Exercise Session

Week 05

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Today's Overview

polybox for session material   Mail to TA

Special

- From now on: 90min sessions, no break, earlier lunch
- Please fill out the evaluation survey

Today's Topics

Comments on last [code]expert Exercises

- I'll usually be done with correcting on Sunday morning. Please make sure to have a look at my feedback before the next exercise session
- Give you variables descriptive names
- Use more comments
- Use tabs to indent code
- `unsigned int` representation $\neq$ `int` representation (check Handout02.pdf on the polybox)

Question or Comments re: Exercises?

Learning Objectives Checklist

Now I...

- ☐ have refreshed my memory on how ”two’s complement representation” works
- ☐ know how floating point numbers are stored
- ☐ can convert a non-integer number into its binary representation
- ☐ can compute the elements of the set $F(\beta, p, e_{min}, e_{max})$
- ☐ can perform arithmetic operations in the set $F(\beta, p, e_{min}, e_{max})$
- ☐ know the floating point guidelines and the reasoning behind them

Recap: Binary Representation ...but which one?

Math. Decimal	Math. Binary	`int`	`unsigned int`

Recap: Binary Representation ...but which one?

Math. Decimal	Math. Binary	`int`	`unsigned int`
42_{10}			

Recap: Binary Representation ...but which one?

Math. Decimal	Math. Binary	int	unsigned int
42_{10}	101010_2	101010	101010

Recap: Binary Representation ...but which one?

Math. Decimal	Math. Binary	int	unsigned int
42_{10}	101010_2	101010	101010
-42_{10}			

Recap: Binary Representation ...but which one?

Math. Decimal	Math. Binary	int	unsigned int
42_{10}	101010_2	101010	101010
-42_{10}	-101010_2	1010110	nope

Recap: Binary Representation ...but which one?

Math. Decimal	Math. Binary	`int`	`unsigned int`
42_{10}	101010_2	`101010`	`101010`
-42_{10}	-101010_2	`1010110`	nope

`int` stores numbers with *two's complement representation*.

(Technically, the rest of the binary-representation-numbers would be filled up (to the left) with either `1`'s or `0`'s depending on the type/representation because an `int` stores numbers in a fixed amount of space, but we're not covering this, so don't worry about that.)

Binary Arithmetic

Task

→binary, add, →decimal

Perform the following steps:

1. Convert the integer numbers `a = 4` and `b = 7` into their binary representation (not two's complement)
2. Add them in their binary representation
3. Convert the result into decimal

Binary Arithmetic

Task

Perform the following steps:

1. Convert the integer numbers `a = 4` and `b = 7` into their binary representation (not two's complement)
2. Add them in their binary representation
3. Convert the result into decimal

Solution

$a = 4_{10} = 100_2$

Binary Arithmetic

Task

Perform the following steps:

1. Convert the integer numbers `a = 4` and `b = 7` into their binary representation (not two's complement)
2. Add them in their binary representation
3. Convert the result into decimal

Solution

$a = 4_{10} = 100_2$
$b = 7_{10} = 111_2$

1011

Binary Arithmetic

Task

Perform the following steps:

1. Convert the integer numbers `a = 4` and `b = 7` into their binary representation (not two's complement)
2. Add them in their binary representation
3. Convert the result into decimal

Solution

$a = 4_{10} = 100_2$
$b = 7_{10} = 111_2$
$100_2 + 111_2 = 1011_2$

Binary Arithmetic

Task

Perform the following steps:

1. Convert the integer numbers `a = 4` and `b = 7` into their binary representation (not two's complement)
2. Add them in their binary representation
3. Convert the result into decimal

Solution

$a = 4_{10} = 100_2$
$b = 7_{10} = 111_2$
$100_2 + 111_2 = 1011_2$
$1011_2 = 11_{10}$

Expressions

Task

Evaluate the following expressions:

1. `(5 < 4) < 1` → true
2. `true > false`

1 > 0 → true

Expressions

Task

Evaluate the following expressions:

1. `5 < 4 < 1`
2. `true > false`

Solution 1

`5 < 4 < 1`

Expressions

Task

Evaluate the following expressions:

