Semiconductor Devices Summary
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1. Constants & Various

Constants (@300K)
\[
\begin{align*}
\varepsilon_0 &= 8.854 \times 10^{-12} \text{ F/m} \quad m_0 = 9.11 \times 10^{-31} \text{ kg} \\
k &= 1.38 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K} \\
kT/q &= 0.0259 \text{ V, } \frac{q}{kT} = 38.61 \frac{1}{V}, kT = 25.9 \text{ meV} \\
1 \text{ eV} &= 1.602 \times 10^{-19} J \quad q = 1.602 \times 10^{-19} \text{ As} \\
\text{Silicon (@300K)} &\rightarrow 4 \text{ valence electrons} \\
n_i^2 &= 9.3 \times 10^{19} / \text{cm}^6 \quad n_i = 9.65 \times 10^9 / \text{cm}^3 \\
N_c &= 2.86 \times 10^{19} / \text{cm}^3 \quad N_V = 2.66 \times 10^{15} / \text{cm}^3 \\
\varepsilon_s &= 11.8 \times \varepsilon_0 \quad \nu_{th} \approx 10^7 \text{ cm/s} \\
\varepsilon_{ox} &= 3.9 (\text{SiO}_2) \quad E_g = 1.12 \text{ eV} \quad \chi_S = 4.05 \text{ V} \\
\text{Quantum physics} &\quad h = 6.625 \times 10^{-34} \text{ Js} = \lambda \cdot p \\
\omega &= \frac{2\pi}{\tau} = v \cdot k \\
k &= \frac{2\pi}{\lambda} \quad p = \frac{h}{2\pi} \cdot k = h' \cdot k \\
\frac{1}{2} m \nu_{th} &= \frac{3}{2} kT \rightarrow \nu_{th} = \sqrt{3kT/m_0} \approx 10^7 \text{ cm/s} \\
\text{Electronics} \quad R = \rho \cdot \frac{\text{Length}}{\text{Area}} \ , \rho \text{ conductivity (\Omega/cm)} \\
E_{kin} + E_{pot} &= \text{constant} \\
V = -\frac{E_{pot}}{q} = -\int E \, dx \ , \quad E = -\frac{dV}{dx}
\end{align*}
\]

2. Crystals and Current Carriers

Semi-Conductor: Conductivity controllable over orders of magnitude by means of:
Impurities (doping), light, temperature, EM-fields

Coordination number: number of nearest neighbors

Simple Metals: coord. number > no. of valence electrons
Transition Metals: bonds covalent-like, harder
Covalent Bonding: hybridization of s- & p-orbitals, stiff
\[ \rightarrow \] tetrahedral bonding: coord. number = 4, 8N states
s-orbitals: 2 allowed states; p-orbitals: 6 allowed states
Partially filled/empty bands conduct currents!

Band gap: between valence and conduction band

Intrinsic carriers
No doping, pure semiconductor, created by heat
\[ n_0 = p_0 = n_i \sim \frac{1}{E_g} \]

\[ E_g: \text{ Silicon } 1.12 \text{ eV}, \quad \text{GaAs } 1.42 \text{ eV} @ 300 \text{ K} \]

Extrinsic carriers

Donors (n-type): give electrons (P, As, Sb)
Acceptors (p-type): give holes (B, Al, Ga, In)
Overall, solid is neutral: one fixed charge, one free

\[ p_0 = \frac{n_i^2}{N_D}, \quad n_0 = \frac{n_i^2}{N_A} \]

Fermi Dirac Statistics

\[ F(E) = \text{probability of finding an electron with energy } E \]
\[ F(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \approx e^{-(E-E_F)/kT} \quad E >> E_F \]

Fermi level \( E_F \): energy where \( F(E = E_F) = \frac{1}{2} \)

Probability of finding a hole: \( H(E) = 1 - F(E) \)
\[ n_0 = \int_{E_F} F(E) \times d(E_{kin}) \cdot p_0 = \int_{E_F} (1 - F(E)) \times d(E_{kin}) \]

Density of State: \( D(E_{kin}) = \frac{8\pi^2}{h^3} (m^*)^{3/2} (E_{kin})^{1/2} \)

Kinetic energy: \( E_{kin} = \frac{|p|^2}{2m^*} = \frac{(p_x)^2 + (p_y)^2 + (p_z)^2}{2m^*} \)

i) Fermi levels in all regions will line up
ii) Far away from transition, Fermi level is like without junction (material doesn’t “know”)
iii) At Equilibrium/Steady-State, \( E_F \) must be flat (constant) so that no current will be flowing

