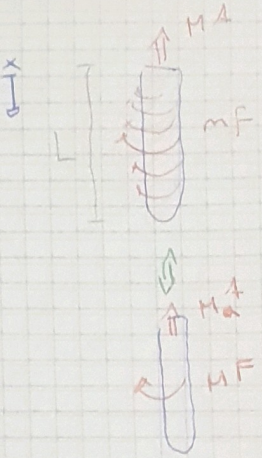
# Übung 9 | S2

1) $m_f = kx \quad \left[\frac{\text{Moment}}{\text{Länge}} = \frac{Nm}{m} = N\right]$




→ Gleichgewichtbedingung: Momentengleichgewicht

$$\Sigma M: \quad M^A = M^F$$

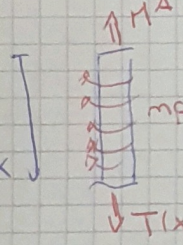
$$M^F = \int_0^L m_f \, dx = \int_0^L kx \, dx =$$

$$\left[k\frac{x^2}{2}\right]_{x=0}^{x=L} = \frac{kL^2}{2} \quad \rightarrow M^A = \frac{kL^2}{2}$$

$$\rightarrow \boxed{k = \frac{2M^A}{L^2}}$$

2.) $$\theta_{max} = \theta(x=L) = \int_0^L \frac{T(x)}{G \cdot I_T(x)} \, dx$$

$$T_T(x) = I_p^{Kreis} = \int_0^{2\pi}\int_0^R \rho^3 \, d\rho \, d\varphi = 2\pi \left[\frac{\rho^4}{4}\right]_{\rho=0}^{\rho=R} = \frac{\pi R^4}{2}$$




$\Sigma M^{im\ Schnittufer}$: $T(x) = M^A - \int_0^x m_f \, dx$

$$= M^A - \int_0^x kx \, dx$$

$$= M^A - k\frac{x^2}{2} = M^A - \frac{2M^A}{L^2}\frac{x^2}{2}$$

$$= M^A\left(1 - \frac{x^2}{L^2}\right)$$

$$\theta_{max} = \int_0^L \frac{M^A\left(1 - \frac{x^2}{L^2}\right)}{G\,\frac{\pi R^4}{2}} \, dx = \frac{2M^A}{\pi R^4 G} \int_0^L 1 - \frac{x^2}{L^2} \, dx =$$

$$\frac{2M^A}{\pi R^4 G}\left[x - \frac{x^3}{3L^2}\right]_0^L = \frac{2M^A}{\pi R^4 G}\left(L - \underbrace{\frac{L^3}{3L^2}}_{=\frac{2}{3}L}\right) = \boxed{\frac{4M^A L}{3\pi R^4 G}}$$

3) $$\theta_{max} \leq \theta_0$$

$$\frac{4M^A_{zul}\,L}{3\pi R^4 G} \leq \theta_0 \rightarrow \boxed{M_{zul} \leq \frac{3\pi R^4 G \theta_0}{4L}}$$