151-0590-00 **Regelungstechnik II** (FS 2008) **Übung 8**

Thema: **Linear Quadratic Regulators**

Ausgabe: 8. 05. 07 Vorbesprechung: 9. 05. 07 Abgabe: 16. 06. 07 Nachbesprechung: 23. 06. 07

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Aufgabe 1 (Sufficient Conditions)

For the existence of a meaningful solution to the LQR problem several conditions must be fulfilled. Determine whether the following state space and weighting matrices are well posed for the problem and furthermore guarantee a solution. Assume full state feedback.

a)

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ -2 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \quad Q = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 1 \end{bmatrix}$$

c)

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad Q = \begin{bmatrix} 4 & 0 \\ 4 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 2 \end{bmatrix}$$

Aufgabe 2 (LQR-Problem)

Given the following second order plant

$$\begin{aligned} \dot{x}_1(t) &= -2x_1(t) + u(t) \\ \dot{x}_2(t) &= -2x_1(t) \end{aligned}$$

with the objective function

$$J = \int_0^\infty \left[x_1^2(t) + x_2^2(t) + u^2(t) \right] dt$$

Assume that full state feedback is possible and that the problem is well posed.

a) Is the plant stable?

b) Find the solution Φ to the algebraic matrix Riccati equation and the corresponding controller matrix K.

c) Determine the poles of the resulting closed-loop system.

Aufgabe 3 (Cart and Inverted Pendulum, Matlab/Simulink)

The cart and inverted pendulum is a nonlinear system. The goal is to maintain the pendulum in the vertical orientation whilst changing the cart position. The open-loop plant is highly unstable. In this exercise full state feedback control will be implemented using a linearized model of the plant.

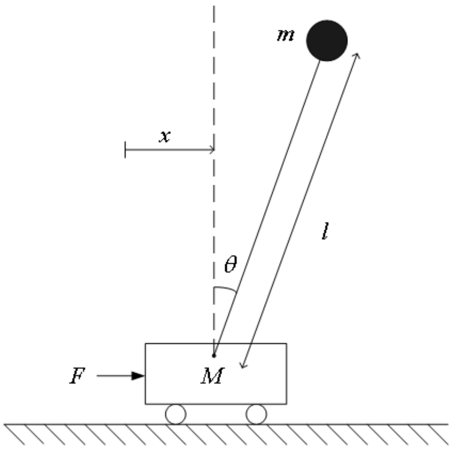

Abbildung 1: Cart and Inverted Pendulum

The nonlinear equations take the form

$$\dot{z}(t) = f(z(t), v(t)) = \begin{pmatrix} z_2 \\ \dfrac{ucos(z_1) - (M+m)gsin(z_1) + ml(cos(z_1)sin(z_1))z_2^2}{mlcos^2(z_1) - (M+m)l} \\ z_4 \\ \dfrac{v + mlsin(z_1)z_2^2 - mgcos(z_1)sin(z_1)}{M + m - mcos^2(z_1)} \end{pmatrix}$$

where $z(t) = \begin{pmatrix} \theta(t) \\ \dot{\theta}(t) \\ d(t) \\ \dot{d}(t) \end{pmatrix}$ and $v(t) = F$.

These can be linearized around the equilibrium position giving the state-space matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{mg}{M} & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -\dfrac{1}{Ml} \\ 0 \\ \dfrac{1}{M} \end{bmatrix}$$

a) Design an LQR controller with suitably designed weighting matrices to maintain the pendulum in the upright position (regulation of the cart position is not important). Using the provided nonlinear Simulink model and Matlab script run a simulation from an initial angle $\theta(0) = 0.1\ rad$. Which weighting produces the best performance? Plot the response.

b) The cart must now move from position $d = 0m$ to $d = 1m$ over 5 seconds (a ramp function). Extend your controller to include integral action and modify the weighting matrices. Run the simulation again with suitably chosen weighting and plot the response. Give a physical interpretation for the limited tracking performance of the cart in this case.