Datastructures and Algorithms

Introduction, Logistics, Asymptotics ($\mathcal{O}$, Ω, Θ)

Adel Gavranović — ETH Zürich — 2025

Overview

- Learning Objectives
- Exercise Management
- Repetition Theory
- Examples and Quiz on Theory
- Quiz on Asymptotic Running Time of Program Fragments
- Formulas and their Derivation*
- Past Exam Questions
- Tips for **code** expert

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Material

Webpage

Mail

1. Intro

Hello, World!

Welcome!

←

2. Follow-up

Follow-up from last session

Follow-up from last session

- This is where I explain things I missed (or messed up) in last week's session

3. Feedback regarding **code** expert

General things regarding **code** expert

General things regarding **code** expert

- This is where I mention very common mistakes that were made in the exercises on **code** expert
- Emails are welcome too

Any questions regarding **code** expert on your part?

Any questions regarding **code** expert on your part?

- This is where you'll have a chance to ask thigs regarding **code** expert that you think might be relevant for the class (hint: it almost always is)

4. Learning Objectives

Objectives

Objectives

- ☐ Know how the course is built up
- ☐ Understand the definitions of $\mathcal{O}$, Ω, and Θ
- ☐ Understand the uses of $\mathcal{O}$, Ω, and Θ

5. Summary

Getting on the same page

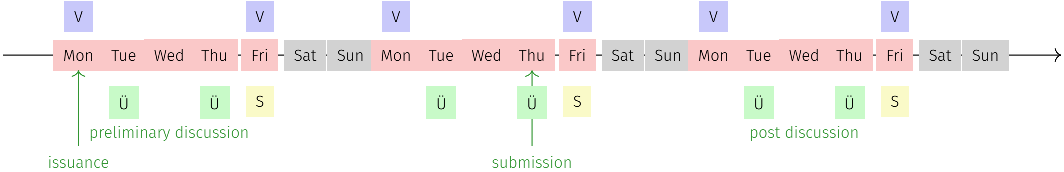
RAModel
vs.
PMModel
(Pointer machine)

Getting on the same page

- This is where we could talk about what happened during the week if you want to

6. Exercise Management

Exercises



- Exercises available at lecture time.
- Preliminary discussion in the following recitation session
- Submit the exercise at the lecture two weeks later. Exception: for the first exercise you only have one week to finish.
- Dicussion of the exercise in the recitation session after the deadline. Feedback within a week after discussion.

7. Repetition Theory

←

Warm-up

- What is a problem?

Warm-up

- What is a problem?
- What is an algorithm?

Warm-up

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- What is an algorithm?

  ➜ well-defined computing procedure to compute output data from input data.

Warm-up

- What is a problem?
- What is an algorithm?
  - ➜ well-defined computing procedure to compute output data from input data.
- What is a program?

Warm-up

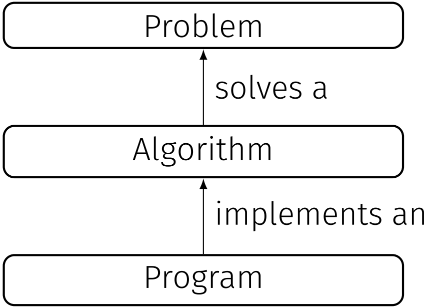
- What is a problem?
- What is an algorithm?

  ➜ well-defined computing procedure to compute output data from input data.

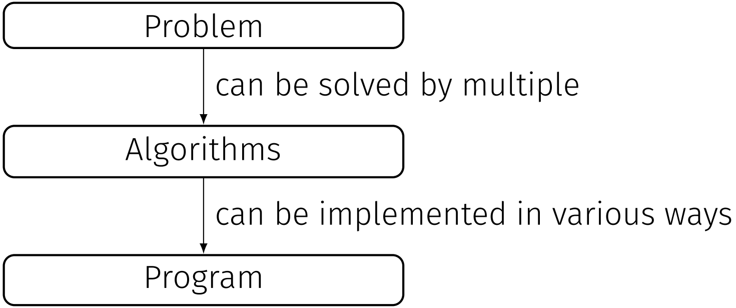
- What is a program?

