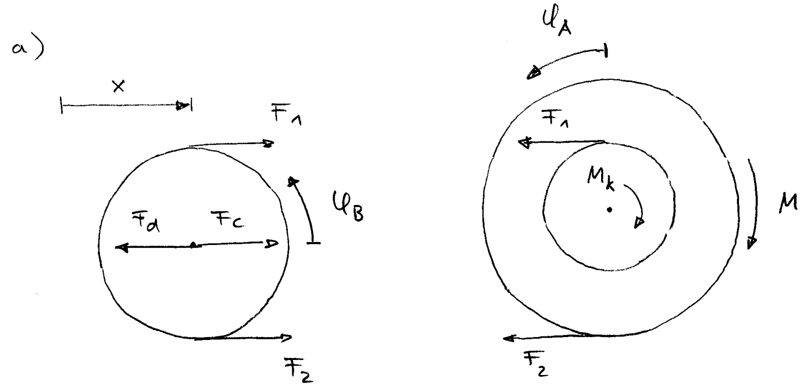
Aufgabe 1

a)



b)

Impulssatz Rolle B:

$$m_B \ddot{x} = F_c - F_d + F_1 + F_2$$

Spinsatz Rolle B:

$$\Theta_B \ddot{\varphi}_B = -3rF_1 + 3rF_2$$

Spinsatz Rolle A:

$$\Theta_A \ddot{\varphi}_A = 2rF_1 - 4rF_2 - M_k - M$$

c)

Kraftgesetze:

$$F_c = -cx \quad ; \quad M_k = k\varphi_A$$

$$F_d = d(\dot{x} - \dot{e})$$

Bindungsgleichungen:

$$\left.\begin{aligned} -2r\varphi_A &= -3r\varphi_B + x \\ 4r\varphi_A &= 3r\varphi_B + x \end{aligned}\right\} \quad \varphi_A = \varphi_B = \frac{x}{r}$$

d) Kraftgesetze einsetzen

$$m_B\ddot{x} = -cx - d\dot{x} + d\dot{e} + F_1 + F_2 \qquad (1)$$

$$\frac{\Theta_B}{3r}\ddot{\varphi}_B = -F_1 + F_2 \qquad (2)$$

$$\frac{\Theta_A}{2r}\ddot{\varphi}_A = F_1 - 2F_2 - \frac{k}{2r}\varphi_A - \frac{M}{r} \qquad (3)$$

Zwangskräfte eliminieren: (1)+3·(2)+2(3):

$$m_B\ddot{x} + \frac{\Theta_B}{r}\ddot{\varphi}_B + \frac{\Theta_A}{r}\ddot{\varphi}_A + cx + d\dot{x} + \frac{k}{r}\varphi_A = d\dot{e} - \frac{M}{r}$$

Bindungsgleichungen einsetzen

$$\underline{\underline{\left(m_B + \frac{\Theta_B}{r^2} + \frac{\Theta_A}{r^2}\right)\ddot{x} + d\dot{x} + \left(c + \frac{k}{r^2}\right)x = d\dot{e} - \frac{M}{r}}}$$

e)

$$\omega_0 = 2\sqrt{\frac{c^*}{m^*}} \quad ; \quad \Omega = 2\sqrt{\frac{c^*}{m^*}} \quad ; \quad \hat{F} = A$$

$$D = \frac{1}{2} d \frac{1}{\sqrt{mc}} = \frac{3}{4} \quad ; \quad \eta = \frac{\Omega}{\omega_0} = 1$$

$$V(\eta, D) = \frac{1}{\sqrt{(1-\eta^2)^2 + (2D\eta)^2}} = \frac{2}{3}$$

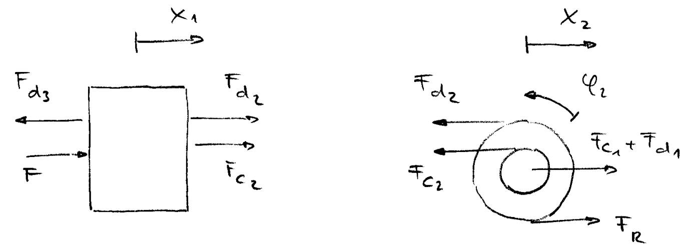
$$\varphi(\eta, D) = \frac{\pi}{2}$$

$$\underline{\underline{y_p(t) = \frac{A}{4c^*} \cdot \frac{2}{3} \cdot \cos\left(2\sqrt{\frac{c^*}{m^*}}\, t - \frac{\pi}{2}\right)}}$$

1

Aufgabe 2

a)



b)

Impulssatz Klotz:

$$m_A \ddot{x}_1 = F_{d_2} + F_{c_2} - F_{d_3} + F$$

Impulssatz Scheibe:

$$m_B \ddot{x}_2 = F_{c_1} + F_{d_1} + F_R - F_{d_2} - F_{c_2}$$

Spinsatz Scheibe:

$$\frac{1}{2} m_B R^2 \ddot{\varphi}_2 = R F_{d_2} + r F_{c_2} + R F_R$$

c)