Carrier concentration
\[ N_c = \frac{4\sqrt{2\pi m^* kT}}{h^3}, \quad N_V = \frac{4\sqrt{2\pi m^* kT}}{h^3} \]

\[ n_0 = N_c \cdot e^{-E_E-E_F/kT} = n_i \cdot e^{-E_i-E_F/kT} \]

\[ p_0 = N_V \cdot e^{-E_E-E_V/kT} = n_i \cdot e^{-E_i-E_F/kT} \]

\[ n_i^2 = N_V \cdot N_c \cdot e^{-E_E/kT}, \quad E_G = E_c - E_V \]

Constant product: \[ n_0 \cdot p_0 = n_i^2 \]

Carrier “Freeze-Out”: \( T << 0°C \)

“Extrinsic Region”: donors ionized

“Intrinsic Region”: doping irrelevant
3. Carrier transport

**Diffusion current:** concentration gradients from high to low concentration

\[ J_n = q D_n \frac{dn}{dx}, \quad J_p = -q D_p \frac{dp}{dx} \]

**Drift current:** electric field holes with field, electrons against it

\[ J_n = n q \mu \vec{E}, \quad J_p = p q \mu \vec{E} \]

**Total current:**

\[ J_n = n q \mu \vec{E} \] + \[ q D_n \frac{dn}{dx}, \quad J_p = p q \mu \vec{E} - q D_p \frac{dp}{dx} \]

**Conductivity**

\[ J_{drift,\text{tot}} = \sigma E \rightarrow \sigma = n q \mu_n + p q \mu_p \]

**Einstein relation**

\[ D_n = \frac{kT}{q} \mu_n, \quad D_p = \frac{kT}{q} \mu_p \]

**In PN Junction:** only diffusion currents (flat bands)

\[ \frac{dn}{dx} = n_{po}(e^{qV_F/kT} - 1) \quad \frac{dp}{dx} = p_{po}(e^{qV_F/kT} - 1) \]

\[ J_n = q D_n \frac{dn}{dx} \quad J_p = -q D_p \frac{dp}{dx} = J_s (e^{qV_F/kT} - 1) \]

\[ J_s = \left[ \frac{q D_n n_{po}}{L_n} \right] (e^{qV_F/kT} - 1) \]

**Reverse breakdown**

1. **Band-to-Band Tunneling (Zener)** applies when both sides are heavily doped
2. **Avalanche Multiplication** strong electric field creates large kinetic energy to the carriers, so that they ionize others via collision

\[ n_p = N_D * e^{-\frac{q(V_{bi}-V)}{kT}} = n_{po} * e^{-\frac{qV_F}{kT}} \]

\[ p_n = N_A * e^{-\frac{q(V_{bi}-V)}{kT}} = p_{po} * e^{-\frac{qV_F}{kT}} \]

\[ V_{bi} = \frac{kT}{q} \ln \left( \frac{N_A * N_D}{(n_i)^2} \right) = \frac{1}{2} E_{max} * W \]

\[ V_{bi} = \frac{kT}{q} \ln \left( \frac{N_A * N_D}{(n_i)^2} \right) = \frac{1}{2} E_{max} * \frac{W}{\varepsilon_s} \]

\[ W = x_p + x_n = \sqrt{\frac{2q (1/N_A + 1/N_P)}{(V_{bi} - V_{apply})}} \]

\[ W \approx \sqrt{\frac{2 \varepsilon_s}{q N_D V_{bi}}} \quad N_D \ll N_A \]

4. PN Junction

**Built-in voltage**

Band-bending that balances drift & diffusion currents

\[ V_{bi} = \frac{kT}{q} \ln \left( \frac{N_A * N_D}{(n_i)^2} \right) = \frac{1}{2} E_{max} * W \]

**Forward Bias:** reduce band bending, less difference more minority carriers -> minority carrier injection

**Reverse Bias:** increase band bending, less minority

Band-bending = presence of an electric field

**Conduction Band Edge:** \( E_{po} \) of electrons

**Valence Band Edge:** \( E_{po} \) of holes

**Diode currents:** minority carriers

\[ n_{po} = \left( \frac{n_i}{N_A} \right)^2 = N_D * e^{-\frac{qV_{bi}}{kT}} \]

\[ p_{po} = \left( \frac{n_i}{N_D} \right)^2 = N_A * e^{-\frac{qV_{bi}}{kT}} \]

