \frac{1}{8} \right) = \frac{eL_1^3}{6}$$

6.) Extra: I_z berechnen:



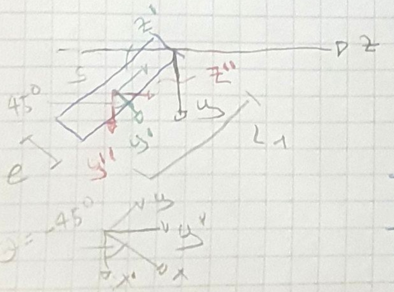
$$\rightarrow I_z^2 = I_z^3 = I_z^4 = I_z^5$$

$$I_z^{tot} = I_z^2 \cdot 4 + I_z^1$$

$$I_1^1 = \frac{L_2 \cdot (\sqrt{2}\, e)^3}{12} = \frac{L_2 \sqrt{2} \cdot 2 e^3}{12} \approx 0 \quad \text{weil } e \ll h \qquad \approx \frac{4}{6} e L_1^3 = \frac{2}{3} e L_1^3$$

Hauptachsen

$$I_{z'} = \frac{L_2 e^3}{12} \approx 0, \quad I_{y'} = \frac{e L_1^3}{12}, \quad C_{yz} = 0$$



$$I_{z''} = I_{y'} \sin^2(\alpha) + \cancel{I_{z'}} \cos^2(\alpha) - 2\, C_{yz} \sin(\theta) \cos(\theta)$$

$$I_{z''} = \frac{e L_1^3}{12} \left(\frac{\sqrt{2}}{2} \right)^2 (-1)^2 = \frac{e L_1^3}{24}$$