151-0590-00 **Regelungstechnik II** (FS 2008) **Musterlösung**

Übung 9 **State Observer, LQG, Matlab/Simulink**

C.B., 26. Mai 2008

Aufgabe 1 (State Observer)

The system matrices are:

$$A = \begin{bmatrix} 0 & 3 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 2 \end{bmatrix}, \quad D = 0 .$$

a) For the design of the state observer the following Riccati equtaion is used:

$$0 = \frac{1}{q}\,\Psi C^T C \Psi - \Psi A^T - A\Psi - BB^T ,$$

where the matrix Ψ is symmetric and positive definite. And so,

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 2 \begin{bmatrix} \psi_1 & \psi_2 \\ \psi_2 & \psi_3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} \psi_1 & \psi_2 \\ \psi_2 & \psi_3 \end{bmatrix} - \begin{bmatrix} \psi_1 & \psi_2 \\ \psi_2 & \psi_3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix}$$
$$- \begin{bmatrix} 0 & 3 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \psi_1 & \psi_2 \\ \psi_2 & \psi_3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \end{bmatrix}$$
$$= 8 \begin{bmatrix} \psi_2^2 & \psi_2\psi_3 \\ \psi_2\psi_3 & \psi_3^2 \end{bmatrix} - \begin{bmatrix} 6\psi_2 & \psi_1+\psi_2+3\psi_3 \\ \psi_1+\psi_2+3\psi_3 & 2\psi_2+2\psi_3 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix},$$

from which ψ_1, ψ_2 and ψ_3 can be found:

$$\begin{cases} 8\psi_2^2 - 6\psi_2 - 1 = 0 \rightarrow \psi_2 = \frac{6 \pm \sqrt{36+4\cdot 8\cdot 1}}{2\cdot 8} \rightarrow \psi_2 = 0.8904 \text{ or } -0.1404 \\ 8\psi_3^2 - 2\psi_2 - 2\psi_3 - 4 = 0 \rightarrow \psi_3 = \frac{2 \pm \sqrt{4+4\cdot 8\cdot(4+2\psi_2)}}{2\cdot 8} \rightarrow (\psi_3 = 0.9842 \text{ or } -0.7342) \text{ or } (\psi_3 = 0.8182 \text{ or } -0.5682) \\ 8\psi_2\psi_3 - \psi_1 - \psi_2 - 3\psi_3 - 2 = 0 \rightarrow \psi_1 = 1.1677 \end{cases}$$

The transpose of the observer gain matrix is:

$$L^T = \frac{1}{q}\,C\Psi = 2\cdot\begin{bmatrix} 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1.1677 & 0.8904 \\ 0.8904 & 0.9842 \end{bmatrix} = \begin{bmatrix} 3.5616 & 3.9368 \end{bmatrix} .$$

The characteristic polynomial is:

$$\begin{aligned} \det\{sI - [A - LC]\} &= \det\left\{ \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \left(\begin{bmatrix} 0 & 3 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 3.5616 \\ 3.9368 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 \end{bmatrix} \right) \right\} \\ &= \det \begin{bmatrix} s & 4.1232 \\ -1 & s+6.8736 \end{bmatrix} = s(s+6.8736) + 4.1232 \\ &= s^2 + 6.8736s + 4.1232 \rightarrow s_1 = -0.6640 \text{ and } s_2 = -6.2096 \end{aligned}$$

b) For the design of the state observer with noise the following Riccati equtaion is used:

$$0 = A\cdot P + P\cdot A^T - P\cdot C^T \cdot R_y^{-1} \cdot C \cdot P + B \cdot R_u \cdot B^T ,$$

where the matrix P is symmetric and positive definite and $R_u = 4$ and $R_y = 1$. So,

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 1 \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} - \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix}$$
$$- \begin{bmatrix} 0 & 3 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot 4 \cdot \begin{bmatrix} 1 & 2 \end{bmatrix}$$
$$= 4 \begin{bmatrix} p_2^2 & p_2 p_3 \\ p_2 p_3 & p_3^2 \end{bmatrix} - \begin{bmatrix} 6p_2 & p_1 + p_2 + 3p_3 \\ p_1 + p_2 + 3p_3 & 2p_2 + 2p_3 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ 8 & 16 \end{bmatrix},$$

from which p_1, p_2 and p_3 can be found:

$$\begin{cases} 4p_2^2 - 6p_2 - 4 = 0 \rightarrow p_2 = \frac{6 \pm \sqrt{36+4 \cdot 4 \cdot 4}}{2 \cdot 4} \rightarrow p_2 = 2 \text{ or } -0.5 \\ 4p_3^2 - 2p_2 - 2p_3 - 16 = 0 \rightarrow p_3 = \frac{2 \pm \sqrt{4+4 \cdot 4 \cdot (16+2p_2)}}{2 \cdot 4} \rightarrow (p_3 = 2.5 \text{ or } -2) \text{ or } (p_3 = 2.2026 \text{ or } -1.7026) \\ 4p_2 p_3 - p_1 - p_2 - 3p_3 - 8 = 0 \rightarrow p_1 = 2.5 \end{cases}$$

The observer gain matrix is:

$$L = PC^T R_y^{-1} = \begin{bmatrix} 2.5 & 2 \\ 2 & 2.5 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \end{bmatrix} \cdot 1 = \begin{bmatrix} 4 \\ 5 \end{bmatrix}.$$

Aufgabe 2 (Cart and Inverted Pendulum with Observer, Matlab/Simulink)

a) Matlab Code:

```
 % Initial conditions

x_0=[0.1 0 0 0]';

xhat_0=[0 0 0 0]';

% Weighting matrices

Q=eye(4)
Q(1,1)=1000;

r=0.001;

q=1;

% Observer gain

[L]=lqr(A',C',B*B',q);
L=L';

% Controller gain

[K]=lqr(A,B,Q,r);
```

b) Matlab Code:

```
 % Initial conditions

x_0=[0 0 0 0]';

xhat_0=[0 0 0 0]';

% Weighting matrices

Q=eye(4);

r=0.001;

q=1;

% Observer gain

[L]=lqr(A',C',B*B',q);
L=L';

% Controller gain

[K]=lqr(A,B,Q,r);

% Integral action

gamma=500;

A_tilde=[A zeros(4,1); -C 0];

B_tilde=[B; 0];

C_tilde=[sqrt(Q) zeros(4,1); zeros(1,4) gamma];

K_tilde=lqr(A_tilde,B_tilde,C_tilde'*C_tilde,R);
K=K_tilde(1:4);
KI=-[K_tilde(5)];
```

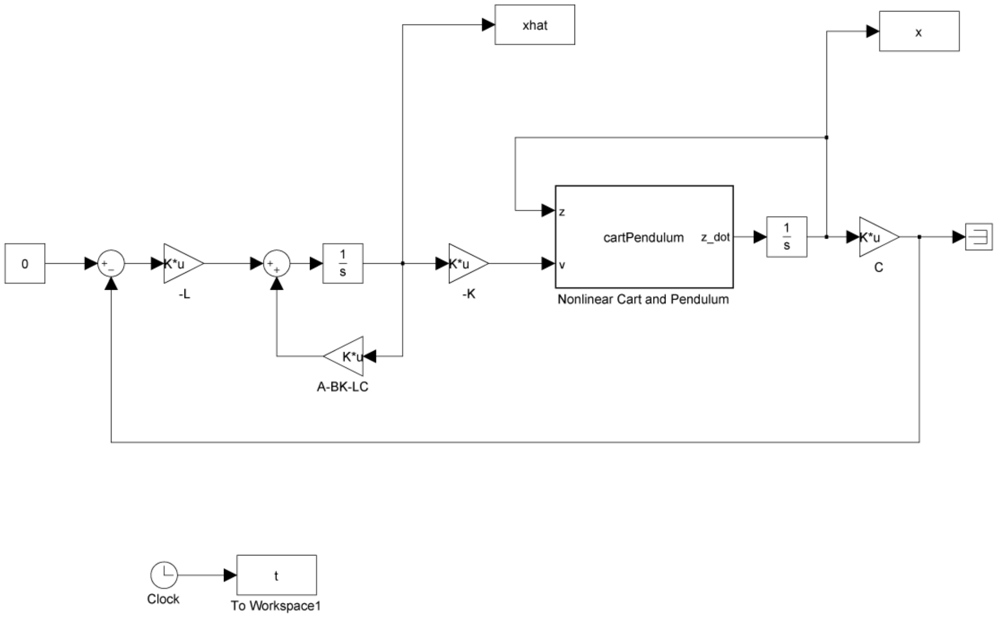

Abbildung 1: Simulink Model (a)