  ➜ Concrete implementation of an algorithm

Problems, Algorithms and Programs



Warm-up


Problem
can be solved by multiple
Algorithms
can be implemented in various ways
Program

Efficiency

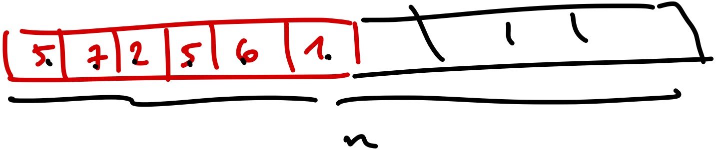
Program	Computing time	Measurable value on an actual machine.
Algorithm	Cost	Number of elementary operations
Problem	Complexity	Minimal (asymptotic) cost over all algorithms that solve the problem.

"$P \in O(n^2)$"

Efficiency

Program	Computing time	Measurable value on an actual machine.
Algorithm	Cost	Number of elementary operations
Problem	Complexity	Minimal (asymptotic) cost over all algorithms that solve the problem.

→ Estimation of cost or computing time depending on the input size, denoted by n.



Asymptotic behavior

- What are $\Omega(g(n))$, $\Theta(g(n))$, $\mathcal{O}(g(n))$?

Asymptotic behavior

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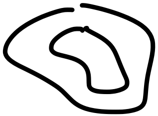
→ Sets of functions!

Asymptotic behavior

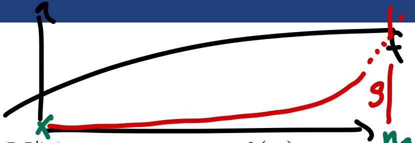
- What are $\Omega(g(n))$, $\Theta(g(n))$, $\mathcal{O}(g(n))$?

→ Sets of functions!

subset	$A \subseteq B$
proper subset	$A \subsetneq B$
intersection	$A \cap B$



Asymptotic behavior



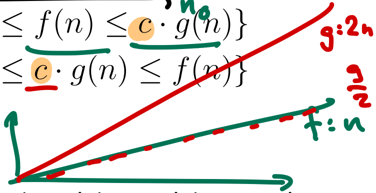
Given: function $g : \mathbb{N} \to \mathbb{R}$.

Definition:

$$\mathcal{O}(g) = \{f : \mathbb{N} \to \mathbb{R} | \exists c > 0, n_0 \in \mathbb{N} | \forall n \geq n_0 : 0 \leq f(n) \leq c \cdot g(n)\}$$

$$\Omega(g) = \{f : \mathbb{N} \to \mathbb{R} | \exists c > 0, n_0 \in \mathbb{N} | \forall n \geq n_0 : 0 \leq c \cdot g(n) \leq f(n)\}$$

$$\Theta(g) = \mathcal{O}(g) \cap \Omega(g)$$



Intuition:

$f \in \mathcal{O}(g)$: f grows asymptotically **not faster** than g. Algorithm with running time f is **not worse** than any other algorithm with g.

$f \in \Omega(g)$: f grows asymptotically **not slower** than g. Algorithm with running time f is **not better** than any other algorithm with g.

$f \in \Theta(g)$: f grows asymptotically **as fast** as g. Algorithm with running time f is **as good as** any other algorithm with g.

Used less often

Given: function $g : \mathbb{N} \to \mathbb{R}$.
Definition:

$$\mathcal{O}(g) = \{f : \mathbb{N} \to \mathbb{R} | \exists c > 0, n_0 \in \mathbb{N} | \forall n \geq n_0 : 0 \leq f(n) \leq c \cdot g(n)\}$$

$$o(g) = \{f : \mathbb{N} \to \mathbb{R} | \forall c > 0 \; \exists n_0 \in \mathbb{N} | \forall n \geq n_0 : 0 \leq f(n) \leq c \cdot g(n)\}$$

$$\Omega(g) = \{f : \mathbb{N} \to \mathbb{R} | \exists c > 0, n_0 \in \mathbb{N} | \forall n \geq n_0 : 0 \leq c \cdot g(n) \leq f(n)\}$$

$$\omega(g) = \{f : \mathbb{N} \to \mathbb{R} | \forall c > 0 \; \exists n_0 \in \mathbb{N} | \forall n \geq n_0 : 0 \leq c \cdot g(n) \leq f(n)\}$$

$f \in o(g)$: f grows much slower than g

$f \in \omega(g)$: f grows much faster than g

Useful information for the exercise

Theorem 1

1. $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \Rightarrow f \in \mathcal{O}(g)$, $\mathcal{O}(f) \subsetneq \mathcal{O}(g)$.
2. $\lim_{n\to\infty} \frac{f(n)}{g(n)} = C > 0$ (C *constant*) $\Rightarrow f \in \Theta(g)$.
3. $\frac{f(n)}{g(n)} \underset{n\to\infty}{\longrightarrow} \infty \Rightarrow g \in \mathcal{O}(f)$, $\mathcal{O}(g) \subsetneq \mathcal{O}(f)$.