**Under Forward-Bias:** **Shockley Boundary Conditions**

\[ n_p = N_D * e^{\frac{q(V_{bi}-V)}{kT}} = n_{po} * e^{\frac{qV_F}{kT}} \]

**Poisson Equation**

\[ \frac{dE}{dx} = \frac{\rho}{\varepsilon_r * \varepsilon_o} = \frac{\rho}{\varepsilon_s} \rightarrow V_{bi} = -\int_{-x_p}^{x_n} E(x) \, dx \]

**Depletion approximation**

\[ |E_{max}| = |E(x = 0)| = \frac{q N_A x_p}{\varepsilon_s} = \frac{q N_D x_n}{\varepsilon_s} \]

\[ W = x_p + x_n = \sqrt{\frac{2q (1/N_A + 1/N_P)}{(V_{bi} - V_{apply})}} \]

**Neutrality:** \( N_A x_P = N_D x_N \) (same areas)

**One-Sided junction:** only depletion on lightly-doped side

\[ W \approx \sqrt{\frac{2 \varepsilon_s}{q N_D V_{bi}}} \quad N_D \ll N_A \]

**Depletion capacitance**

\[ C_j = \frac{dQ}{dV} = \frac{\varepsilon_s}{W (V_{bi}, V_{apply})} = \sqrt{\frac{q \varepsilon_s \varepsilon_o}{2 V_{bi} (N_A + N_D)}} \]
4. Generation and Recombination

Recombination brings the system back to equilibrium.

**Non-equilibrium concentration:**
\[ n = n_0 + \Delta n, \quad p = p_0 + \Delta p, \quad \Delta n = \Delta p \]

Recombination rate (even at Non-equilibrium):
\[ R = \beta \ast (n \ast p) \]

**Thermal generation rate**
\[ G_{th} = R_{th} = \beta \ast (n_{no} \ast p_{no}) \]

External excitation (e.g. Light) gives additional term:
\[ G = G_L + G_{th} \rightarrow \frac{dp_n}{dt} = G_L + G_{th} - R \]

**Direct recombination**

Direct recombination across the bandgap results in the emission of a photon with energy \( E_L = \hbar \ast f \).

**Net generation rate U**
\[ U = \beta \ast (n \ast p - n_i^2) = G_L = R - G_{th} \]

Under low-level injection: \( p_{n0} \ll n_{no} \), \( \Delta p \ll n_{no} \)
\[ U = \frac{\Delta p}{\tau_p} = \frac{1}{\beta n_{no}} \]
\( \tau : \) Minority carrier lifetime (how fast decay)

**Example: Lesson 5, p.7**
- **Light ON**
  \[ G_L = U = \frac{p_n - p_{n0}}{\tau_p} \rightarrow p_n = p_{n0} + \tau_p G_L \]
- **Light OFF:**
  \[ G_L = 0 \rightarrow \frac{dp_n}{dt} = G_{th} - R = -\frac{p_n - p_{n0}}{\tau_p} \]
  \[ \rightarrow p_n(t) = p_{n0} + \tau_p G_L e^{-t/\tau_p} \]

**Indirect recombination** *(Neamen: p.223)*

G-R Centers in the Gap (defect states near midgap)
These "traps" facilitate the return of an electron

**G/R Centers:** most effective if \( E_i \) near intrinsic \( E_i \)

\[ N_i : \text{Density of Recombination Centers} \]
\[ \sigma : \text{Recombination Center cross section} \]

**Generation and Recombination**

\[ e_n = \nu_{th} \sigma_n n_i e^{(E_i - E_f)/kT} \]
**Electron emission prob.**

\[ e_p = \nu_{th} \sigma_p n_i e^{(E_f - E_i)/kT} \]
**Hole emission probability**

\[ R_a = n N_i (1 - f) \ast \nu_{th} \sigma_n \]
**Electron capture rate**

\[ R_b = e \ast n N_i \]
**Electron emission rate**

\[ R_c = p N_i (f) \ast \nu_{th} \sigma_p \]
**Hole capture rate**

\[ R_d = e \ast p N_i (1 - f) \]
**Hole emission rate**

Surface recombination: "dangling bonds" at surface

**Continuity equation**

\[ \frac{dn}{dt} = \frac{1}{q} \frac{dE}{dx} + (G_n - R_n), \quad \frac{dp}{dt} = -\frac{1}{q} \frac{dE}{dx} + (G_p - R_p) \]

\[ \frac{dn_p}{dt} = n_p \nu_p \mu_p \frac{dE}{dx} + \mu_p^2 \frac{dn_p}{dx^2} + D_n \frac{d^2n_p}{dx^2} + G_n - \frac{n_p - n_{p0}}{\tau_n} \]

\[ \frac{dp_n}{dt} = -p_n \nu_p \mu_p \frac{dE}{dx} + \mu_p^2 \frac{dp_n}{dx^2} + D_p \frac{d^2p_n}{dx^2} + G_p - \frac{p_n - p_{p0}}{\tau_p} \]