Kraftgesetze:

$$F_{c_1} = -c_1 x_2$$

$$F_{d_1} = -d_1(\dot{x}_2 + \dot{e})$$

$$F_{c_2} = c_2(x_2 - r\varphi_2 - x_1)$$

$$F_{d_2} = d_2(\dot{x}_2 - R\dot{\varphi}_2 - \dot{x}_1)$$

$$F_{d_3} = d_3 \cdot \dot{x}_1$$

Bindungsgleichung:

$$x_2 = -R\varphi_2$$

2

d) Kraftgesetze einsetzen und Zwangskräfte eliminieren

$$m_A \ddot{x}_1 = 2d_2\dot{x}_2 - d_2\dot{x}_1 + c_2\left(1+\frac{r}{R}\right)x_2 - c_2 x_1 - d_3\dot{x}_1 + F$$

$$m_B\ddot{x}_2 + \frac{1}{2}m_B\ddot{x}_2 = -2d_2\dot{x}_2 + d_2\dot{x}_1 - \frac{r}{R}c_2\left(1+\frac{r}{R}\right)x_2 + \frac{r}{R}c_2 x_1 - d_1\dot{x}_2 - d_1\dot{e} - c_1 x_2 - 2d_2\dot{x}_2 + d_2\dot{x}_1 - c_2\left(1+\frac{r}{R}\right)x_2 + c_2 x_1$$

$$\begin{pmatrix} m_A & 0 \\ 0 & \frac{3}{2}m_B \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} d_2+d_3 & -2d_2 \\ -2d_2 & d_1+4d_2 \end{pmatrix}\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{pmatrix} c_2 & -c_2\left(1+\frac{r}{R}\right) \\ -c_2\left(1+\frac{r}{R}\right) & c_1+c_2\left(1+\frac{r}{R}\right)^2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} F \\ -d_1\dot{e} \end{pmatrix}$$

e)

$$D = \alpha M + \beta K$$

$$\Rightarrow \alpha = \frac{2d}{m} \; ; \quad \beta = \frac{d}{c} \; ; \quad \text{Bequemlichkeit gilt.}$$

f) Eigenwerte des zugehörigen MK-Systems:

$$\det(M\lambda^2 + K) = 0$$

$$\det\begin{pmatrix} m\lambda^2 + c & -2c \\ -2c & 2m\lambda^2 + 9c \end{pmatrix} = 0$$

$$2m^2\lambda^4 + 11mc\lambda^2 + 5c^2 = 0$$

$$\lambda^2_{1,2,3,4} = \frac{-11mc \pm \sqrt{121m^2c^2 - 40m^2c^2}}{4m^2} = \begin{cases} = -\frac{1}{2}\frac{c}{m} \\ = -5\frac{c}{m} \end{cases}$$

$$\lambda^2_{1,2} = -\frac{1}{2}\frac{c}{m} \; ; \quad \lambda^2_{3,4} = -5\frac{c}{m}$$

"Eigenvektor" $\vec{u}_{1,2}$ zu $\lambda_{1,2}^2$ und $\vec{u}_{3,4}$ zu $\lambda_{3,4}^2$:

$$c\begin{pmatrix} \frac{1}{2} & -2 \\ -2 & 8 \end{pmatrix} \vec{u}_{1,2} = 0 \quad \Rightarrow \quad \vec{u}_{1,2} = \begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix}$$

$$c\begin{pmatrix} -4 & -2 \\ -2 & -1 \end{pmatrix} \vec{u}_{3,4} = 0 \quad \Rightarrow \quad \vec{u}_{3,4} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

Beim Mode mit "EV" $\vec{u}_{3,4}$ schwingen die Massen gegeneinander.

g) Um die Differentialgleichungen zu entkoppeln wählen wir folgende Koordinatentransformation :

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{4} & -2 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} ; \quad U = \begin{pmatrix} 1 & 1 \\ \frac{1}{4} & -2 \end{pmatrix}$$

$$U^T M U \ddot{\vec{\xi}} + U^T D U \dot{\vec{\xi}} + U^T K U \vec{\xi} = U^T F$$

$$U^T M U = U^T \begin{pmatrix} m & 0 \\ 0 & 2m \end{pmatrix} \begin{pmatrix} 1 & 1 \\ \frac{1}{4} & -2 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{4} \\ 1 & -2 \end{pmatrix} \begin{pmatrix} m & m \\ \frac{m}{2} & -4m \end{pmatrix} = \begin{pmatrix} \frac{9}{8}m & 0 \\ 0 & 9m \end{pmatrix}$$

$$U^T D U = dU^T \begin{pmatrix} 3 & -2 \\ -2 & 13 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ \frac{1}{4} & -2 \end{pmatrix} = d\begin{pmatrix} 1 & \frac{1}{4} \\ 1 & -2 \end{pmatrix} \begin{pmatrix} \frac{5}{2} & 7 \\ \frac{5}{4} & -28 \end{pmatrix} = \begin{pmatrix} \frac{45}{16}d & 0 \\ 0 & 63d \end{pmatrix}$$

$$U^T K U = cU^T \begin{pmatrix} 1 & -2 \\ -2 & 9 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ \frac{1}{4} & -2 \end{pmatrix} = c\begin{pmatrix} 1 & \frac{1}{4} \\ 1 & -2 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 5 \\ \frac{1}{4} & -20 \end{pmatrix} = \begin{pmatrix} \frac{9}{16}c & 0 \\ 0 & 45c \end{pmatrix}$$

$$U^T F = \begin{pmatrix} 1 & \frac{1}{4} \\ 1 & -2 \end{pmatrix} \begin{pmatrix} F^* \\ -e^* \end{pmatrix} = \begin{pmatrix} F^* - \frac{e^*}{4} \\ F^* + 2e^* \end{pmatrix}$$

$$\frac{9}{8} m \ddot{\xi}_1 + \frac{45}{16} d \dot{\xi}_1 + \frac{9}{16} c \xi_1 = F^*(t) - \frac{e^*(t)}{4}$$

$$9m \ddot{\xi}_2 + 63d \dot{\xi}_2 + 45c \xi_2 = F^*(t) + 2e^*(t)$$