$f(n) \quad 3n^2$

$g(n) \quad 2n^2 \quad \to \quad \frac{3}{2} \quad \frac{2}{3} \quad \frac{f}{g} = \frac{n^2}{n^3}$

$g \in O(3n^2)$

$g \in \Theta(f) \longleftrightarrow f \in \Theta(g)$

←

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Theorem 1

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2. $\lim_{n\to\infty} \frac{f(n)}{g(n)} = C > 0$ (C constant) $\Rightarrow f \in \Theta(g)$.
3. $\frac{f(n)}{g(n)} \underset{n\to\infty}{\to} \infty \Rightarrow g \in \mathcal{O}(f), \mathcal{O}(g) \subsetneq \mathcal{O}(f)$.

Example 2

1. $\lim_{n\to\infty} \frac{n}{n^2} = 0 \Rightarrow n \in \mathcal{O}(n^2), \mathcal{O}(n) \subsetneq \mathcal{O}(n^2)$.
2. $\lim_{n\to\infty} \frac{2n}{n} = 2 > 0 \Rightarrow 2n \in \Theta(n)$.
3. $\frac{n^2}{n} \underset{n\to\infty}{\to} \infty \Rightarrow n \in \mathcal{O}(n^2), \mathcal{O}(n) \subsetneq \mathcal{O}(n^2)$.

Property

$$f_1 \in \mathcal{O}(g), f_2 \in \mathcal{O}(g) \Rightarrow f_1 + f_2 \in \mathcal{O}(g)$$

←

8. Examples and Quiz on Theory

←

Examples

$$\mathcal{O}(g) = \{f : \mathbb{N} \to \mathbb{R} \mid \exists c > 0, \exists n_0 \in \mathbb{N} : \forall n \geq n_0 : 0 \leq f(n) \leq c \cdot g(n)\}$$

$f(n)$	$f \in \mathcal{O}(?)$	Example
$3n + 4$	$O(n)$	$c =$
$2n$		
$n^2 + 100n$		
$n + \sqrt{n}$		

$2n$ → $O(n)$ $O(3n)$ $O(n^3)$

$c = 2$

$n_0 = 0$

$f \in O(g)$

Examples

$$\mathcal{O}(g) = \{f : \mathbb{N} \to \mathbb{R} |\ \exists c > 0, \exists n_0 \in \mathbb{N} : \forall n \geq n_0 : 0 \leq f(n) \leq c \cdot g(n)\}$$

$f(n)$	$f \in \mathcal{O}(?)$	Example
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$2n$	$\mathcal{O}(n)$	$c = 2, n_0 = 0$
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$n^2 + 100n$	$\mathcal{O}(n^2)$	$c = 2, n_0 = 100$
$n + \sqrt{n}$		

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$2n$	$\mathcal{O}(n)$	$c = 2, n_0 = 0$
$n^2 + 100n$	$\mathcal{O}(n^2)$	$c = 2, n_0 = 100$
$n + \sqrt{n}$	$\mathcal{O}(n)$	$c = 2, n_0 = 1$

Examples

Examples

- $n \in \mathcal{O}(n^2)$?

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$\leftarrow$

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- $\mathcal{O}(n) \subseteq \mathcal{O}(n^2)$?

$\leftarrow$

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- $\Theta(n) \subseteq \Theta(n^2)$?