**Steady State:** Quantities are Time Independent
**Zero Field:** fields in neutral regions are approx. zero

**Generation:** deficiency of minority carriers
**Recombination:** excess of minority carriers

**Exp: Steady State surface Generation**

**Long diode:** semi-infinite, exponential decay \( L \ll W \)

\[ p_n(0) = \text{const}, \quad p_n(x \rightarrow \infty) = p_{n0} \]

\[ p_n(x) = p_{n0} + [p_n(0) - p_{n0}] \ast e^{-x/L_p} \]

\[ I_p(x_n) = -q D_p \frac{dp_n}{dx} \bigg|_{x_n} = \frac{q D_p n_{p0}}{L_p} \left( e^{-x/L_p} - 1 \right) \]

**Short diode:** finite, linear decay \( L \gg W \)

\[ p_n(0) = \text{const}, \quad p_n(x = W) = p_{n0} \]

\[ p_n(x) = p_{n0} + [p_n(0) - p_{n0}] \left[ \frac{\sinh \left( \frac{W - x}{L_p} \right)}{\sinh \left( \frac{W}{L_p} \right)} \right] \]

**Minority Carrier Diffusion Length:**
\[ L_p = \sqrt{D_p \tau_p} \]

**Quasi-Fermi Levels**

Under bias, the equilibrium Fermi level splits into 2 distinct quasi-Fermi levels that describe carrier statistics in each diode region

\[ n(x) = N_C e^{-\left( E_C(x) - E_Fn(x) \right)/kT}, \quad p(x) = N_V e^{-\left( E_Fp - E_V(x) \right)/kT} \]

\[ n(x)p(x) = N_C N_V e^{\frac{E_Fp - E_Fn}{kT}} e^{(E_Fp - E_Fn)/kT} \]

\[ E_Fp - E_Fn = q V_F \]
Carrier Profile through Depletion Region

Forward Bias

Reverse Bias

Capacitance in depletion region

Depletion capacity per unit square \([ F / cm^2 ]\)

\[ C_A = \frac{C}{A} = \frac{\varepsilon_0 \varepsilon_r}{W}, \quad \text{W: depletion width} \]

Non idealsities

\[ n(x) = N_c e^{-\frac{E(x)}{kT}} \]

\[ p(x) = N_v e^{-\frac{E(x)}{kT}} \]

\[ n(x) p(x) = N_c N_v e^{-\frac{E(x)}{kT}} = N_i^2 e^{\frac{qV_P}{kT}} \]

Generation currents

Reverse bias

Carrier Deficit  \(\rightarrow\)  Generation current

\[ I_{gen} = \int_0^W qG \, dx = \frac{q n_i}{\tau_g} W, \quad G = \frac{n_i}{\tau_g} \]

\[ J_{RT} = J_S + J_{gen} = \left[ \frac{qD_p}{N_A L_n} + \frac{qD_n}{N_D L_P} \right] n_i^2 + q W n_i \]

Forward bias

Carrier Excess  \(\rightarrow\)  Recombination current

\[ U_{max} = \sigma_0 N_c n_i^2 \left( e^{\frac{qV_F}{kT}} - 1 \right) \approx \frac{1}{2} \frac{1}{v_{th}} \sigma_0 N_c n_i e^{qV_F/kT} \]

\[ J_{rec} = \int_0^W q U \, dx = \frac{q W n_i}{2 \tau_r} e^{\frac{qV_F}{kT}} \]

\[ J_{FT} = \left[ \frac{qD_p}{N_A L_n} + \frac{qD_n}{N_D L_P} \right] n_i^2 e^{qV_F/kT} + \frac{q W n_i}{2 \tau_r} e^{qV_F/2kT} \]

Ideality Factor \(\eta\): characterizes Diode Forward Current Ideality

Materials with longer recombination lifetime have better ideality

\[ I_{FT} = J_S e^{\frac{qV_F}{kT}} + J_{rec} \sim \exp \left[ \frac{q V_F}{\eta kT} \right] \]

