

→ Power in a capacitor:

$$P_c = V_c \cdot i_c$$

$$P_m = \frac{\omega C V_m^2}{2}$$

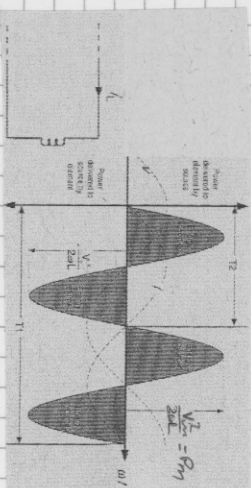
$$= V_m \cdot \sin(\omega t + \theta) \cdot \omega C V_m \cos(\omega t + \theta)$$

$$= \frac{\omega C V_m^2}{2} \sin(2\omega t + 2\theta)$$

$$P_c = \frac{\omega C V_m^2}{2} \int_0^{2\pi} \sin(2\omega t + 2\theta) dt = 0$$

mittlere Leistung

• Power in an Inductor



The net flow of power to the pure inductor is zero over a full cycle

$$P_L = V_L \cdot i_L$$

$$= \frac{V_m}{\omega L} \cdot \sin(\omega t + \theta) \cdot \cos(\omega t + \theta)$$

$$= -\frac{V_m^2}{2\omega L} \sin(2\omega t + 2\theta)$$

$$P_m = \frac{V_m^2}{2\omega L}$$

$$P_L = -\frac{V_m^2}{2\omega L} \int_0^{2\pi} \sin(2\omega t + 2\theta) dt = 0$$

mittlere Leistung

• Power in an Impedance

First an example:

$$P = i \cdot V = \frac{\omega C \cdot V_m^2}{2} \cdot [\cos(\theta) - \cos(2\omega t + \theta)]$$

$$\theta = 90^\circ - \arctan(\omega R C); \cos(\theta) = \frac{\omega R C}{1 + (\omega R C)^2}$$

$$\bar{P} = \frac{(V_m \cos(\theta))^2}{2R} = \frac{V_R^2}{2R}$$

Instan. Leistung:

$$P_R(t) = V_R(t) \cdot I_R(t)$$

The general calculation for the power in an impedance goes similarly

$$P = i \cdot V = \frac{V_m \cos(\theta)}{2} \cdot \frac{V_m \sin(\theta)}{2} \cdot \sin(2\omega t)$$

$$\theta = -\arctan\left(\frac{X_i}{R}\right) \quad \left[P_{\text{inst}} = V_m i_m \cos(\theta) - \cos(2\omega t + \theta) \right]$$

→ für $\bar{P}$ = RMS Amplitude

Apparent Power (Scheinleistung)

$$S = \frac{V_m \cdot I_m}{2} = \frac{V_m^2}{2|Z|} = \frac{I_m^2 |Z|}{2} \quad [VA] \quad S = I_{\text{eff}} \cdot V_{\text{eff}}$$

RMS Amplitude (Root Mean Square, Effektiv)

$$I_{\text{eff}} = \frac{I_m}{\sqrt{2}} \quad V_{\text{eff}} = \frac{V_m}{\sqrt{2}}$$

Average Power (Wirkleistung)

$$P = \cos(\theta) \cdot S \quad \theta = \arctan\left(\frac{X_i}{R}\right) = \arctan\left(\frac{\omega L}{R}\right)$$

Reactive Power (Blindleistung)

$$Q = S \cdot \sin(\theta)$$

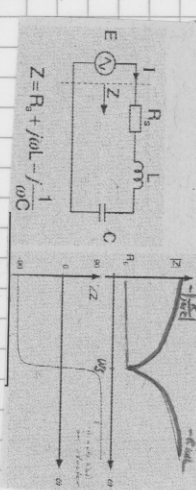
Power Factor (Leistungsfaktor)

$$\text{PF} = \cos(\theta) = \frac{P}{S}$$

Systems with low power factor need to be driven by equipment with high rating

→ Resonant Circuits

• Series Resonant Circuit



$$\text{resonant frequency: } f_s = \frac{\omega_s}{2\pi} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$$

at resonance:

$$\left[\frac{V}{R_s} \right]$$

power factor:

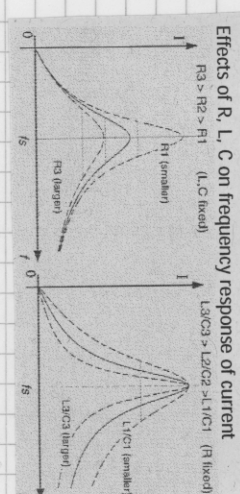
$$\text{PF} = \frac{P}{S} = 1$$

quality factor:

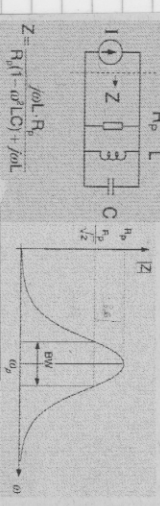
$$Q = \frac{\text{reactive power}}{\text{average power}} = \frac{1}{R_s} \sqrt{\frac{L}{C}} = \frac{\omega_s L}{R_s}$$

bandwidth:

$$BW_{-3dB} = \frac{R_s}{L} = \frac{\omega_s}{Q} \quad Q = \frac{1}{R_s \omega_s C}$$



• Parallel Resonant Circuit



resonant frequency:

$$f_P = \frac{\omega_P}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

at resonance:

$$Z = R_P$$

quality factor:

$$Q = \frac{R_P}{\omega_P L} = R_P \omega_P C$$

bandwidth:

$$BW_{-3dB} = \frac{\omega_P}{Q} = \frac{1}{CR_P}$$

Spulengüte (Quality-Factor-Spule)

$$Q_L = \frac{\omega_P L}{R_L} \quad \text{in Serie}$$

$$R'_L = R_L (1 + Q_L^2)$$

→ von allen Spulen!

$$R'_L = R_L (1 + Q_L^2) \rightarrow \left[\frac{\omega_P L}{R_L} \right]^2 = C$$

Neuer Widerstand $R'_S = R_L || R_P$

→ Power in a capacitor:

$$P_c = V_c \cdot i_c$$

$$P_m = \frac{V_c V_m}{2}$$

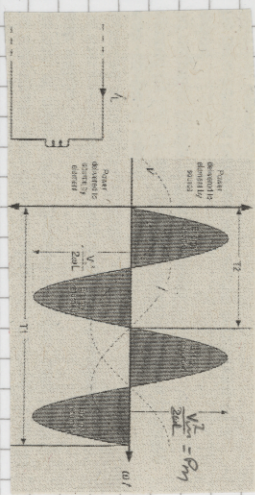
$$= V_m \cdot \sin(\omega t + \theta) \cdot \omega C V_m \cos(\omega t + \theta)$$

$$= \frac{\omega C V_m^2}{2} \sin(2\omega t + 2\theta)$$

$$P_c = \frac{\omega C V_m^2}{2} \int_0^{2\pi} \sin(2\omega t + 2\theta) dt = 0$$

Wahlweise Leistung

Power in an Inductor



The net flow of power to the pure inductor is zero over a full cycle.

$$P_L = V_L \cdot i_L$$

$$= -\frac{V_m}{\omega L} \cdot \sin(\omega t + \theta) \cdot \cos(\omega t + \theta)$$

$$= -\frac{V_m^2}{2\omega L} \sin(2\omega t + 2\theta)$$

$$P_m = \frac{V_m^2}{2\omega L}$$

$$P_L = -\frac{V_m^2}{2\omega L} \int_0^{2\pi} \sin(2\omega t + 2\theta) dt = 0$$