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- $2n^2 \in \mathcal{O}(n)$? is wrong: $\frac{2n^2}{n} = 2n \underset{n\to\infty}{\to} \infty$!
- $\mathcal{O}(n) \subseteq \mathcal{O}(n^2)$? is correct
- $\Theta(n) \subseteq \Theta(n^2)$? is wrong $n \notin \Omega(n^2) \supset \Theta(n^2)$

Quiz

$1 \in \mathcal{O}(15) \quad ?$

Quiz

$1 \in \mathcal{O}(15)$? ✓ better $1 \in \mathcal{O}(1)$

Quiz

$1 \in \mathcal{O}(15)$?	✓ better $1 \in \mathcal{O}(1)$
$2n + 1 \in \Theta(n)$?	

Quiz

$1 \in \mathcal{O}(15)$?	✓ better $1 \in \mathcal{O}(1)$
$2n + 1 \in \Theta(n)$?	✓

Quiz

$1 \in \mathcal{O}(15)$?	✓ better $1 \in \mathcal{O}(1)$
$2n + 1 \in \Theta(n)$?	✓
$\sqrt{n} \in \mathcal{O}(n)$?	

←

Quiz

$1 \in \mathcal{O}(15)$?	✓ better $1 \in \mathcal{O}(1)$
$2n + 1 \in \Theta(n)$?	✓
$\sqrt{n} \in \mathcal{O}(n)$?	✓

Quiz

$1 \in \mathcal{O}(15)$?	✓ better $1 \in \mathcal{O}(1)$
$2n + 1 \in \Theta(n)$?	✓
$\sqrt{n} \in \mathcal{O}(n)$?	✓
$\sqrt{n} \in \Omega(n)$?	

←

Quiz

$1 \in \mathcal{O}(15)$?	✓ better $1 \in \mathcal{O}(1)$
$2n + 1 \in \Theta(n)$?	✓
$\sqrt{n} \in \mathcal{O}(n)$?	✓
$\sqrt{n} \in \Omega(n)$?	✗

←

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$2n + 1 \in \Theta(n)$?	✓
$\sqrt{n} \in \mathcal{O}(n)$?	✓
$\sqrt{n} \in \Omega(n)$?	✗
$n \in \Omega(\sqrt{n})$?	

←

Quiz

$1 \in \mathcal{O}(15)$?	✓ better $1 \in \mathcal{O}(1)$
$2n + 1 \in \Theta(n)$?	✓
$\sqrt{n} \in \mathcal{O}(n)$?	✓
$\sqrt{n} \in \Omega(n)$?	✗
$n \in \Omega(\sqrt{n})$?	✓

$\leftarrow$

Quiz

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$\sqrt{n} \in \mathcal{O}(n)$?	✓
$\sqrt{n} \in \Omega(n)$?	✗
$n \in \Omega(\sqrt{n})$?	✓
$\sqrt{n} \notin \Theta(n)$?	

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$n \in \Omega(\sqrt{n})$?	✓
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$\sqrt{n} \in \Omega(n)$?	✗
$n \in \Omega(\sqrt{n})$?	✓
$\sqrt{n} \notin \Theta(n)$?	✓
$\mathcal{O}(\sqrt{n}) \subset \mathcal{O}(n)$?	

←

Quiz

$1 \in \mathcal{O}(15)$?	✓ better $1 \in \mathcal{O}(1)$
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$\sqrt{n} \in \Omega(n)$?	✗
$n \in \Omega(\sqrt{n})$?	✓
$\sqrt{n} \notin \Theta(n)$?	✓
$\mathcal{O}(\sqrt{n}) \subset \mathcal{O}(n)$?	✓
$2^n \notin \mathcal{O}(\exp(n))$?	

$2^n \notin \mathcal{O}(2.7^n)$

Quiz

$1 \in \mathcal{O}(15)$?	✓ better $1 \in \mathcal{O}(1)$
$2n + 1 \in \Theta(n)$?	✓
$\sqrt{n} \in \mathcal{O}(n)$?	✓
$\sqrt{n} \in \Omega(n)$?	✗
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$\mathcal{O}(\sqrt{n}) \subset \mathcal{O}(n)$?	✓
$2^n \notin \mathcal{O}(\exp(n))$?	✗

←

Quiz

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$n \in \Omega(\sqrt{n})$?	✓
$\sqrt{n} \notin \Theta(n)$?	✓
$\mathcal{O}(\sqrt{n}) \subset \mathcal{O}(n)$?	✓
$2^n \notin \mathcal{O}(\exp(n))$?	✗

A good strategy?

… Then I simply buy a new machine!

A good strategy?

