Lösung E&M Sommer 2014

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1 Anregung von Oberflächenwellen

1. $\theta > \arcsin\sqrt{\frac{\varepsilon_3}{\varepsilon_1}}$

2. $k_{2z}^2 = k_0^2(\varepsilon_2 - \varepsilon_1 \sin^2\theta) < 0$

3.
$$\mathbf{E_0}(\mathbf{r}, t) = Re\{E_0 \begin{bmatrix} -\cos\theta \\ 0 \\ \sin\theta \end{bmatrix} e^{i(k_x x + k_{1z} z)} e^{-i\omega t}\}$$

4. -

5.
$$\mathbf{H_0}(\mathbf{r}, \mathbf{t}) = -\frac{k_x}{\omega\mu_0 \sin\theta} E_0 \; e^{ikr}\mathbf{e_y}$$
$$H_0 = \frac{k_x}{\omega\mu_0 \sin\theta} E_0$$

6.
$$\mathbf{E_r} = \frac{E_r}{\sqrt{k_x^2 + k_{1z}^2}} e^{ikr} \begin{bmatrix} k_{1z} \\ 0 \\ k_x \end{bmatrix}$$
$$\mathbf{E_t} = \frac{E_t}{\sqrt{k_x^2 + k_{3z}^2}} e^{ikr} \begin{bmatrix} -k_{3z} \\ 0 \\ k_x \end{bmatrix}$$
$$\mathbf{H_r} = -\frac{E_r \sqrt{k_x^2 + k_{1z}^2}}{\omega\mu_0} e^{ikr}\mathbf{e_y}$$
$$\mathbf{H_r} = -\frac{E_t \sqrt{k_x^2 + k_{3z}^2}}{\omega\mu_0} e^{ikr}\mathbf{e_y}$$

$$with \quad H_r = \frac{\sqrt{k_x^2 + k_{1z}^2}}{\omega\mu_0} E_r, \quad H_t = \frac{\sqrt{k_x^2 + k_{3z}^2}}{\omega\mu_0} E_t$$

7.

$$E_1^{\|} = E_3^{\|}: \quad \frac{(E_r - E_0)k_{1z}}{\sqrt{k_{1z}^2 + k_x^2}} = -\frac{E_t\ k_{3z}}{\sqrt{k_{3z}^2 + k_x^2}} \tag{1}$$

$$H_1^{\|} = H_3^{\|}: \quad \frac{(E_0 + E_r)}{\sqrt{k_x^2 + k_{3z}^2}} = \frac{E_t}{\sqrt{k_x^2 + k_{1z}^2}} \tag{2}$$

Using the boundary conditions for the D-Field yields the same equation as (2). Writing the equations for the two unknowns E_r, E_t in matrix form gives us:

$$\begin{pmatrix} k_{1z}\frac{1}{\sqrt{\varepsilon_1}} & k_{3z}\frac{1}{\sqrt{\varepsilon_3}} \\ \sqrt{\varepsilon_1} & -\sqrt{\varepsilon_3} \end{pmatrix} \begin{pmatrix} E_r \\ E_t \end{pmatrix} = \begin{pmatrix} k_{1z}\frac{1}{\sqrt{\varepsilon_1}} \\ -\sqrt{\varepsilon_1} \end{pmatrix} E_0$$

where we use $\quad k_{3z} = \sqrt{\varepsilon_3 \frac{\omega^2}{c^2} - k_x^2}$

8. 4 unknowns $E_{1,r}, E_{2,t}, E_{2,r}, E_{3,t}$ require 4 boundary conditions.

9.

$$k_{2z} = \sqrt{\varepsilon_2 \frac{\omega^2}{c^2} - k_x(\theta)^2}$$
$$k_{3z} = \sqrt{\varepsilon_3 \frac{\omega^2}{c^2} - k_x(\theta)^2}$$

$$\theta = \arcsin\sqrt{\frac{\varepsilon_2\varepsilon_3^2 - \varepsilon_2^2\varepsilon_3}{\varepsilon_1(\varepsilon_3^2 - \varepsilon_2^2)}}$$

2 Lorentz'scher Lichtpuls

1.

$$\mathbf{E}(x = 0, y = 0, z, t) = \frac{E_0}{1 + (\frac{z}{z_0})^2} \cos\left(\frac{\omega_0}{c} z - \omega_0 t\right)$$

2.

$$Gauss\ (divergence - free): \quad \nabla * \mathbf{E}(\mathbf{r}, t) = 0$$
$$Wave\ equation: \quad \nabla^2 \mathbf{E}(\mathbf{r}, t) - \frac{1}{c^2}\frac{d^2}{dt^2}\mathbf{E}(\mathbf{r}, t) = 0$$

3.

$$\hat{\mathbf{E}}(z, w) = \frac{E_0\ z_0}{4c} e^{\frac{i\omega z}{c}} \left(e^{-\frac{2\omega_0 z_0}{c}} + 1\right)$$

The function is constant for $z = 0$ (as the exponential term is 1 independent of ω).