Reverse Breakdown of Diode:

i) Band-to-Band Tunneling (Zener)

applies when both sides are heavily doped

ii) Avalanche Multiplication

strong electric field creates large kinetic energy to the carriers, so that they ionize others via collision

Ohmic losses

Ohmic losses reduce the internal voltage that actually appears across the depletion; at low current levels negligible

\[ I \approx I_S e^{\frac{qV_A}{kT}} \]

Real PN Junction Diode

a) Recombination in Depletion Region

b) Ideal Injection  \((\eta = 1, \text{60mV/dec})\)

c) High-Level Injection  \((\eta \rightarrow 2)\)

d) Series resistance effects (ohmic loss)

e) Generation in Depletion region

a) Recombination

b) Ideal Injection

c) High-Level Injection

d) Series resistance effects

e) Generation in Depletion region
5. Bipolar Junction Transistor (BJT)

BJT is a Minority Carrier Device and acts as an ideal current source (I_{Collector} does not vary with V_{CB})

### Ideal currents

- Injection from Emitter into Base
- No Generation/Recombination in Base Layer
- Neglect Diode Leakage Current

\[
\begin{align*}
I_E &= I_{pE} = q \frac{d_{nE}}{W} \ v_{BE} \ \text{for } v_{BE} > 0 \\
I_B &= I_{pB} = q \frac{d_{nB}}{W} \ v_{BE} \ \text{for } v_{BE} > 0 \\
I_C &= I_{pC} = g \ \frac{d_{nE}}{W} \ v_{BE} \ \text{for } v_{BE} > 0
\end{align*}
\]

Common Emitter current gain

\[
\beta = \frac{I_C}{I_E} = I_{pC} = I_{pE} = q \frac{d_{nE}}{W} \ v_{BE} \ \text{for } v_{BE} > 0
\]

For NPN:

\[
\begin{align*}
I_E &= I_{pE} = q \frac{d_{nE}}{W} \ v_{BE} \\
I_B &= I_{pB} = q \frac{d_{nB}}{W} \ v_{BE} \\
I_C &= I_{pC} = q \frac{d_{nE}}{W} \ v_{BE}
\end{align*}
\]

Emitter doping must be higher than base doping:

\[
I_{pC} \gg I_{nE} \Rightarrow N_{AE} \gg N_{DB}
\]

Doping Ration most powerful factor to reach gain

**Gummel-Characteristics:** $60\text{mV}/\text{dec gain in } I/V$

**Transconductance**

Collector current: \( I_C = I_E \frac{v_{BE}}{v_{BE} - 1} \)

Transconductance: \( g_m = \frac{I_C}{v_{BE} - 1} \)

### Non-ideal currents

<table>
<thead>
<tr>
<th>Non-ideal currents</th>
<th>NPN</th>
<th>PNP</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_{ext} )</td>
<td>-q \delta_{oE} \ \frac{d_{nE}}{W} \ v_{BE} \ \text{for } v_{BE} &gt; 0</td>
<td>-q \delta_{oE} \ \frac{d_{nE}}{W} \ v_{BE} \ \text{for } v_{BE} &gt; 0</td>
</tr>
<tr>
<td>( J_{int} )</td>
<td>-q \delta_{oB} \ \frac{d_{nB}}{W} \ v_{BE} \ \text{for } v_{BE} &gt; 0</td>
<td>-q \delta_{oB} \ \frac{d_{nB}}{W} \ v_{BE} \ \text{for } v_{BE} &gt; 0</td>
</tr>
<tr>
<td>( J_{bc} )</td>
<td>q \delta_{bc} \ \frac{d_{nE}}{W} \ v_{BE} \ \text{for } v_{BE} &gt; 0</td>
<td>q \delta_{bc} \ \frac{d_{nE}}{W} \ v_{BE} \ \text{for } v_{BE} &gt; 0</td>
</tr>
<tr>
<td>( J_{np} )</td>
<td>q \delta_{np} \ \frac{d_{nB}}{W} \ v_{BE} \ \text{for } v_{BE} &gt; 0</td>
<td>q \delta_{np} \ \frac{d_{nB}}{W} \ v_{BE} \ \text{for } v_{BE} &gt; 0</td>
</tr>
<tr>
<td>( J_{pC} )</td>
<td>\text{No base recombination}</td>
<td>\text{No base recombination}</td>
</tr>
</tbody>
</table>

\[ W_{E} \ll I_{BE} \Rightarrow J_{BE} = 0 \]

\[ W_{E} \ll I_{BE} \Rightarrow J_{BE} = 0 \]

\[ J_{E} = I_{E} + I_{BE} \]

\[ I_{E} = I_{E} + I_{BE} \]

\[ I_{BE} = I_{BE} \]