... Then I simply buy a new machine! If today I can solve a problem of size n, then with a 10 or 100 times faster machine I can solve ...

Komplexität	(speed $\times 10$)	(speed $\times 100$)
$\log_2 n$		
n		
n^2		
2^n		

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Komplexität	(speed $\times 10$)	(speed $\times 100$)
$\log_2 n$	$n \to n^{10}$	$n \to n^{100}$
n		
n^2		
2^n		

←

A good strategy?

... Then I simply buy a new machine! If today I can solve a problem of size n, then with a 10 or 100 times faster machine I can solve ...

Komplexität	(speed $\times 10$)	(speed $\times 100$)
$\log_2 n$	$n \to n^{10}$	$n \to n^{100}$
n	$n \to 10 \cdot n$	$n \to 100 \cdot n$
n^2		
2^n		

←

A good strategy?

... Then I simply buy a new machine! If today I can solve a problem of size n, then with a 10 or 100 times faster machine I can solve ...

Komplexität	(speed $\times 10$)	(speed $\times 100$)
$\log_2 n$	$n \rightarrow n^{10}$	$n \rightarrow n^{100}$
n	$n \rightarrow 10 \cdot n$	$n \rightarrow 100 \cdot n$
n^2	$n \rightarrow 3.16 \cdot n$	$n \rightarrow 10 \cdot n$
2^n		

A good strategy?

... Then I simply buy a new machine! If today I can solve a problem of size n, then with a 10 or 100 times faster machine I can solve ... [1]

Komplexität	(speed $\times 10$)	(speed $\times 100$)
$\log_2 n$	$n \to n^{10}$	$n \to n^{100}$
n	$n \to 10 \cdot n$	$n \to 100 \cdot n$
n^2	$n \to 3.16 \cdot n$	$n \to 10 \cdot n$
2^n	$n \to n + 3.32$	$n \to n + 6.64$

[1] To see this, you set $f(n') = c \cdot f(n)$ ($c = 10$ or $c = 100$) and solve for n'

←

9. Quiz on Asymptotic Running Time of Program Fragments

←

Asymptotic Running Times with Θ

```
void run(int n){
  for (int i = 1; i<n; ++i){
     op();
} }
```

How often is `op()` called as a function of n?

$$\sum_{i=1}^{n-1} 1 = n-1 \in \Theta(n)$$

run(n)

Asymptotic Running Times with Θ

```
void run(int n){
  for (int i = 1; i<n; ++i)
  op();
}
```

How often is `op()` called as a function of n?

$$\sum_{i=1}^{n-1} 1 = n - 1 \in \Theta(n)$$

Asymptotic Running Times with Θ

```
void run(int n){
  for (int i = 1; i<n; ++i)
    for (int j = 1; j<n; ++j)
      op();
}
```

How often is `op()` called as a function of n?

$$\sum_{i=1}^{n-1} \sum_{j=1}^{n-1} 1 = (n-1)^2 =$$

←

Asymptotic Running Times with Θ

```
void run(int n){
  for (int i = 1; i<n; ++i)
    for (int j = 1; j<n; ++j)
      op();
}
```

How often is `op()` called as a function of n?

$$\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} 1 = \sum_{i=1}^{n-1}(n-1) = (n-1)\cdot(n-1) \in \Theta(n^2)$$

$n^2 + (\ldots)$

$\Theta(n^2)$

Asymptotic Running Times with Θ

```
void run(int n){
  for (int i = 1; i<n; ++i)
    for (int j = i; j<n; ++j)
      op();
}
```

How often is `op()` called as a function of n?

Asymptotic Running Times with Θ

```
void run(int n){
  for (int i = 1; i<n; ++i)
    for (int j = i; j<n; ++j)
      op();
}
```

How often is `op()` called as a function of n?

$$\sum_{i=1}^{n-1}\sum_{j=i}^{n-1} 1 = \sum_{i=1}^{n-1}(n-i) = \sum_{i=1}^{n-1} i = \frac{n(n-1)}{2} \in \Theta(n^2)$$

$\frac{1}{2}n^2 + \dots$

Asymptotic Running Times with Θ

```
void run(int n){
  for (int i = 1; i<n; ++i){
    op();
    for (int j = i; j<n; ++j)
      op();
    }
}
```

How often is `op()` called?