1. `5 < 4 < 1`
2. `true > false`

Solution 1

```
5 < 4 < 1
(5 < 4) < 1
```

Expressions

Task

Evaluate the following expressions:

1. `5 < 4 < 1`
2. `true > false`

Solution 1

```
5 < 4 < 1
(5 < 4) < 1
false < 1
```

Expressions

Task

Evaluate the following expressions:

1. `5 < 4 < 1`
2. `true > false`

Solution 1

```
5 < 4 < 1
(5 < 4) < 1
false < 1
0 < 1
```

Expressions

Task

Evaluate the following expressions:

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Solution 1

```
5 < 4 < 1
(5 < 4) < 1
false < 1
0 < 1
true
```

Expressions

Task

Evaluate the following expressions:

1. `5 < 4 < 1`
2. `true > false`

Solution 1

```
5 < 4 < 1
(5 < 4) < 1
false < 1
0 < 1
true
```

Solution 2

```
true > false
```

Expressions

Task

Evaluate the following expressions:

1. `5 < 4 < 1`
2. `true > false`

Solution 1

```
5 < 4 < 1
(5 < 4) < 1
false < 1
0 < 1
true
```

Solution 2

```
true > false
1 > 0
```

Expressions

Task

Evaluate the following expressions:

1. `5 < 4 < 1`
2. `true > false`

Solution 1

```
5 < 4 < 1
(5 < 4) < 1
false < 1
0 < 1
true
```

Solution 2

```
true > false
1 > 0
true
```

Questions?

Binary Representation

binary	1	1	1	1	.	1	1	1
decimal	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}
	8	4	2	1	.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

Binary Representation

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decimal	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}
	8	4	2	1	.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

Tasks

1. 11.01_2 in decimal
2. 101.1_2 in decimal
3. 7.125_{10} in binary
4. 4.375_{10} in binary
5. 1.1 in binary

Binary Representation

binary	1	1	1	1	.	1	1	1
decimal	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}
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Tasks

1. 11.01_2 in decimal
2. 101.1_2 in decimal
3. 7.125_{10} in binary
4. 4.375_{10} in binary
5. 1.1 in binary

Solutions

1. $2 + 1 + 0 + \frac{1}{4} = 3.25_{10}$

Binary Representation

binary	1	1	1	1	.	1	1	1
decimal	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}
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Tasks

1. 11.01_2 in decimal
2. 101.1_2 in decimal
3. 7.125_{10} in binary
4. 4.375_{10} in binary
5. 1.1 in binary

Solutions

1. $2 + 1 + 0 + \frac{1}{4} = 3.25_{10}$
2. $4 + 0 + 1 + \frac{1}{2} = 5.5_{10}$

Binary Representation

binary	1	1	1	1	.	1	1	1
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Tasks

1. 11.01_2 in decimal
2. 101.1_2 in decimal
3. 7.125_{10} in binary
4. 4.375_{10} in binary
5. 1.1 in binary

Solutions

1. $2 + 1 + 0 + \frac{1}{4} = 3.25_{10}$
2. $4 + 0 + 1 + \frac{1}{2} = 5.5_{10}$
3. 111.001_2

Binary Representation

binary	1	1	1	1	.	1	1	1
decimal	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}
	8	4	2	1	.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

Tasks

1. 11.01_2 in decimal
2. 101.1_2 in decimal
3. 7.125_{10} in binary
4. 4.375_{10} in binary
5. 1.1 in binary

Solutions

1. $2 + 1 + 0 + \frac{1}{4} = 3.25_{10}$
2. $4 + 0 + 1 + \frac{1}{2} = 5.5_{10}$
3. 111.001_2
4. 100.011_2

Binary Representation

binary	1	1	1	1	.	1	1	1
decimal	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}
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Tasks

1. 11.01_2 in decimal
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3. 7.125_{10} in binary
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5. 1.1 in binary