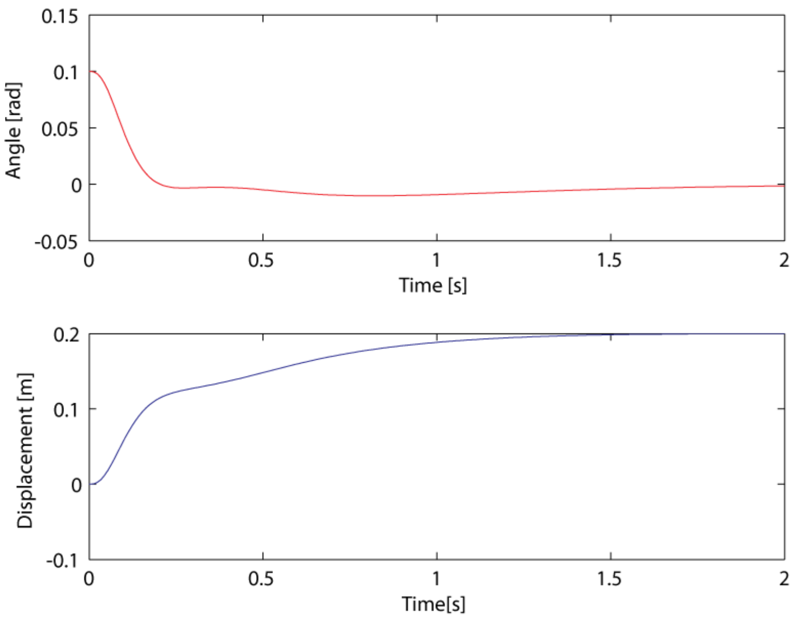

Abbildung 2: Response (a)

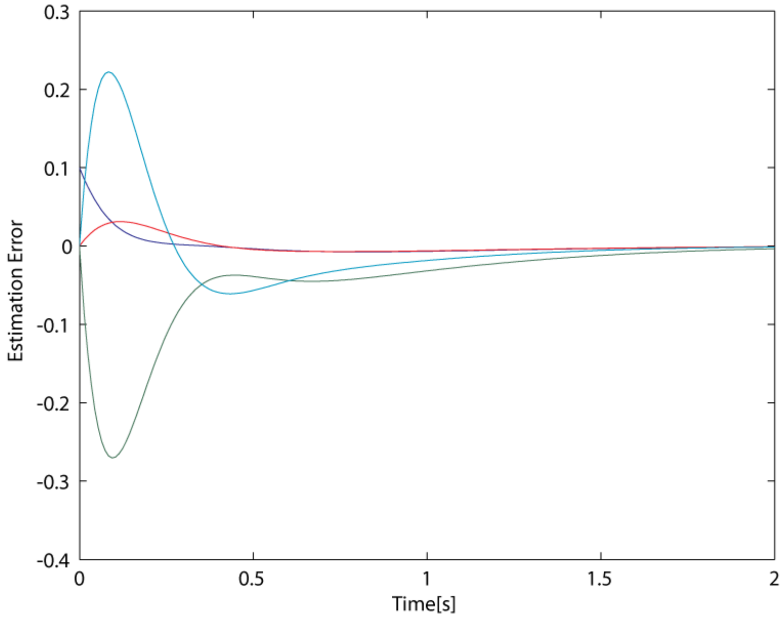

Abbildung 3: Estimation error (a)