Asymptotic Running Times with Θ

```
void run(int n){
  for (int i = 1; i<n; ++i){
    op();
    for (int j = i; j<n; ++j)
      op();
    }
}
```

How often is `op()` called?

$\Theta(n)$ $\Theta(n^2)$

$$\sum_{i=1}^{n-1}\left(1+\sum_{j=i}^{n-1} 1\right) = \sum_{i=1}^{n-1}(1+(n-i)) = \overbrace{n-1}^{\Theta(n)} + \overbrace{\frac{n(n-1)}{2}}^{\Theta(n^2)} \in \Theta(n^2)$$

Asymptotic Running Times with Θ

```
void run(int n){
  for (int i = 1; i<n; ++i){
    op();
    for (int j = 1; j<i*i; ++j)
      op();
    }
}
```

How often is `op()` called?

Asymptotic Running Times with Θ

```
void run(int n){
  for (int i = 1; i<n; ++i){
    op();
    for (int j = 1; j<i*i; ++j)
      op();
  }
}
```

How often is `op()` called?

$$\sum_{i=1}^{n-1}\left(1+\sum_{j=1}^{i^2-1} 1\right) = \sum_{i=1}^{n-1}\left(1+i^2-1\right) = \sum_{i=1}^{n-1} i^2 \in \Theta(n^3)$$

←

Asymptotic Running Times with Θ

```
void run(int n){
  for(int i = 1; i <= n; ++i)
    for(int j = 1; j*j <= n; ++j)
      for(int k = n; k >= 2; --k)
        op();
}
```

How often is `op()` called as a function of n?

Asymptotic Running Times with Θ

```
void run(int n){
  for(int i = 1; i <= n; ++i)
    for(int j = 1; j*j <= n; ++j)
      for(int k = n; k >= 2; --k)
        op();
}
```

How often is `op()` called as a function of n?

$$\sum_{i=1}^{n} \sum_{j=1}^{\lfloor\sqrt{n}\rfloor} n-1 \in \Theta\left(\sum_{i=1}^{n} n^{3/2}\right) = \Theta(\sqrt{n^5})$$

←

Asymptotic Running Times with Θ

```
int f(int n){
    i=1;
    while (i <= n*n*n){
        i = i*2;
        op();
    }
    return i;
}
```

How often is `op()` called as a function of n?

←

Asymptotic Running Times with Θ

```
int f(int n){
    i=1;
    while (i <= n*n*n){
        i = i*2;
        op();
    }
    return i;
}
```

How often is `op()` called as a function of n?

$\log_2(n^3) = 3\log_2(n)$

$$|\{i \in \mathbb{N} : 2^i \leq n^3\}| \in \Theta(\log_2 n^3) = \Theta(\log n) = \Theta(\log_{42}(n))$$

10. Formulas and their Derivation*

Sums

$$\sum_{i=0}^{n} i = ?$$

Sums

$$\sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2}$$

←

Sums

$$\sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2}$$

Why?

←

Sums

$$\sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2}$$

Why?

Intuition

$$1 + ... + 100 = (1 + 100) + (2 + 99) + (3 + 98) + ... + (50 + 51)$$

←

Sums

$$\sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2}$$

Why?

Intuition

$$1 + ... + 100 = (1 + 100) + (2 + 99) + (3 + 98) + ... + (50 + 51)$$

More formally?

Sums

$$\sum_{i=0}^{n}(n-i) = ?$$

Sums

$$\sum_{i=0}^{n}(n-i) = \sum_{i=0}^{n} i$$

←

Sums

$$\sum_{i=0}^{n}(n-i) = \sum_{i=0}^{n} i$$

$$\Rightarrow 2 \cdot \sum_{i=0}^{n} i = \sum_{i=0}^{n} i + \sum_{i=0}^{n}(n-i)$$

$$= \sum_{i=0}^{n}(i + (n-i)) = \sum_{i=0}^{n} n = (n+1) \cdot n$$

Sums

$$\sum_{i=0}^{n}(n-i) = \sum_{i=0}^{n} i$$

$$\begin{aligned}\Rightarrow 2 \cdot \sum_{i=0}^{n} i &= \sum_{i=0}^{n} i + \sum_{i=0}^{n}(n-i) \\ &= \sum_{i=0}^{n}(i + (n-i)) = \sum_{i=0}^{n} n = (n+1) \cdot n\end{aligned}$$

←

Sums

$$\sum_{i=0}^{n} i^2 = ?$$

←

Sums

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Sums

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

This you do not need to know by heart. But you should know that it is a polynomial of third degree.

