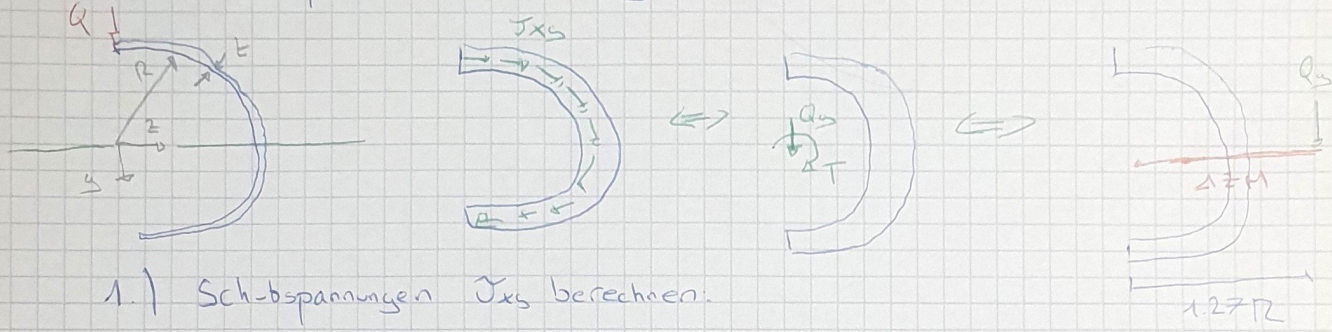
Übung 9 | S1

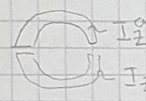
→ Schubmittelpunkt liegt auf Symmetrieachsen



1.) Schubspannungen τ_{xs} berechnen:

$$\tau_{xs}(x,s) = -\frac{Q_y}{I_z}\frac{H_z(s)}{e(s)} \qquad Q_y = Q \qquad e(s) = t$$

$$I_z = \frac{1}{2}\frac{\pi}{4}\left((R+t)^4 - R^4\right) = \frac{\pi}{8}\left((R+t)^2(R+t)^2 - R^4\right)$$



$$I_z^a = I_z^b \rightarrow 2I_z^a = I_z^{ges}$$

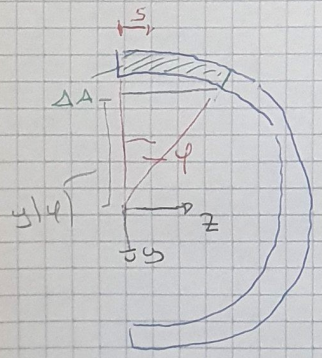
$$= \frac{\pi}{8}\left((R^2+2Rt+t^2)^2 - R^4\right) \qquad t \ll R$$

$$= \frac{\pi}{8}\left(R^4 + 4R^2t^2 + t^4 + 4R^3t + 2R^2t^2 + 4Rt^3 - R^4\right)$$

$$= \frac{\pi}{8}\,4R^3t = \frac{\pi R^3 t}{2}$$

$$H_z(s) = \int_0^s y(\varphi)\,e(\varphi)\,t\,R\,d\varphi = -tR^2\int_0^s \cos(\varphi)\,d\varphi = -tR^2\left[\sin(\varphi)\right]_{\varphi=0}^{\varphi=s}$$

$$= -tR^2\sin(s)$$



$$y(\varphi) = -R\cos(\varphi)$$

$$\tau_{xs}(x,s) = -\frac{Q\cdot 2}{\pi R^2 t}\,\frac{-tR^2\sin(s)}{t} = \frac{2Q}{\pi Rt}\sin(s) \qquad s=\varphi$$

Torsionsmoment berechnen

$$3)\quad T = \Delta x \cdot F = R\cdot\iint_A \tau_{xs}\,dA = R\int_0^{\pi}\int_R^{R+t}\frac{2Q}{\pi Rt}\sin(\varphi)\,\rho\,d\rho\,d\varphi$$

$$= \frac{2Q}{\pi t}\int_0^{\pi}\int_R^{R+t}\sin(\varphi)\,d\rho\,d\varphi = \frac{2Q}{\pi t}\int_0^{\pi}\sin(\varphi)\left[\frac{\rho^2}{2}\right]_{\rho=R}^{\rho=R+t}d\varphi$$

$$= \frac{2Q}{\pi t 2}\int_0^{\pi}\sin(\varphi)\left[R^2+2Rt+t^2-R^2\right]d\varphi = -\frac{2Rt\,Q}{\pi t}\left[\cos(\varphi)\right]_{\varphi=0}^{\varphi=\pi}$$

$$= -\frac{2RQ}{\pi}(-1-1) = \frac{4RQ}{\pi}$$

4.) $\Delta z_H = \frac{T}{a_y} = \frac{4QR}{\pi Q} = \boxed{\frac{4R}{\pi}} \approx 1.27R$