Wahlweise Leistung

Power in an Impedance

First an example:

$$P = i \cdot V = \frac{V_c \cdot V_m}{2f(1 + (\omega RC)^2)} \cdot [\cos(\theta) - \cos(2\omega t + \theta)]$$

$$\theta = 90^\circ - \arctan(\omega RC); \cos(\theta) = \frac{\omega RC}{1 + (\omega RC)^2}$$

$$P = \frac{(V_m \cos(\theta))^2}{2R} = \frac{V_c^2}{2R}$$

$$P_R(t) = V_R(t) \cdot I_R(t)$$

The general calculation for the power in an impedance goes similarly

$$P = i_c \cdot V_c = \frac{[V_m \cos(\theta)]^2}{2R} [1 - \cos(2\omega t)] + \frac{V_m \sin(\theta)^2}{2R} \sin(2\omega t)$$

$$\theta = \arctan\left(\frac{Z_i}{Z_R}\right)$$

for $Z_i = \text{RMS Amplitude}$

Apparent Power (Scheinleistung)

$$S = \frac{V_m \cdot V_m}{2} = \frac{V_m^2}{2} = \frac{V_m^2}{Z}$$

[VA] S = $V_{\text{eff}} \cdot I_{\text{eff}}$

RMS Amplitude (Root Mean Square, Effektiv)

$$I_{\text{eff}} = \frac{I_m}{\sqrt{2}} \quad V_{\text{eff}} = \frac{V_m}{\sqrt{2}}$$

Average Power (Wirkleistung)

$$P = \cos(\theta) \cdot S$$

$$\theta = \arctan\left(\frac{X(\omega)}{R(\omega)}\right)$$

Reactive Power (Blindleistung)

$$Q = S \cdot \sin(\theta)$$

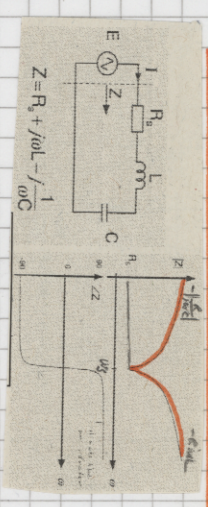
Power Factor (Leistungsfaktor)

$$\text{PF} = \cos(\theta) = \frac{P}{S}$$

Systems with low power factor need to be driven by equipment with high ratings

Resonant Circuits

Series Resonant Circuit



Resonant Frequency:

$$f_s = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$$

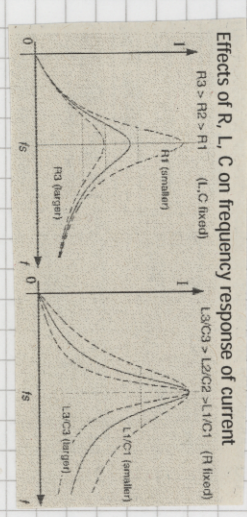
$$\Gamma = \frac{Z}{R_s}$$

$$\text{power factor: } \text{PF} = \frac{P}{S} = 1$$

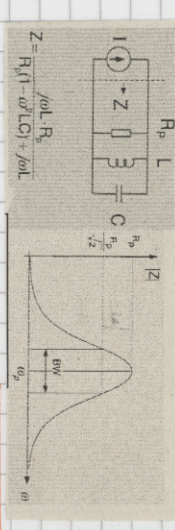
$$\text{quality factor: } Q = \frac{\text{Reactive Power}}{\text{Average Power}} = \frac{1}{R_s} \frac{V_c^2}{I^2} = \frac{V_c \cdot V_c}{R_s \cdot I^2}$$

$$\text{bandwidth: } BW_{3dB} = \frac{R_s}{L} = \frac{\omega_s}{Q}$$

$$Q = \frac{1}{R_s \cdot \omega_s \cdot C}$$



Parallel Resonant Circuit



$$\text{resonant frequency: } f_p = \frac{\omega_p}{2\pi} = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$

$$\text{at resonance: } Z = R_p$$

$$\text{quality factor: } Q = \frac{R_p}{\omega_p L} = Q_{\text{parallel C}}$$

$$\text{bandwidth: } BW_{3dB} = \frac{\omega_p}{Q} = \frac{1}{CR_p}$$

Spangweite (Quality-factor-Span)

$$Q_L = \frac{\omega_p L}{R_L} \quad \text{in Serie}$$

$$R_L' = R_L (1 + Q_L^2)$$

Q von allen Spulen!