Sums

How do you derive something like this?

Sums

How do you derive something like this? Interesting Trick: On the one hand

$$\sum_{i=0}^{n} i^3 - \sum_{i=1}^{n} (i-1)^3 = \sum_{i=0}^{n} i^3 - \sum_{i=0}^{n-1} i^3 = n^3,$$

Sums

How do you derive something like this? Interesting Trick: On the one hand

$$\sum_{i=0}^{n} i^3 - \sum_{i=1}^{n} (i-1)^3 = \sum_{i=0}^{n} i^3 - \sum_{i=0}^{n-1} i^3 = n^3,$$

on the other hand

$$\begin{aligned}\sum_{i=0}^{n} i^3 - \sum_{i=1}^{n} (i-1)^3 &= \sum_{i=1}^{n} i^3 - \sum_{i=1}^{n} (i-1)^3 \\ &= \sum_{i=1}^{n} i^3 - (i-1)^3 = \sum_{i=1}^{n} 3 \cdot i^2 - 3 \cdot i + 1\end{aligned}$$

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = ? \qquad \log_a(x \cdot y) = ?$$

$$\frac{a^x}{a^y} = ? \qquad \log_a \frac{x}{y} = ?$$

$$a^{x \cdot y} = ? \qquad \log_a x^y = ?$$

$$\log_a n! = ?$$

$$\log_b x = ? \qquad a^{\log_b x} = ?$$

←

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = ?$$

$$a^{x \cdot y} = ?$$

$$\log_b x = ?$$

$$\log_a (x \cdot y) = ?$$

$$\log_a \frac{x}{y} = ?$$

$$\log_a x^y = ?$$

$$\log_a n! = ?$$

$$a^{\log_b x} = ?$$

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$a^{x \cdot y} = ?$$

$$\log_b x = ?$$

$$\log_a (x \cdot y) = ?$$

$$\log_a \frac{x}{y} = ?$$

$$\log_a x^y = ?$$

$$\log_a n! = ?$$

$$a^{\log_b x} = ?$$

$\leftarrow$

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$a^{x \cdot y} = (a^x)^y$$

$$\log_b x = ?$$

$$\log_a (x \cdot y) = ?$$

$$\log_a \frac{x}{y} = ?$$

$$\log_a x^y = ?$$

$$\log_a n! = ?$$

$$a^{\log_b x} = ?$$

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$a^{x \cdot y} = (a^x)^y$$

$$\log_b x = ?$$

$$\log_a (x \cdot y) = ?$$

$$\log_a \frac{x}{y} = ?$$

$$\log_a x^y = ?$$

$$\log_a n! = ?$$

$$a^{\log_b x} = ?$$

←

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y} \qquad \log_a (x \cdot y) = \log_a x + \log_a y$$

$$\frac{a^x}{a^y} = a^{x-y} \qquad \log_a \frac{x}{y} = ?$$

$$a^{x \cdot y} = (a^x)^y \qquad \log_a x^y = ?$$

$$\log_a n! = ?$$

$$\log_b x = ? \qquad a^{\log_b x} = ?$$

←

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$a^{x \cdot y} = (a^x)^y$$

$$\log_b x = ?$$

$$\log_a (x \cdot y) = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^y = ?$$

$$\log_a n! = ?$$

$$a^{\log_b x} = ?$$

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$a^{x \cdot y} = (a^x)^y$$

$$\log_b x = ?$$

$$\log_a(x \cdot y) = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^y = y \log_a x$$

$$\log_a n! = ?$$

$$a^{\log_b x} = ?$$

$\leftarrow$

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$a^{x \cdot y} = (a^x)^y$$

$$\log_a(x \cdot y) = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^y = y \log_a x$$

$$\log_a n! = \sum_{i=1}^{n} \log i$$

$$\log_b x = ?$$

$$a^{\log_b x} = ?$$

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$a^{x \cdot y} = (a^x)^y$$

$$\log_a(x \cdot y) = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^y = y \log_a x$$