3 Abstrahlung nahe einer dielektrischen Kugel

1.

$$\mathbf{E}(r=d) = \frac{\omega^2 \mu_0 \mu \ p}{4\pi d} e^{ikd} \left[1 + \frac{ikd-1}{k^2 d^2} \right] \mathbf{n_z}$$

$$\mathbf{p}_{ind} = \alpha \ \mathbf{E}(r=d) = \alpha \frac{\omega^2 \mu_0 \mu \ p}{4\pi d} e^{ikd} \left[1 + \frac{ikd-1}{k^2 d^2} \right] \mathbf{n_z}$$

2.

$$\mathbf{E_\infty}^{ind}(r, \theta; d, p) = \frac{\omega^2 \mu_0 \mu}{16\pi^2 \ rd \ \varepsilon_0 \varepsilon} \alpha \ k^2 \ p \ e^{ik(R+d(1-\sin\theta))} \left[1 + \frac{ikd-1}{k^2 d^2} \right] \sin\theta - \mathbf{e}_\theta$$

3.

$$\mathbf{E_\infty}^{tot}(r, \theta, p, d) = \frac{p \sin\theta}{4\pi\varepsilon_0\varepsilon} \frac{e^{ikr}}{r} k^2 \left[1 + \frac{\alpha k^2}{4\pi\varepsilon_0\varepsilon d} e^{ikd(1-\sin\theta)} \right] - \mathbf{e}_\theta$$

The constructive interference of the fields can be found by looking at the phase term:

$$\theta = \arcsin\left(1 - \frac{2\pi}{kd} n\right), \quad \theta_0 = \frac{\pi}{2}, \theta_1 = \frac{\pi}{6}, \theta_2 = 0$$

4.

$$\theta_2 = \pi - \theta_1$$

$$\theta_3 = -(\theta_1 - \arcsin\left(\frac{d}{r} \sin\left(\frac{\pi}{2} - \theta_1\right)\right))$$

$$\theta_4 = \pi - \theta_3 = \pi + (\theta_1 - \arcsin\left(\frac{d}{r} \sin\left(\frac{\pi}{2} - \theta_1\right)\right))$$

5.

$$\bar{P} = \frac{\alpha^2 (\mu_0 \mu)^2 \omega^5 k^3}{192 \ \pi^3 d^2 \varepsilon_0 \varepsilon} p^2$$

6.

$$\mathbf{E}_{ind}(r=0) = \alpha \ p \frac{k^4}{16\pi^2 (\varepsilon_0\varepsilon)^2} \frac{e^{i2kd}}{d^2} - \mathbf{e_z}$$

7.

$$\frac{P}{P_0} = 1 - \frac{3k\alpha}{8\pi\varepsilon_0\varepsilon} \frac{\sin(2kd)}{d^2}$$

4 Ausbreitung und Beugung von Wellen

1.
$$z_0 = \frac{\pi}{\lambda}(\alpha^2 + \beta^2)$$

2. Fresnel contains a further term $\frac{x'^2+y'^2}{2R}$ which improves the approximation.
   With Fresnel, the field has cylindrical wave fronts, while with the Fraunhofer approximation we get plane wave fronts. For further information, see this link.

3.
$$\hat{\mathbf{E}}_1(k_x, k_y, z = 0) = \frac{\alpha}{\pi} sinc(k_x\alpha)\frac{\beta}{\pi} sinc(k_y\beta)e^{+ix_0k_x}\ \mathbf{E_0}$$
$$\hat{\mathbf{E}}_2(k_x, k_y, z = 0) = \frac{\alpha}{\pi} sinc(k_x\alpha)\frac{\beta}{\pi} sinc(k_y\beta)e^{-ix_0k_x}\ \mathbf{E_0}e^{i\phi}$$

4.
$$\mathbf{E}_\infty = -2\pi i\sqrt{k^2 - k_x^2 - k_y^2}\frac{e^{ikr}}{r}\frac{\alpha\beta}{\pi^2} sinc(k_x\alpha)sinc(k_y\beta)\ \mathbf{E_0}(e^{ix_0k_x} + e^{-ix_0k_x}e^{i\phi})$$

   where $r = \sqrt{x^2 + y^2 + z^2}$

5.
$$I_{coh}(r) = \frac{4\pi^2(k^2 - k_x^2 - k_y^2)}{r^2}\frac{\alpha^2\beta^2}{\pi^2} sinc^2(k_x\alpha)sinc^2(k_y\beta)|\mathbf{E_0}|^2 \quad * \quad 4\cos^2(x_0k_x - \frac{\phi}{2})$$
$$I_{coh}(r) = \frac{4\pi^2(k^2 - k_x^2 - k_y^2)}{r^2}\frac{\alpha^2\beta^2}{\pi^2} sinc^2(k_x\alpha)sinc^2(k_y\beta)|\mathbf{E_0}|^2 \quad * \quad 2$$
$$\frac{I_{coh}}{I_{inc}} = 2\cos^2(x_0k_x - \frac{\phi}{2})$$

6. Coherent case : the cosine modulation of the interference is shifted by the phase of the fields (just as in FS15 Exercise 10, Task 1).

   Incoherent case : no change occurs, as no influence on the intensity of either one of the fields.