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\[ I_{BE} = I_{BE} \]
"Early" – effect:
In practice, the $I_C$ depends on $V_{BC}$. The depletion region becomes wider with increasing BC reverse bias, decreasing the undepleted base width, which increases $I_C$. To avoid this, the base doping must be higher than collector:

$$N_{DB} \gg N_{AC}$$

Cutoff frequency

Unity Current Gain Cutoff frequency: $\beta(f_T) = 1$

Measured with Short-Circuit load ($R_L = 0$)

$$f_T = \frac{g_m r_n}{2 \pi C_B^O}$$

Transistor behaves as a Low-Pass

$$h_f(\omega) = \frac{I_C}{I_B} = \frac{g_m r_n}{1 + j \omega r_n C_P} = \beta(\omega)$$

$$h_f(\omega = 0) = g_m r_n$$

Power Gain from Amplifier

For power gain: $R_{out} \to \infty \ Leftrightarrow V_A \to \infty$

$$G_A = \beta^2 \frac{R_L}{R_{in}} \left( \frac{V_A + V_{CE}}{I_C R_L + 1} \right)$$

Used power: $P = V_{CE} * I_C$

Cost of power gain: $P_D = V_{CE} * I_C - P_{out}$

Maximum Power gain (with impedance matching)

$$G_p = \frac{1}{2 \pi} \frac{f_T}{B R_B C_{BC}}, \quad f_{MAX} = \frac{f_T}{\sqrt{B R_B C_{BC}}}$$

Additional Delay terms

$\tau_B$ : Base Transit Time (diffusion across the base)

$\tau_C$ : Collector Signal Delay Time (through depletion)

$$\tau_B = \frac{Q_B}{I_C} = \frac{W_B^2}{2 D n}, \quad \tau_C = \frac{W_C}{2 v_{sat}}$$

Cutoff frequency including delay terms

$$f_T = \sqrt{\beta^2 - 1} f_{BR} \approx \alpha_0 f_{AR} = \frac{1}{2 \pi \tau_T}$$

Where $\tau_T$ is the transit time / sum of all delays

$$\tau_T \approx \frac{C_h}{g_m} + \tau = \frac{C_p}{g_m} + \tau_1 + \tau_2 + ...$$

Kirk-Effect ("Base spreading")

At high currents, the electron density $n_C$ in the collector becomes comparable to the donor density (npn BJT). Therefore, it cannot be neglected for the calculation of the E-Field in the collector:

$$E(x) = \frac{q}{\varepsilon_S} [(N_{DC} - n_C)x + E_{depletion}]$$

Base spread (increases $\tau_B$, reduces $\beta$)

$$W_K = W_C \left[ 1 - \frac{V_{CB} V_{CA}}{(n_C/N_{DC}) - 1} \right]$$

Kirk effect threshold current

$$J_K = q * v_{sat} * N_{DC} \left( 1 + \frac{2 \varepsilon_S V_{CB}}{q N_{DC} W_C^2} \right)$$
Base drift field

Carrier transport across the base can be aided by introducing an electric field, such as by non-flat base doping profiles / grading the doping.

P-type base with width $W_B$ with an electric field:

$$n_B(x) = -\frac{I_n W_B}{q D_n} \frac{1 - e^{-\frac{x}{W_B}}}{\eta}, \quad \eta = \frac{W_B}{x_0}$$

$\eta$ : accelerating field factor / grading

$$\tau_B = \frac{W_B^2}{D_n} \left( \frac{1 + e^{-\eta}}{\eta^2} \right), \quad \tau_B(\eta = 0) = \frac{W_B^2}{2D_{NB}}$$

NPN base charge: $Q_B = \int_0^{W_B} -q n_B \, dx \quad [C/cm^2]$

Heterojunction Bipolar Transistor (HBT)

Different materials and bandgaps for emitter & base

$$\beta = \beta_{BJT} \frac{n_{IB}^2}{n_{IE}^2} = \beta_{BJT} e^{\frac{\Delta E_F}{kT}}$$

6. MOSFET

In contrast to BJTs majority devices (majority carrier)

N-Channel: electrons, P-Channel: holes

Depletion Mode: channel present at equilibrium
Enhancement Mode: no channel at equilibrium