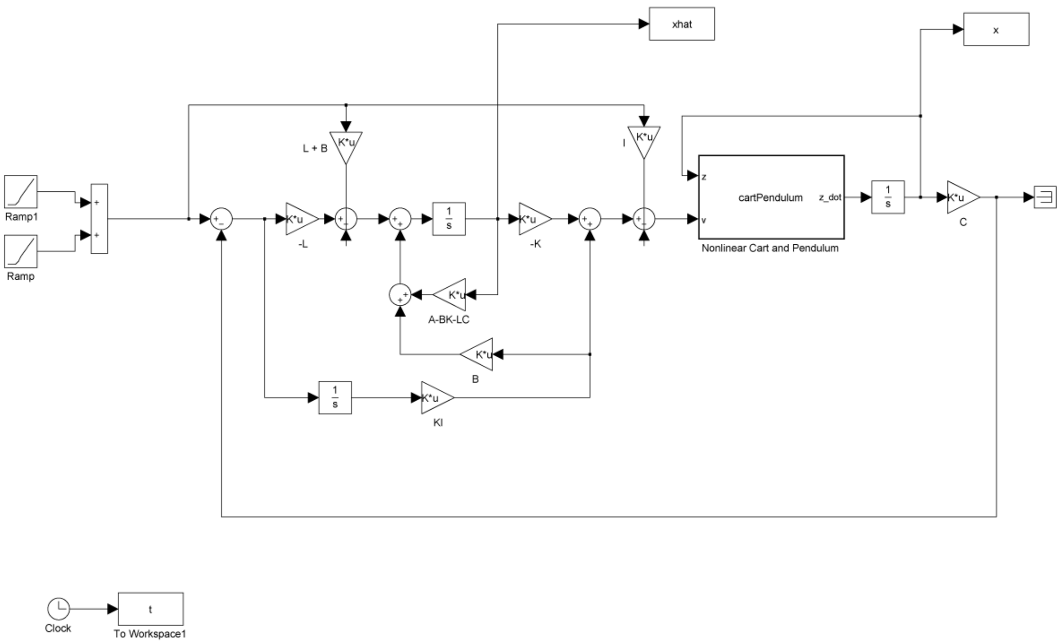

Abbildung 4: Simulink Model (b)

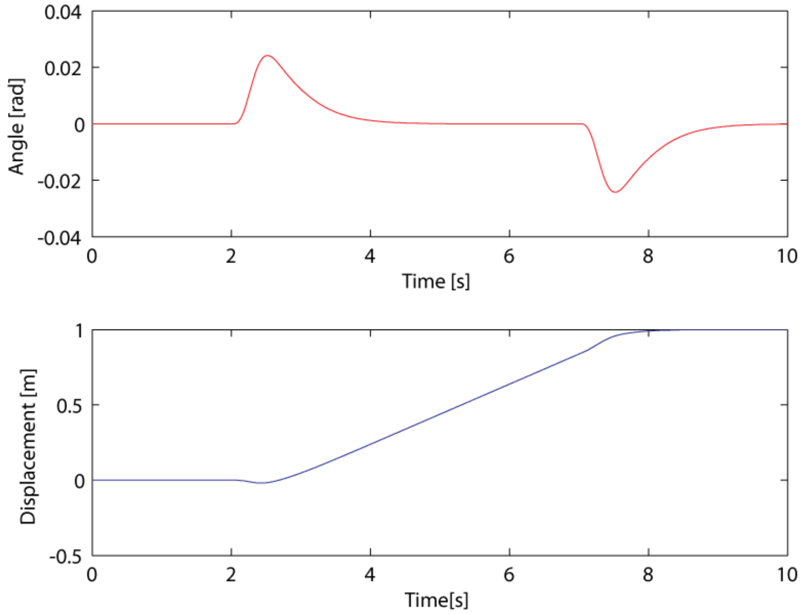

Abbildung 5: Response (b)

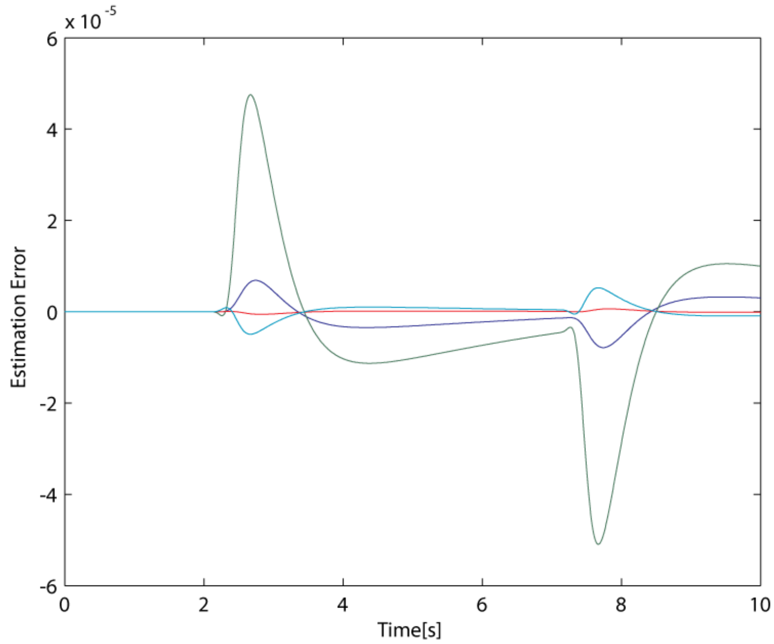

Abbildung 6: Estimation error (b)