$$R_L' = R_L (1 + Q_L^2) \rightarrow R_L' \parallel R_p$$

$$R_s = R_L' \parallel R_p$$

10. Filters and the Operational Amplifiers

Any combination of passive (R, L, or C) or active (transistor or op-amp) elements designed to select or reject a band of frequencies is called a filter

Lowpass Filters

$\frac{V_o}{V_i} = \frac{1}{1 + j\omega RC}$
 $\left| \frac{V_o}{V_i} \right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$
 $f_c = \frac{1}{2\pi RC}$

stopband
passband
 f_c
 $\omega = f$

Circuit: $V_i \rightarrow R \rightarrow \text{op-amp} \rightarrow C \rightarrow V_o$

$\angle \frac{V_o}{V_i} = -\arctan(\omega RC)$
 $\left| \frac{V_o}{V_i} \right| = 20 \log_{10} \left| \frac{V_o}{V_i} \right| = -10 \log_{10} [1 + (\omega RC)^2]$

$\frac{V_o}{V_i} = \frac{1}{1 + j\omega RC}$

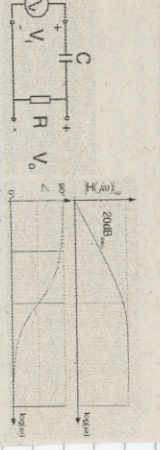
(KCL): $\frac{V_i - V_o}{R} + j\omega C V_o + j\omega C V_o = 0$

$H(j\omega) = \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC + (j\omega RC)^2}$

Highpass - Filters

$H(j\omega) = \frac{V_o}{V_i} = \frac{j\omega RC}{1 + j\omega RC}$
 $\left| H(j\omega) \right| = \frac{j\omega RC}{1 + j\omega RC}$

stopband
passband
 f_c



$H(j\omega) = \frac{j\omega RC}{1 + (j\omega RC)^2}$
 $\angle H(j\omega) = 90^\circ - \arctan(\omega RC)$

$H(\omega) = \frac{V_o}{V_i} = \frac{(j\omega RC)^2}{1 + 3j\omega RC + (j\omega RC)^2}$

stopband
passband
 f_c

Bandpass Filters

$H(j\omega) = \frac{j\omega RC}{1 + 3j\omega RC + (j\omega RC)^2}$
 $\left| H(j\omega) \right| = \frac{1}{\sqrt{1 + 9(\omega RC)^2}}$

stopband
passband
 f_c

A bandpass filter is a combination of highpass and lowpass filters

Circuit: $V_i \rightarrow C \rightarrow R \rightarrow \text{op-amp} \rightarrow C \rightarrow R \rightarrow V_o$

$H(j\omega) = \frac{j\omega RC}{1 + 3j\omega RC + (j\omega RC)^2}$
 $\left| H(j\omega) \right| = \frac{1}{\sqrt{1 + 9(\omega RC)^2}}$

$\left| H(j\omega) \right| = 1$
 $BW = \frac{1}{RC} = \frac{1}{RC}$

$H(j\omega) = \frac{j\omega \frac{1}{RC}}{1 + j\omega \frac{1}{RC} + (j\omega)^2 \frac{1}{RC^2}} = \frac{j\omega \frac{1}{RC}}{\omega^2 + j\omega \frac{1}{RC} + (j\omega)^2}$

Operational Amplifier (op-amp)

Symbol
 Equivalent Circuit

Virtual Ground Concept: If the output voltage V_o of an op-amp is finite, then

$V_i = \lim_{A \rightarrow \infty} \frac{V_o}{A} = 0$

Inverting Amplifier

$I_1 + I_2 = \frac{V_i}{R_1} + \frac{V_{out}}{R_2} = \frac{V_i}{R_1} + \frac{V_o}{R_2} = 0$
 $\frac{V_o}{V_i} = -\frac{R_2}{R_1}$

Non-inverting Amplifier

$\frac{V_{out} - V_{in}}{R_2} = \frac{V_{in}}{R_1} \Leftrightarrow \frac{V_{out}}{R_2} = V_{in} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$
 $\frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$

