Lösung E&M Sommer 2013

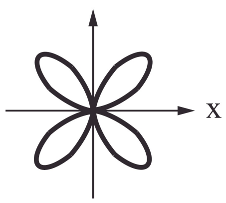
1 Quadrupolstrahlung

1.
$$\mathbf{E} = -\omega^2 \mu_0 p \frac{\exp(ikr)}{4\pi r} \sin\theta \hat{\theta}$$
$$\mathbf{H} = \frac{k}{\omega \mu_0} E_\theta \hat{\phi}$$

2.
$$E_\theta = -\sin\theta \mu_0 \omega^2 p \frac{\exp(ikr)}{4\pi r} \exp(-ikd\cos\theta)$$
$$H_\phi = -\sin\theta k \omega p \frac{\exp(ikr)}{4\pi r} \exp(-ikd\cos\theta)$$
$$P = \frac{k\omega^3 \mu_0 p^2}{12\pi}$$

3.
$$E_\theta = 2ik\omega^2 \mu_0 p d \sin\theta \cos\theta \frac{\exp(ikr)}{4\pi r}$$

4.
$$I = \frac{k^5 \omega p^2 d^2}{8\pi^2 \varepsilon_0} \frac{\sin^2\theta \cos^2\theta}{r^2}$$
$$\theta_{max} = \pm\frac{\pi}{4}; \pm\frac{3\pi}{4}$$

5.


6.
$$dW/dt = 0$$

Phasendifferenz führt zu destruktiver Interferenz.

2 Fabry-Pérot Resonanzen

1. Für evaneszente Wellen muss gelten: $\varepsilon < \sin^2\theta$. Da dies nicht erfüllt werden kann, existiert kein Winkel, welcher evaneszente Wellen hervorruft.

2. $$\mathbf{E_0}(\mathbf{r},t) = E_0 \begin{bmatrix}\cos\theta \\ 0 \\ \sin\theta\end{bmatrix} e^{ik(x\sin\theta - z\cos\theta)-i\omega t}$$

3. $$\mathbf{H_0}(\mathbf{r},t) = \sqrt{\frac{\varepsilon_0}{\mu_0}} E_0 \begin{bmatrix}0 \\ -1 \\ 0\end{bmatrix} e^{ik(x\sin\theta - z\cos\theta)-i\omega t}$$

4. Inhomogenes Problem (hängt von θ ab), 4 Unbekannte, 4 Gleichungen nötig

$$\mathbf{E}_{upper} = E_0 \begin{bmatrix}\cos\theta \\ 0 \\ \sin\theta\end{bmatrix} e^{ikx\sin\theta - ikz\cos\theta} + E_r \begin{bmatrix}-\cos\theta \\ 0 \\ \sin\theta\end{bmatrix} e^{ikx\sin\theta + ikz\cos\theta}$$

$$\mathbf{E}_{middle} = E_{t1} \begin{bmatrix}\cos\theta' \\ 0 \\ \sin\theta'\end{bmatrix} e^{i\sqrt{\varepsilon}kx\sin\theta' - i\sqrt{\varepsilon}kz\cos\theta'} + E_{t2} \begin{bmatrix}-\cos\theta' \\ 0 \\ \sin\theta'\end{bmatrix} e^{i\sqrt{\varepsilon}kx\sin\theta' + i\sqrt{\varepsilon}kz\cos\theta'}$$

$$\mathbf{E}_{lower} = E_t \begin{bmatrix}\cos\theta \\ 0 \\ \sin\theta\end{bmatrix} e^{ikx\sin\theta - ikz\cos\theta}$$

5. $$\begin{pmatrix} \cos\theta & \cos\theta' & -\cos\theta' & 0 \\ -\sin\theta & \varepsilon\sin\theta' & \varepsilon\sin\theta' & 0 \\ 0 & \cos\theta' e^{ik\sqrt{\varepsilon}d\cos\theta'} & -\cos\theta' e^{-ik\sqrt{\varepsilon}d\cos\theta'} & -\cos\theta e^{ikd\cos\theta'} \\ 0 & \varepsilon\sin\theta' e^{ik\sqrt{\varepsilon}d\cos\theta'} & \varepsilon\sin\theta' e^{-ik\sqrt{\varepsilon}d\cos\theta'} & -\sin\theta e^{ikd\cos\theta'} \end{pmatrix} \begin{pmatrix} E_r \\ E_{t1} \\ E_{t2} \\ E_t \end{pmatrix} = E_0 \begin{pmatrix}\cos\theta \\ \sin\theta \\ 0 \\ 0\end{pmatrix}$$

6. $$d = \frac{\lambda}{2\sqrt{\varepsilon - \sin^2\theta}}$$

7. $$\lambda' = \frac{\lambda}{m} = \frac{2d\sqrt{\varepsilon - \sin^2\theta}}{m}, \quad m = 1,2,3...$$

3 Reflexion im Wellenleiter

1. $TE_{01}; \quad k_z = \sqrt{(\omega/c)^2 - (\pi/L_y)^2}$

2.
$$k_z^{(r)} = -k_z$$
$$k_z^{(t)} = \sqrt{(\omega/c)^2\varepsilon - (\pi/L_y)^2}$$