Solutions

1. $2 + 1 + 0 + \frac{1}{4} = 3.25_{10}$
2. $4 + 0 + 1 + \frac{1}{2} = 5.5_{10}$
3. 111.001_2
4. 100.011_2
5. $1.1_{10} = 1.000110..._2$

Binary Representation

My way of transforming decimal into binary

1. calculate the numbers before the decimal point

Binary Representation

My way of transforming decimal into binary

1. calculate the numbers before the decimal point
2. write that number_2 down. now look at the number_{10}.rest

Binary Representation

My way of transforming decimal into binary

1. calculate the numbers before the decimal point
2. write that number_2 down. now look at the number_{10}.rest
3. can you substract $\frac{1}{2^n}$ from number_{10}?

Binary Representation

My way of transforming decimal into binary

1. calculate the numbers before the decimal point
2. write that number_2 down. now look at the number_{10}.rest
3. can you substract $\frac{1}{2^n}$ from number_{10}?
   if possible, subtract $\frac{1}{2^n}$ from the number_{10} and add a 1 at the end of your number_2

Binary Representation

My way of transforming decimal into binary

1. calculate the numbers before the decimal point
2. write that $number_2$ down. now look at the $number_{10}$.rest
3. can you substract $\frac{1}{2^n}$ from $number_{10}$?
   if possible, subtract $\frac{1}{2^n}$ from the $number_{10}$ and add a 1 at the end of your $number_2$
   else add a 0 at the end of your $number_2$

Binary Representation

0,5
0,25
0,12 - -

My way of transforming decimal into binary

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2. write that number_2 down. now look at the number_{10}.rest
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   if possible, subtract $\frac{1}{2^n}$ from the number_{10} and add a 1 at the end of your number_2
   else add a 0 at the end of your number_2
4. if $\text{number}_{10} = 0$, stop and check your solution
   else, `n++;` and go to 3. again

Binary Representation

My way of transforming decimal into binary

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   if possible, subtract $\frac{1}{2^n}$ from the number_{10} and add a 1 at the end of your number_2
   else add a 0 at the end of your number_2
4. if $\text{number}_{10} = 0$, stop and check your solution
   else, `n++;` and go to 3. again
5. check solution again and remember $2^{-2} \neq \frac{1}{2}$ (common mistake)

Normalized Floating Point Systems

$F^*(\beta, p, e_{min}, e_{max})$

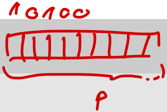
$*$ normalized ($b_0 \neq 0$)

Normalized Floating Point Systems

$F^*(\beta, p, e_{min}, e_{max})$

$*$	normalized ($b_0 \neq 0$)
$\beta \geq 2$	base

Normalized Floating Point Systems



$F^*(\beta, p, e_{min}, e_{max})$

$*$	normalized ($b_0 \neq 0$)
$\beta \geq 2$	base
$p \geq 1$	precision (number of places)

Normalized Floating Point Systems

$F^*(\beta, p, e_{min}, e_{max})$

$*$	normalized ($b_0 \neq 0$)
$\beta \geq 2$	base
$p \geq 1$	precision (number of places)
e_{min}	smallest possible exponent

Normalized Floating Point Systems

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$*$	normalized ($b_0 \neq 0$)
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Normalized Floating Point Systems

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$*$	normalized ($b_0 \neq 0$)
$\beta \geq 2$	base
$p \geq 1$	precision (number of places)
e_{min}	smallest possible exponent
e_{max}	largest possible exponent

...describes numbers of the form:

$$\pm d_0.d_1 d_2 d_3 \ldots d_{p-1} \cdot \beta^e$$

$d_0 \neq 0$ p $\sim 4-5$

Normalized Floating Point Systems

$F^*(\beta, p, e_{min}, e_{max})$

$*$	normalized ($b_0 \neq 0$)
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$d_i \in \{0, \ldots, b-1\}$

Normalized Floating Point Systems

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$d_0 \neq 0$

Normalized Floating Point Systems

$F^*(\beta, p, e_{min}, e_{max})$

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...describes numbers of the form:

$$\pm d_0.d_1 d_2 d_3 \ldots d_{p-1} \cdot \beta^e$$

0, 1, ... 9, a, b ... f

$d_i \in \{0, \ldots, b-1\}$
$d_0 \neq 0$
$e \in [e_{min}, e_{max}]$

Questions?

