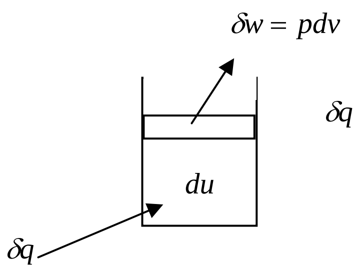
Q. What is specific heat at constant pressure, c_p?

Okay, we could probably stop at c_v, which as is essentially a measure of internal energy. But scientists have found that another definition for specific heat is also convenient.

What if, instead of keeping the previous system at constant volume, we somehow allowed the system to remain at constant *pressure*? We could accomplish this if we set up a piston-cylinder device like that shown below:



$$\delta q - \delta w = du + d(\cancel{KE}) + d(\cancel{PE})$$

$$\Rightarrow \delta q = du + pdv$$

Now, it turns out that the heat that goes into this system is also directly proportional to the system's change in temperature:

$$\delta q (= du + pdv) \propto dT$$

To make this relation an equality, we'll again insert a proportionality constant:

$$\delta q (= du + pdv) = c_p dT$$

We call this constant the *specific heat at constant pressure*. Also, the combination of terms $U + PV$ is so common in Thermodynamics that engineers give this a special name and symbol, *enthalpy* (H) or *specific enthalpy* (h=H/m).

Q. What if the process is not constant pressure? Can we still use c_p?

Technically, the answer is no. We have to allow for the possibility that changes in pressure might affect the enthalpy. Like for specific internal energy, we'll say that the specific enthalpy is a function of two variables, thus time temperature and pressure:

$$h = h(T, p) \quad .$$

Then a more general form for the change in specific enthalpy would be, from calculus,

$$\boxed{dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp \quad .}$$

We will call the first partial derivative the *specific heat at constant pressure*,

$$\boxed{c_p \equiv \left(\frac{\partial h}{\partial T}\right)_p \quad .}$$

(The other partial derivative has no name.) **So h is, in general, a function of temperature and pressure.**

But wait. Under certain conditions the specific enthalpy is approximately a function only of temperature. This is true when the substance can be approximated as either an **incompressible substance** or an **ideal gas**. In these cases the partial derivative, $(\partial h/\partial p)_T$, is negligible, and so

$$dh = \left(\frac{dh}{dT}\right)dT$$

and

$$c_p \equiv \left(\frac{dh}{dT}\right).$$

Note that the partial derivatives have been replaced by ordinary derivatives because h is now a function only of T. We also drop the subscript p because it doesn't matter anymore whether the system is at constant pressure. Then we can write,

$$dh = c_p dT \ .$$