3.
$$\mathbf{H_i} = E_0 \frac{e^{ik_z z}}{\omega\mu_0}\left(k_z \sin\left(\frac{y\pi}{L_y}\right)\mathbf{n_y} + i\frac{\pi}{L_y}\cos\left(\frac{y\pi}{L_y}\right)\mathbf{n_z}\right)$$
$$\mathbf{H_r} = E_r \frac{e^{-ik_z z}}{\omega\mu_0}\left(-k_z \sin\left(\frac{y\pi}{L_y}\right)\mathbf{n_y} + i\frac{\pi}{L_y}\cos\left(\frac{y\pi}{L_y}\right)\mathbf{n_z}\right)$$
$$\mathbf{H_t} = E_t \frac{e^{ik_z^{(t)} z}}{\omega\mu_0}\left(k_z^{(t)} \sin\left(\frac{y\pi}{L_y}\right)\mathbf{n_y} + i\frac{\pi}{L_y}\cos\left(\frac{y\pi}{L_y}\right)\mathbf{n_z}\right)$$

4. Grenzbedingungen:
$$E_0 + E_r = E_t$$
$$k_z E_0 - k_z E_r = k_z^{(2)} E_t$$

   Somit finden wir für die Koeffizienten:
$$r = \frac{\sqrt{(\omega/c)^2 - (\pi/L_y)^2} - \sqrt{(\omega/c)^2\varepsilon - (\pi/L_y)^2}}{\sqrt{(\omega/c)^2 - (\pi/L_y)^2} + \sqrt{(\omega/c)^2\varepsilon - (\pi/L_y)^2}}$$
$$t = \frac{2\sqrt{(\omega/c)^2 - (\pi/L_y)^2}}{\sqrt{(\omega/c)^2 - (\pi/L_y)^2} + \sqrt{(\omega/c)^2\varepsilon - (\pi/L_y)^2}}$$

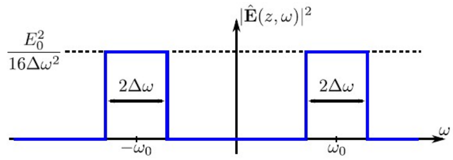
5.
$$\frac{c\pi}{\sqrt{\varepsilon}L_y} < \omega < \frac{c\pi}{L_y}$$

4 Elektromagnetischer Puls in einem Dispersiven Medium

1.
$$\mathbf{E}(z,t) = E_0 \cos\left[\frac{\omega_0}{c}(z - ct)\right] \frac{\sin\left[\frac{\Delta\omega}{c}(z-ct)\right]}{\frac{\Delta\omega}{c}(z-ct)}\mathbf{n_x}$$

2. -

3.
$$\hat{\mathbf{E}}(z,\omega) = \frac{e^{i\omega z/c}}{4\Delta\omega}\left[R_{\Delta_\omega}(\omega_0) + R_{\Delta_\omega}(-\omega_0)\right]\mathbf{n_x}$$
$$R_{\Delta_x}(x_0) = \begin{cases} 1, & x_0 \in [-\Delta_x, \Delta_x] \\ 0, & \text{else} \end{cases}$$

4. 

5. -

6. -

7. $$\hat{\mathbf{E}}_r(z,\omega) = -r^p(\omega)\frac{E_0 e^{-i\omega z/c}}{4\Delta\omega}\left[R_{\Delta_\omega}(\omega_0) + R_{\Delta_\omega}(-\omega_0)\right]\mathbf{n_x}$$
$$r^p(\omega) = \frac{\sqrt{\varepsilon}-\sqrt{\mu}}{\sqrt{\varepsilon}+\sqrt{\mu}}$$
Bei senkrechtem Einfall kann auch s-Polarisation verwendet werden: $r^s = -r^p$

8. Inverse Fourier-Trafo, $D = [-\omega_0 - \Delta\omega, -\omega_0 + \Delta\omega] \cup [\omega_0 - \Delta\omega, \omega_0 + \Delta\omega]$
$$E_r(z,t) = -\frac{E_0}{4\Delta\omega}\mathbf{n_x}\int_D r^p(\omega)e^{-i\omega(z/c+t)}d\omega$$

5 Ausbreitung und Beugung von Feldern

1. $z \approx \lambda/5$

2. $$\mathbf{E}_\infty\left(\frac{x}{r},\frac{y}{r},\frac{z}{r}\right) = -\frac{8ize^{ikr}}{\pi k r^2}\mathbf{E_0}\frac{\sin\left(\frac{\pi x}{2r}\right)}{(x/r)^3 - 4(x/r)}\frac{\sin\left(\frac{\pi y}{2r}\right)}{(y/r)^3 - 4(y/r)}$$

3. $$\mathbf{E}(x,y,z) = \iint_{-\infty}^{\infty}\frac{4}{\pi^2k^2}\mathbf{E_0}\frac{\sin\left(\frac{\pi k_x}{2k}\right)}{(k_x/k)^3 - 4(k_x/k)}\frac{\sin\left(\frac{\pi k_y}{2k}\right)}{(k_y/k)^3 - 4(k_y/k)}e^{i\left[k_x x + k_y y + z\sqrt{k^2-k_x^2-k_y^2}\right]}dk_x dk_y$$