$$\log_a n! = \sum_{i=1}^{n} \log i$$

$$\log_b x = \log_b a \cdot \log_a x$$

$$a^{\log_b x} = ?$$

←

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$a^{x \cdot y} = (a^x)^y$$

$$\log_a (x \cdot y) = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^y = y \log_a x$$

$$\log_a n! = \sum_{i=1}^{n} \log i$$

$$\log_b x = \log_b a \cdot \log_a x \qquad a^{\log_b x} = x^{\log_b a}$$

$n, \; 3n^5, \; \log_n(3n), \; 2^{50}$

$\to \; 2^{50}, \; \log_n(3n), \; n, \; 3n^5$

←

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y} \qquad \log_a(x \cdot y) = \log_a x + \log_a y$$

$$\frac{a^x}{a^y} = a^{x-y} \qquad \log_a \frac{x}{y} = \log_a x - \log_a y$$

$$a^{x \cdot y} = (a^x)^y \qquad \log_a x^y = y \log_a x$$

$$\log_a n! = \sum_{i=1}^{n} \log i$$

$$\log_b x = \log_b a \cdot \log_a x \qquad a^{\log_b x} = x^{\log_b a}$$

To see the last line, replace $x \to a^{\log_a x}$

Comparisons

$$\frac{n^2}{2^n} \xrightarrow[n\to\infty]{} ?$$

←

Comparisons

$$\frac{n^2}{2^n} \xrightarrow[n\to\infty]{} 0$$

Comparisons

$$\frac{n^{10000}}{2^n} \xrightarrow[n\to\infty]{} ?$$

Comparisons

$$\frac{n^{10000}}{2^n} \xrightarrow[n\to\infty]{} 0$$

Comparisons

$d > 1, c > 0$

$$\frac{n^c}{d^n} \underset{n\to\infty}{\longrightarrow} ?$$

←

Comparisons

$d > 1, c > 0$

$$\frac{n^c}{d^n} \xrightarrow[n \to \infty]{} 0$$

←

Comparisons

$d > 1, c > 0$

$$\frac{n^c}{d^n} \xrightarrow[n\to\infty]{} 0$$

because

$$\frac{n^c}{d^n} = \frac{2^{\log_2 n^c}}{2^{\log_2 d^n}} = 2^{c \cdot \log_2 n - n \log_2 d}$$

Comparisons

$$\frac{n}{\log n} \xrightarrow[n\to\infty]{} ?$$

$\leftarrow$

Comparisons

$$\frac{n}{\log n} \xrightarrow[n \to \infty]{} \infty$$

←

Comparisons

$$\frac{n \log n}{\sqrt{n}} \xrightarrow[n \to \infty]{} ?$$

←

Comparisons

$$\frac{n \log n}{\sqrt{n}} \xrightarrow[n \to \infty]{} \infty$$

←

Comparisons

$$\frac{\log_2 n^2}{\sqrt{n}} \xrightarrow[n\to\infty]{} ?$$

Comparisons

$$\frac{\log_2 n^2}{\sqrt{n}} \xrightarrow[n \to \infty]{} 0$$

←

Comparisons

$$\frac{\log_2 n^2}{\sqrt{n}} \xrightarrow[n\to\infty]{} 0$$

$$\log_2 n^2 = 2\log_2 n$$

$$\sqrt{n} = n^{1/2} = 2^{\log_2 n^{1/2}} = \left(\sqrt{2}\right)^{\log_2 n}$$

$$\frac{\log n^2}{\sqrt{n}} = 2\frac{\log_2 n}{\left(\sqrt{2}\right)^{\log_2 n}}$$

which behaves because of $\log_2 n \to \infty$ for $n \to \infty$ like $2\frac{n}{\left(\sqrt{2}\right)^n}$

11. Past Exam Questions

Past Exam Questions

←

Past Exam Questions

- If time allows, this is where we could have a look at old exam questions and go over them together

Past Exams

12. Tips for **code** expert

Tips for upcoming **code** expert exercises

Task "Taskname"

←

Tips for upcoming **code** expert exercises

Task "Taskname"

- This is where I give you hints and tips for the upcoming **code** expert exercises

←

13. Outro

General Questions?

General Questions?

- This is where you can ask general questions regarding the course or bring up things I didn’t cover during the session

See you next time!

Have a nice week!

←