Basic characteristics

Inversion layer has thickness $X$, charge density $Q_n$

$$Q_n = -q n X \equiv -C_{OX} \left( V_{GS} - V_T - V(x) \right)$$

$$X = \frac{C_{OX} \left( V_{GS} - V_T - V(x) \right)}{q n}, \quad z: \text{width}$$

$$I_{CH} = I_B = \frac{\mu_n C_{OX} Z}{2} \left[ 2(V_{GS} - V_T) V_{DS} - V_{DS}^2 \right]$$

Pinch-Off

Pinch-off when $V_{DS} \geq V_{GS} - V_T$ at drain side

Linear region: $V_{DS} < V_{GS} - V_T$

$$I_{CH} = I_B = \frac{\mu_n C_{OX} Z}{2} \left[ 2(V_{GS} - V_T) V_{DS} - V_{DS}^2 \right]$$

Saturation region: $V_{DS} \geq V_{GS} - V_T$

$$I_{DSat} = \frac{\mu_n C_{OX} Z}{2} \left( V_{GS} - V_T \right)^2$$

V(x): channel voltage; $V(0) = V_S = 0$, $V(L) = V_{DS}$

$V_T$: threshold voltage for strong inversion

Structure

Drain-Source voltage $V_{DS}$: low for uniform channel
Gate-Source voltage $V_{GS}$: large enough for channel

Channel is built by minority carriers between S & D

Sheet resistance

$$R_S = \rho * \frac{\text{Length}}{\text{Area}} = \frac{\rho}{\text{Thickness}} \left[ \Omega/m^2 \right]$$

$$R_S(x) = \frac{1}{\mu_n C_{OX} \left( V_{GS} - V_T - V(x) \right)}$$
Transconductance in Saturation region

\[ g_m = \frac{dI_{DS}}{dV_{GS}} = \frac{\mu_n C_{OX} Z}{L} (V_{GS} - V_T) \]

Channel length modulation (L12P2)

Assume \( \Delta L \ll L \): channel length independent of \( V_{DS} \).

When we cannot assume \( \Delta L \ll L \), we have a short-channel MOSFET whose drain-current increases with increasing \( V_{DS} \). Like Early for BJTs.

At Equilibrium, the Fermi Level must be constant!

As the metal workfunction differs from the semiconductor workfunction, there will be bandbending.

Flatband voltage: Gate voltage that makes them flat.

Reducing the channel length increases:
- transconductance \( g_m \)
- operation speed
- device density

But \( V_T \) decreases (threshold voltage shift)

Electron affinity: \( \chi = E_0 - E_C \) [eV]

Work function: \( \Phi = E_0 - E_F \) [eV]

Vacuum level: \( E_0 \)

Band diagramm

General potential: \( q \psi(x) = E_i - E_c(x) \)

Bulk potential: \( q \psi_B(x) = E_i - E_F \)

\[ \psi_B = \frac{kT}{q} \ln \left( \frac{N_A}{n_i} \right) \]

\[ p_p = n_i \frac{e^{-(E_F - E_i)/kT}}{e^{(E_F - E_i)/kT}} = n_i e^{(E_F - E_i)/kT} \]

\[ n_p = n_i \frac{e^{-(E_F - E_i)/kT}}{e^{(E_F - E_i)/kT}} \]

Midgap: \( \psi_S = \psi_B \), \( n_p = p_p = n_i \)

Depletion region width

\[ \psi(x) = \psi_S \left( 1 - \frac{x}{W} \right)^2, \quad \psi_S = \frac{qN_A W^2}{2\epsilon_S} \]

Depletion width: \( W = \sqrt{\frac{2\epsilon_S \psi_S}{qN_A}} \)

Max. at inversion: \( W_{max} = \sqrt{\frac{2\epsilon_S \psi_B}{qN_A}}, \psi_S = 2\psi_B \)

Electric field: \( E_S(x) = \frac{qN_A}{\epsilon} (W - x) \)
Ideal gate voltage relationship

\[ V_G = V_{ox} + \Psi_S = d \cdot E_{ox} + \Psi_S \]

\[ V_G : \text{Potential drop across oxide} \& \text{semiconductor} \]

\[ V_{ox} = \sqrt{\frac{2 q \varepsilon N_A \Psi_S}{C_{ox}}} \quad \text{and} \quad C_{ox} = \frac{\varepsilon_{ox}}{d} \]

\[ V_G = \sqrt{\frac{2 q \varepsilon N_A \Psi_S}{C_{ox}}} + \Psi_S \]

