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151-0590-00 **Regelungstechnik II** (FS 2008) **Übung 10**

Thema: **LQG/LTR**

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Aufgabe 1 (LQG/LTR)

Consider an Otto engine running stationarily at 900 rpm with a torque of 10 Nm. The three inputs are the demanded throttle (Drosselklappe) position $\alpha_{th,d}$, the amount of injected fuel F_λ and the ignition time α_i (Zündzeitpunkt)[1]. The outputs of the system are the engine speed n and the air to fuel ratio λ. A nonlinear model of the engine has been modeled and then linearized around the operation point. The inputs of the linear system are the deviations from the nominal values:

$$v_1(t) = \Delta\alpha_{th,d}(t), \quad v_2(t) = \Delta F_\lambda(t), \quad v_3(t) = \Delta\alpha_i(t).$$

The outputs of the linear system are:

$$w_1(t) = \Delta n(t), \quad w_2(t) = \Delta\lambda(t)$$

The four state variables describe the deviations from the nominal points

$$z_1(t) = \Delta\alpha_{th}(t), \quad z_2(t) = \Delta p_s(t), \quad z_3(t) = \Delta n(t), \quad z_4(t) = \Delta\lambda(t).$$

where p_s is the pressure in the intake manifold (Saugrohr). The resulting state space model can be written as:

$$\dot{z}(t) = \begin{bmatrix} -25 & 0 & 0 & 0 \\ 0.261 & -4 & -0.002 & 0 \\ 618 & 7640 & -3.07 & 0 \\ -0.384 & 5.89 & 0.003 & -2.1 \end{bmatrix} \cdot z(t) + \begin{bmatrix} 25 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 3384 & 78.056 \\ 0 & -2.1 & 0 \end{bmatrix} \cdot v(t)$$
$$w(t) = C_p \cdot z(t).$$

An LQG/LTR controller has to be designed such that the following specifications are met:

- Cross-over frequency of the control system is about 1 rad/s.
- For robustness reasons the largest singular value of the sensitivity of the control system should always be smaller than 3 dB for all frequencies.
- The frequency spectrum of the measurement noise “starts” at 20 rad/s, therefore the loop gain should be below -20 dB at this frequency.
- No static errors in the air to fuel ratio as well as no stationary corrections of the ignition time.

[1] The ignition time is specified in the crank angle domain (Kurbelwellenwinkel)

Proceed according to the following steps:

a) Find the matrix C_p.

b) Normalize the system with the nominal values:

$$z_{1,0} = 1^\circ, \quad z_{2,0} = 0.05\,\text{bar}, \quad z_{3,0} = 200\,\text{rpm}, \quad z_{4,0} = 0.05, \quad v_{1,0} = 1^\circ, \quad v_{2,0} = 0.05, \quad v_{3,0} = 1.44^\circ$$

c) To avoid static errors and corrections the plant is augmented with a PI-element with $T_i = 2$ s at each of the inputs $\tilde{u}_1$ and $\tilde{u}_2$ of the nominal plant. The PI-elements have the following state space representation:

$$\begin{bmatrix} \dot{x}_5(t) \\ \dot{x}_6(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_5(t) \\ x_6(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}$$

$$\begin{bmatrix} \tilde{u}_1(t) \\ \tilde{u}_2(t) \\ \tilde{u}_3(t) \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_5(t) \\ x_6(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}$$

where $\tilde{u}(t)$ is the input vector of the nominal plant and $u(t)$ is the input vector of the PI-elements. Compute the system matrices A, B and C of the augmented system.

d) Analyze the plant and conditions for the LQR problem.

e) Compute the observer gain matrix and choose q such that the given specifications are met.

f) Calculate the state feedback matrix and choose the "loop transfer recovery" parameter r such that the specifications of the system are "recovered".

g) Compute the "de-normalized" system matrices of the resulting controller.

h) Simulate the response of the linear, physical system for an initial deviation of the rotational speed Δn of +500 rpm.

Use the prepared Matlab® files[2] for the questions **b)** to **h)**.

[2] *http://www.imrt.ethz.ch/education/lectures/control_systems2/lecture_materials* → Matlab Code