Exercise

Exercise

Are the following numbers in the set $F^*(2, 4, -2, 2)$?

$$0.000 \cdot 2^{1} = 0_{10}$$
$$1.000 \cdot 2^{1} = 2_{10}$$
$$1.001 \cdot 2^{-1} = 0.5625_{10}$$
$$1.0001 \cdot 2^{-1} = 0.53125_{10}$$
$$1.111 \cdot 2^{-2} = 0.46875_{10}$$
$$1.111 \cdot 2^{5} = 60_{10}$$

Exercise

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Are the following numbers in the set $F^*(2, 4, -2, 2)$?

$$
\begin{aligned}
0.000 \cdot 2^1 &= 0_{10} \\
1.000 \cdot 2^1 &= 2_{10} \\
1.001 \cdot 2^{-1} &= 0.5625_{10} \\
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1.111 \cdot 2^5 &= 60_{10}
\end{aligned}
$$

Solutions

is in F^*

$$
\begin{aligned}
1.000 \cdot 2^1 &= 2_{10} \\
1.001 \cdot 2^{-1} &= 0.5625_{10} \\
1.111 \cdot 2^{-2} &= 0.46875_{10}
\end{aligned}
$$

Exercise

Exercise

Are the following numbers in the set $F^*(2,4,-2,2)$?

$$
\begin{aligned}
0.000 \cdot 2^1 &= 0_{10} \\
1.000 \cdot 2^1 &= 2_{10} \\
1.001 \cdot 2^{-1} &= 0.5625_{10} \\
1.0001 \cdot 2^{-1} &= 0.53125_{10} \\
1.111 \cdot 2^{-2} &= 0.46875_{10} \\
1.111 \cdot 2^5 &= 60_{10}
\end{aligned}
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Solutions

is in F^*

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\begin{aligned}
1.000 \cdot 2^1 &= 2_{10} \\
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\end{aligned}
$$

is not in F^*

$0.000 \cdot 2^1$

Exercise

Exercise

Are the following numbers in the set $F^*(2, 4, -2, 2)$?

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1.111 \cdot 2^{-2} &= 0.46875_{10} \\
1.111 \cdot 2^5 &= 60_{10}
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Solutions

is in F^*

$$\begin{aligned}
1.000 \cdot 2^1 &= 2_{10} \\
1.001 \cdot 2^{-1} &= 0.5625_{10} \\
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is not in F^*

$0.000 \cdot 2^1$ not ”normalizable”

$1.0001 \cdot 2^{-1}$

Exercise

Exercise

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Solutions

is in F^*

$$1.000 \cdot 2^1 = 2_{10}$$
$$1.001 \cdot 2^{-1} = 0.5625_{10}$$
$$1.111 \cdot 2^{-2} = 0.46875_{10}$$

is not in F^*

$0.000 \cdot 2^1$ not ”normalizable”

$1.0001 \cdot 2^{-1}$ $5 > p = 4$

$1.111 \cdot 2^5$

Exercise

Exercise

Are the following numbers in the set $F^*(2, 4, -2, 2)$?

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Solutions

is in F^*

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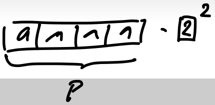
\begin{aligned}
1.000 \cdot 2^1 &= 2_{10} \\
1.001 \cdot 2^{-1} &= 0.5625_{10} \\
1.111 \cdot 2^{-2} &= 0.46875_{10}
\end{aligned}
$$

is not in F^*

$0.000 \cdot 2^1$	not "normalizable"
$1.0001 \cdot 2^{-1}$	$5 > p = 4$
$1.111 \cdot 2^5$	$5 \notin [-2, 2]$

Questions?