Integrator

$\frac{V_o}{R} + j\omega C \cdot V_o = 0 \Rightarrow \frac{V_o}{R} = -\frac{1}{j\omega RC}$
 $\frac{V_o}{R} = -C \frac{\partial V_o}{\partial t} \Rightarrow V_o = \frac{1}{RC} \int V_i dt + K$

Differentiator

$\frac{V_o}{R} + j\omega C \cdot V_o = 0 \Rightarrow \frac{V_o}{R} = -j\omega RC$

Op-amp-based Circuits

1. First-order Lowpass Filters

$H(j\omega) = -\frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega RC}$

$\left| H(j\omega) \right|_{DC} = \frac{R_2}{R_1}$
 $f_c = \frac{1}{2\pi R_2 C}$

2. First-order Highpass Filters

$\left| H(j\omega) \right|_{DC} = \frac{R_2}{R_1}$

2. First-order highpass-filters

$$H(j\omega) = -\frac{j\omega R C_1}{1 + j\omega R C_2}$$

$$|H(j\omega)|_{\omega \rightarrow \infty} = \frac{C_1}{C_2} \quad P_c = \frac{1}{2\pi R C_2}$$

4.1. The Synthesis of Filters

1. Specifications of Filters

2. The Approximation Problem

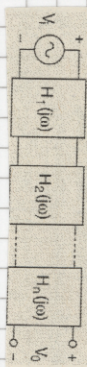
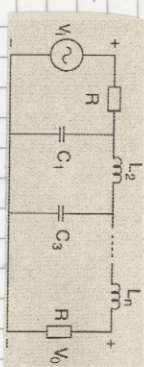
$$|H(\omega)| = \left| \frac{b_n U(\omega)^n + b_{n-1} U(\omega)^{n-1} + \dots + b_1 U(\omega) + b_0}{a_n U(\omega)^n + a_{n-1} U(\omega)^{n-1} + \dots + a_1 U(\omega) + a_0} \right|$$

$$|H(\omega)| = \left| \frac{b_2 U(\omega)^2 + b_1 U(\omega) + b_0}{a_2 U(\omega)^2 + a_1 U(\omega) + a_0} \right| \cdot |H_2(\omega)| \cdot \dots \cdot |H_n(\omega)|$$

BROADBAND SECTION

3. Implementation

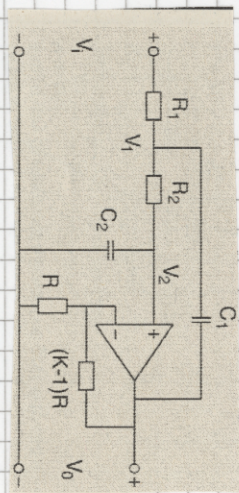
A. Ladder Filters



B. Cascade of Biquadratic Sections

⇒ Biquadratic Filters

• Sallen-Key Lowpass Biquad



$$\frac{V_1 - V_i}{R_1} + \frac{(V_1 - V_0) \cdot j\omega C_1}{1} + \frac{V_1 - V_2}{R_2} = 0$$

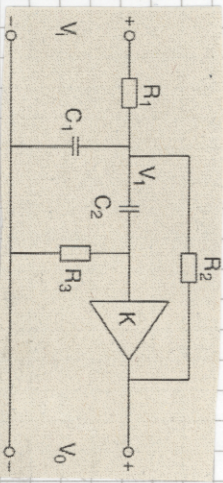
$$V_2 = V_1 \cdot \frac{1}{1 + j\omega R_2 C_2} = \frac{V_0}{K}$$

$$H(j\omega) = \frac{V_0}{V_1} = \frac{K}{(j\omega)^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-K}{R_2 C_2} \right) j\omega + \frac{1}{R_1 R_2 C_1 C_2}}$$

Tip!