Threshold voltage \((\Psi_S = 2 \Psi_B)\)

\[ V_T = \sqrt{\frac{2 q \varepsilon N_A 2 \Psi_B}{C_{ox}}} + 2 \cdot \Psi_B \]

After that, \(W\) is maximal and stays more or less

Non-ideal gate voltage relationship \(\text{(voltage shift)}\)

Bands are not flat due to
1. Workfunction difference \(\Psi_{ms} = (\Phi_m - \Phi_S) / q\)
2. Fixed charges inside the oxide

\[ V_G = V_G^i + V_{FB} \quad \text{\(V_G^i\): ideal gate voltage} \]

\[ V_{FB} = \Psi_{ms} - \frac{1}{\varepsilon_S} \int_{\text{ozone}} x \cdot \rho_{ox}(x) \, dx \quad \varepsilon_S = C_o \cdot d \]

If the charge in the oxide is fixed:

\[ V_{FB} = \Psi_{ms} - \frac{Q_0}{C_{ox}} \quad \text{and} \quad Q_0 \left[ \frac{C}{cm^2} \right] \]

MOS Capacitance

\[ C = \frac{C_{ox} \cdot C_j}{C_{ox} + C_j} \quad \text{\(C_j\): depletion capacitance} \]

\[ C = \frac{1}{\sqrt{1 + \frac{2 \varepsilon_{ox} V}{q N_A \varepsilon_S d}}} \quad \varepsilon_{ox} = \frac{\varepsilon_{ox}}{d} \]

Accumulation: only majority carriers can respond to fast AC signal \(\rightarrow\) added delta-charge

Deep depletion: DC bias changes so fast that minority carriers cannot respond. Therefore, the depletion layer keeps increasing

Subthreshold swing

Subthreshold regime: \(V_G < V_T\)

\[ S = \frac{1}{d} \left( \frac{d \ln(J_0)}{d V_G} \right) \]

Subthr. Swing: how effective can it be turned on / off

7. Various / General

Direct and Indirect Bandgaps

A transition in an indirect bandgap material must necessarily include an interaction with the crystal so that crystal momentum is conserved

Material properties

<table>
<thead>
<tr>
<th>Selected Gate Materials</th>
<th>Work Function (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-Polysilicon</td>
<td>4.0</td>
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<tr>
<td>Al</td>
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<tr>
<td>W</td>
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<tr>
<td>SiO2</td>
<td>9.0</td>
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<tr>
<td>PtSi</td>
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</tr>
<tr>
<td>TiO2</td>
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<table>
<thead>
<tr>
<th>Selected Gate Insulators</th>
<th>Bandgap (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO2</td>
<td>5.3</td>
</tr>
<tr>
<td>Al2O3</td>
<td>5.0</td>
</tr>
<tr>
<td>TiO2</td>
<td>5.3</td>
</tr>
<tr>
<td>HfO2</td>
<td>5.3</td>
</tr>
</tbody>
</table>
**Tipps & Tricks**

Energy: \[ E = \int q \cdot \varepsilon \cdot dx \]
\[ E_{\text{kin}} + E_{\text{pot}} = \text{const.} \]

E Field:
\[ E = \frac{\varepsilon}{\varepsilon_0} \frac{dv}{dx} = -\frac{q}{d} \]

Potential:
\[ V = -\int E \cdot dx = -\frac{1}{q} \varepsilon_0 \]

Charge density:
\[ \rho = \varepsilon_0 \varepsilon_0 \frac{dv}{dx} \]
Charge d. with depletion approximation:
\[ q \cdot (N_A - N_D) \]

---

**Diamond structure**

\[ \frac{a}{4} \]

Diagonale

\[ \frac{a}{2} \]

---

\[ E_{\text{pot}} = E_z \text{ (electron)} \]
\[ E_{\text{pot}} = E_z \text{ (holes)} \]

\[ E = \frac{dV}{dx} = \frac{1}{q} \frac{dE_z}{dx} \]

\[ n = n_0 e^{E_z} \]
\[ p = n_0 e^{E_z} \]

logarithmische Achse:
\[ \log_{10}(n) = \log_{10}(n_0) + \frac{E_z - E_i}{kT} \]
\[ \log_{10}(p) = \log_{10}(n_0) - \frac{E_F - E_i}{kT} \]

\[ j_{\text{diff}} = q \frac{\mu_N}{d} \frac{dn}{dx} \]