Exercises



Exercise

State the following numbers in $F^*(2, 4, -2, 2)$ in decimal

1. the largest number $7.5_{10} = 1.111 \cdot 2^2$
2. the smallest number -7.5_{10}
3. the smallest non-negative number $0.125_{10} = 1.000 \cdot 2^{-2}$ ($\beta^{e_{min}}$)

How many numbers are in $F^*(2, 4, -2, 2)$?

Exercises

Exercise

State the following numbers in $F^*(2, 4, -2, 2)$ in decimal

1. the largest number
2. the smallest number
3. the smallest non-negative number

How many numbers are in $F^*(2, 4, -2, 2)$?

Solution

largest:

Exercises

Exercise

State the following numbers in $F^*(2, 4, -2, 2)$ in decimal

1. the largest number
2. the smallest number
3. the smallest non-negative number

How many numbers are in $F^*(2, 4, -2, 2)$?

Solution

largest: $1.111 \cdot 2^2 = 7.5_{10}$

smallest:

Exercises

Exercise

State the following numbers in $F^*(2, 4, -2, 2)$ in decimal

1. the largest number
2. the smallest number
3. the smallest non-negative number

How many numbers are in $F^*(2, 4, -2, 2)$?

Solution

largest:	$1.111 \cdot 2^2 = 7.5_{10}$
smallest:	$-1.111 \cdot 2^2 = -7.5_{10}$
smallest $>$ 0:	

Exercises

Exercise

State the following numbers in $F^*(2, 4, -2, 2)$ in decimal

1. the largest number
2. the smallest number
3. the smallest non-negative number

How many numbers are in $F^*(2, 4, -2, 2)$?

Solution

largest:	$1.111 \cdot 2^2 = 7.5_{10}$
smallest:	$-1.111 \cdot 2^2 = -7.5_{10}$
smallest > 0:	$1.000 \cdot 2^{-2} = 0.25_{10}$

Normalized Floating Point Systems

Solution

For a fixed exponent there are three digits we can vary freely, and for each number also the negative number is in the set, thus resulting in $2 \cdot 2^3 = 16$ numbers per exponent. There are 5 possible exponents, thus resulting in $5 \cdot 16 = 80$ numbers. Notice that in normalized number systems we cannot count some numbers twice, as we've seen in the lecture that the representation of a number is unique.

Trick

For given $F^*(\beta, p, e_{min}, e_{max})$:

Largest:	$1.11\ldots1 \cdot 2^{e_{max}}$
Smallest:	$-$Largest
Smallest $>$ 0:	$1.00\ldots0 \cdot 2^{e_{min}}$

Questions?

Arithmetic in F^*

Adding floats

1. Bring both numbers to the same exponent
2. Add the significand as binary numbers
3. Re-normalize the sum $1.\cdots\cdot 2^{\cdot\cdot}$
4. Round if necessary

Arithmetic in F^*

Adding floats

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4. Round if necessary

Example

$F^*(2, 6, -2, 3)$
$1.125_{10} + 9.25_{10}$ → $1.001 \cdot 2^0$ +
$1001.01 \cdot 2^0$

Arithmetic in F^*

Adding floats

1. Bring both numbers to the same exponent
2. Add the significand as binary numbers
3. Re-normalize the sum
4. Round if necessary

Example

$F^*(2, 6, -2, 3)$
$1.125_{10} + 9.25_{10}$
$1.001_2 + 1001.01_2$ (already same exponent ($\cdot 2^0$))

Arithmetic in F^*

Adding floats

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$F^*(2, 6, -2, 3)$
$1.125_{10} + 9.25_{10}$
$1.001_2 + 1001.01_2$ (already same exponent ($\cdot 2^0$))
1010.011 (add them like you did in primary school)

Arithmetic in F^*

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$1.125_{10} + 9.25_{10}$
$1.001_2 + 1001.01_2$ (already same exponent ($\cdot 2^0$))
1010.011 (add them like you did in primary school)
$1.010011 \cdot 2^3$ Re-normalizing, adjust e and "cut" for p