$$+ \frac{1}{R_1 R_2 C_1 C_2}$$

• Single-amplifier Bandpass Biquad

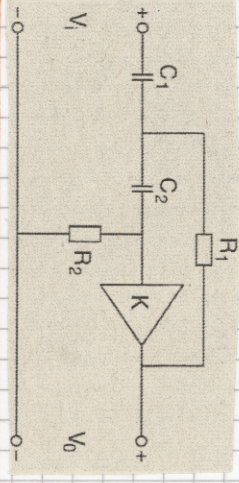


$$\frac{V_1 - V_i}{R_1} + V_1 \cdot j\omega C_1 + (V_1 - V_0) \cdot j\omega C_2 + \frac{V_1 - V_0}{R_2} = 0$$

$$\frac{V_0}{K} = V_1 \cdot \frac{j\omega R_3 C_2}{1 + j\omega R_2 C_2}$$

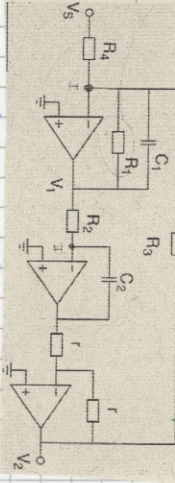
$$H(j\omega) = \frac{K \cdot (j\omega)^2}{(j\omega)^2 + \left(\frac{1}{R_2 C_1} + \frac{1-K}{R_2 C_1} + \frac{1}{R_1 C_1} \right) j\omega + \frac{R_1 + R_2}{R_1 R_2 C_1 C_2}}$$

• Single-amplifier Highpass Biquad



$$H(j\omega) = \frac{K \cdot (j\omega)^2}{(j\omega)^2 + \left(\frac{1}{R_2 C_1} + \frac{1-K}{R_2 C_1} + \frac{1}{R_1 C_1} \right) j\omega + \frac{1}{R_1 R_2 C_1 C_2}}$$

⇒ The Tow-Thomas Biquad



$$\frac{V_1 - V_i}{R_1} + V_1 \cdot j\omega C_1 + \frac{V_1 - V_0}{R_2} = 0 \quad \frac{V_1 - V_0}{R_2} - j\omega C_2 V_2 = 0$$

$$H_{LP}(j\omega) = \frac{V_0}{V_1} = -\frac{R_2 R_4 C_1 C_2}{(j\omega)^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} \right) j\omega + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$H_{HP}(j\omega) = \frac{V_0}{V_1} = \frac{R_2 R_4 C_1 C_2}{(j\omega)^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} \right) j\omega + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$\omega_0^2 = \frac{1}{R_2 R_3 C_1 C_2} \quad Q = \omega_0 R_1 C_1$$

$$H_{LP}(s) = -\frac{R_2}{R_1} \quad H_{HP}(j\omega_0) = -\frac{R_1}{R_2}$$

⇒ Step Response of Second order Systems

• Characteristic Equations

$$s^2 + \frac{\omega_p}{Q} s + \omega_p^2 = 0 \quad s_{1,2} = -\frac{\omega_p}{2Q} \pm j\omega_p \sqrt{1 - \frac{1}{4Q^2}}$$

$$V_s = -\frac{R_2}{R_1} j\omega_p^2 V_s = \begin{cases} 0, & t < 0 \\ \text{step}, & t \geq 0 \end{cases}$$

• Overdamped System

$$\left(\frac{1}{2Q} \right)^2 > 1 \Rightarrow Q < \frac{1}{2}$$

→ Homogeneous Solution

$$V_2(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$K_1 = \frac{s_2}{s_1 - s_2} \quad K_2 = \frac{s_1}{s_2 - s_1}$$

• Critically Damped System

$$Q = \frac{1}{2} \quad s_1 = s_2 = -\omega_p$$

→ Homogeneous Solution

$$V_2(t) = (K_1 + K_2 t) \cdot e^{-\omega_p t}$$

→ Particular Solution: $\omega_p K_1 = -1$

• Underdamped System

$$Q > \frac{1}{2} \quad \left(\frac{1}{2Q} \right)^2 < 1 \quad \omega_d = \omega_p \sqrt{1 - \left(\frac{1}{2Q} \right)^2}$$

Homogeneous Solution

$$V_2(t) = K \cdot e^{-\frac{\omega_p}{2Q} t} \cos(\omega_d t - \theta)$$

Particular Solution:

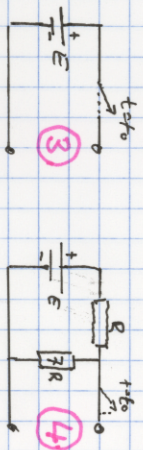
$$\theta = \tan^{-1} \left(\frac{2}{1/(2Q)^2 - 1} \right) \quad K = \sqrt{\frac{(2Q)^2 + 3}{(2Q)^2 - 1}}$$

$$\Rightarrow \text{Marco Weber D-MAT 3. Semester}$$

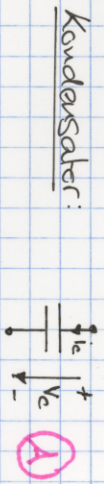
Kapazitäten und Induktivitäten



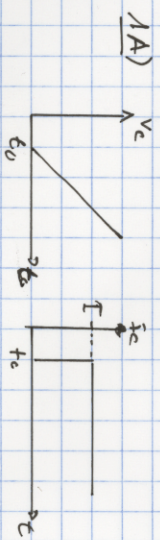
1)



2)

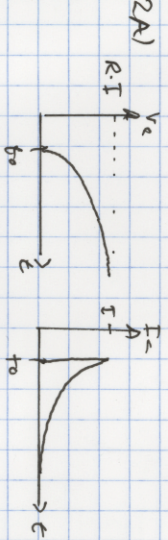


3)



4)

1A) $u_C(t) = 0$ for $t < t_0$
 $u_C(t) = E$ for $t \geq t_0$
 $i_C(t) = 0$ for $t < t_0$
 $i_C(t) = \frac{E}{R}$ for $t \geq t_0$



2A) $u_C(t) = 0$ for $t < t_0$
 $u_C(t) = E \cdot (1 - e^{-\frac{t-t_0}{RC}})$ for $t \geq t_0$
 $i_C(t) = 0$ for $t < t_0$
 $i_C(t) = \frac{E}{R} \cdot e^{-\frac{t-t_0}{RC}}$ for $t \geq t_0$



3A) $u_C(t) = 0$ for $t < t_0$
 $u_C(t) = E$ for $t \geq t_0$
 $i_C(t) = 0$ for $t < t_0$
 $i_C(t) = \frac{E}{R}$ for $t \geq t_0$

4A) $u_C(t) = 0$ for $t < t_0$
 $u_C(t) = E \cdot (1 - e^{-\frac{t-t_0}{RC}})$ for $t \geq t_0$
 $i_C(t) = 0$ for $t < t_0$
 $i_C(t) = \frac{E}{R} \cdot e^{-\frac{t-t_0}{RC}}$ for $t \geq t_0$

Sheet resistance



$R = \frac{\rho}{d} \cdot \frac{l}{b}$
 $R_D = \frac{\rho}{d}$

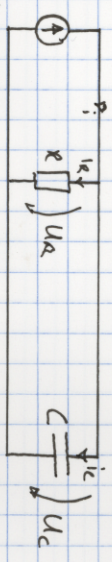
Leistung P

Kondensator mit $U_C \neq 0$

Die Spannung ändert sich periodisch:
 $P_C = \frac{U_{max}}{t_{max}} = \frac{1}{2} C \cdot \frac{U_{max}^2}{t_P}$

Widerstand mit sich veränderndem V

$P = \frac{1}{t_P} \int_{t_0}^{t_0+t_P} \frac{(U_{max} \cdot \frac{t-t_0}{t_P})^2}{R} dt$



i) Effektivwert des Stroms: $I_{eff} = \frac{1}{\sqrt{2}}$

ii) $U_R = U_C(t) = I \cdot \frac{R}{1 + j\omega R C}$

iii) $I_C = U_C \cdot j\omega C = I \cdot \frac{j\omega R C}{1 + j\omega R C}$

iv) Scheinleistung: $S = |I|^* \cdot U$

v) Wirkleistung: nur über R!
 $P = |U_R| \cdot |I|$