Arithmetic in F^*

Adding floats

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4. Round if necessary

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$F^*(2, 6, -2, 3)$
$1.125_{10} + 9.25_{10}$
$1.001_2 + 1001.01_2$ (already same exponent ($\cdot 2^0$))
1010.011 (add them like you did in primary school)
$1.010011 \cdot 2^3$ Re-normalizing, adjust *e* and "cut" for *p*
$1.01010 \cdot 2^3$ Rounding, like decimal: 1 up, 0 down, and carry

Arithmetic in F^*

Adding floats

1. Bring both numbers to the same exponent
2. Add the significand as binary numbers
3. Re-normalize the sum
4. Round if necessary

Example

$F^*(2, 6, -2, 3)$
$1.125_{10} + 9.25_{10}$
$1.001_2 + 1001.01_2$ (already same exponent ($\cdot 2^0$))
1010.011 (add them like you did in primary school)
$1.010011 \cdot 2^3$ Re-normalizing, adjust e and "cut" for p
$1.01010 \cdot 2^3$ Rounding, like decimal: 1 up, 0 down, and carry
$1.01010_2 \cdot 2_2^3 = 1010.10_2 = 10.5_{10}$

Arithmetic in F^*

Adding floats

1. Bring both numbers to the same exponent
2. Add the significand as binary numbers
3. Re-normalize the sum
4. Round if necessary

Example

$F^*(2, 6, -2, 3)$
$1.125_{10} + 9.25_{10}$
$1.001_2 + 1001.01_2$ (already same exponent ($\cdot 2^0$))
1010.011 (add them like you did in primary school)
$1.010011 \cdot 2^3$ Re-normalizing, adjust e and "cut" for p
$1.01010 \cdot 2^3$ Rounding, like decimal: 1 up, 0 down, and carry
$1.01010_2 \cdot 2^3_2 = 1010.10_2 = 10.5_{10} \neq 10.375_{10}$

Why 10.5 and not 10.375?

Simply because the exact number 10.375 *can't* be represented in the F^* we were given. The nearest number that *is* in the set F^* is 10.5. This is why floats can sometimes be dangerous and we must follow the *floating point guidelines*.

(By the way: It's not 10.25, because we're rounding up in this case, even if the difference to from both 10.25 and 10.5 to 10.375 is 0.125.)

Exercise

Exercise

Add $1.001 \cdot 2^{-1} = 0.5625_{10}$
and $1.111 \cdot 2^{-2} = 0.46875_{10}$
in $F^*(2, 4, -2, 2)$.

$10.010 \cdot 2^{-2}$
$1.111 \cdot 2^{-2}$
100.001

Exercise

Exercise

Add $1.001 \cdot 2^{-1} = 0.5625_{10}$
and $1.111 \cdot 2^{-2} = 0.46875_{10}$
in $F^*(2, 4, -2, 2)$.

Solution

1. Bring both to same exponent, say -1
   $1.001 \cdot 2^{-1} + 0.1111 \cdot 2^{-1}$

Exercise

Exercise

Add $1.001 \cdot 2^{-1} = 0.5625_{10}$
and $1.111 \cdot 2^{-2} = 0.46875_{10}$
in $F^*(2, 4, -2, 2)$.

Solution

1. Bring both to same exponent, say -1
   $1.001 \cdot 2^{-1} + 0.1111 \cdot 2^{-1}$
2. Add them as binary numbers: $10.0001 \cdot 2^{-1}$ → $1.00001 \cdot 2^{0}$

$p = 4$

1.000

Exercise

Exercise

Add $1.001 \cdot 2^{-1} = 0.5625_{10}$
and $1.111 \cdot 2^{-2} = 0.46875_{10}$
in $F^*(2, 4, -2, 2)$.

Solution

1. Bring both to same exponent, say -1
   $1.001 \cdot 2^{-1} + 0.1111 \cdot 2^{-1}$
2. Add them as binary numbers: $10.0001 \cdot 2^{-1}$
3. Re-normalize: $1.00001 \cdot 2^{0}$

Exercise

Exercise

Add $1.001 \cdot 2^{-1} = 0.5625_{10}$
and $1.111 \cdot 2^{-2} = 0.46875_{10}$
in $F^*(2, 4, -2, 2)$.

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   $1.001 \cdot 2^{-1} + 0.1111 \cdot 2^{-1}$
2. Add them as binary numbers: $10.0001 \cdot 2^{-1}$
3. Re-normalize: $1.00001 \cdot 2^{0}$
4. Round: $1.000 \cdot 2^{0}$

Exercise

Exercise

Add $1.001 \cdot 2^{-1} = 0.5625_{10}$
and $1.111 \cdot 2^{-2} = 0.46875_{10}$
in $F^*(2, 4, -2, 2)$.

Solution

1. Bring both to same exponent, say -1
   $1.001 \cdot 2^{-1} + 0.1111 \cdot 2^{-1}$
2. Add them as binary numbers: $10.0001 \cdot 2^{-1}$
3. Re-normalize: $1.00001 \cdot 2^{0}$
4. Round: $1.000 \cdot 2^{0} = 1_{10}$

Exercise

Exercise

Add $1.001 \cdot 2^{-1} = 0.5625_{10}$
and $1.111 \cdot 2^{-2} = 0.46875_{10}$
in $F^*(2, 4, -2, 2)$.

Solution

1. Bring both to same exponent, say -1
   $1.001 \cdot 2^{-1} + 0.1111 \cdot 2^{-1}$
2. Add them as binary numbers: $10.0001 \cdot 2^{-1}$
3. Re-normalize: $1.00001 \cdot 2^{0}$
4. Round: $1.000 \cdot 2^{0} = 1_{10} \neq 1.03125_{10}$

Floating Point guidelines

go to Floating_Point_Guidelines.pdf
+ maybe a short live demo on less ugly code

How to Compare Floating Point Numbers

go to Comparing_FP.pdf

How to Compare Floating Point Numbers

go to Comparing_FP.pdf

in short: don't check for equality,
check for "being-within-tolerance"

```
// Example of "equality"-check function for floats
bool equal(double x, double y, double tol){
   double diff = x - y;
   if(diff < 0){
      diff *= -1;
   }
   return (diff < tol);
}
```

Exam Question

YOUR EXAM WILL BE DIFFERENT. DONT'T FORGET THAT

Geben Sie ein möglichst knappes normalisiertes Fliesskommazahlensystem an, mit welchem sich die folgenden dezimalen Werte gerade noch genau darstellen lassen: jede Verkleinerung von p, $e_{\max}$ oder $-e_{\min}$ muss dazu führen, dass mindestens eine Zahl nicht mehr dargestellt werden kann.
Hinweis: p zählt auch die führende Ziffer.
Tipp: Schreiben Sie sich die normalisierte Binärzahldarstellung der Werte auf, wenn sie für Sie nicht offensichtlich ist.

Provide a smallest possible normalized floating point number system that can still represent the following values exactly: any decrease of the numbers p, $e_{\max}$ or $-e_{\min}$ must imply that at least one of the numbers cannot be represented any more.
Hint: p does also count the leading digit.
Tip: Write down the normalized binary representation of the values, if it is not obvious for you.

Werte / *Values*: $2.25,\ \frac{1}{8},\ 0.5,\ 16.5,\ 2^3$

$F^*(\beta, p, e_{\min}, e_{\max})$ mit / *with*

$\beta = 2 \quad , p = \qquad , e_{\min} = \qquad , e_{\max} = \qquad .$

Exam Question

YOUR EXAM WILL BE DIFFERENT. DONT'T FORGET THAT

Geben Sie ein möglichst knappes normalisiertes Fliesskommazahlensystem an, mit welchem sich die folgenden dezimalen Werte gerade noch genau darstellen lassen: jede Verkleinerung von p, $e_{\max}$ oder $-e_{\min}$ muss dazu führen, dass mindestens eine Zahl nicht mehr dargestellt werden kann.
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Werte / *Values*: $2.25,\ \frac{1}{8},\ 0.5,\ 16.5,\ 2^3$

$F^*(\beta, p, e_{\min}, e_{\max})$ mit / *with*

$\beta = 2$, $p = 6$, $e_{\min} = -3$, $e_{\max} = 4$.

Questions?