# Datastructures and Algorithms

Containers, `auto`, Templates, Induction

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# Overview

Learning Objectives
C++ Container Library
Templates Recap
Auto vs Templates
Repetition theory: Induction
Subarray Sum Problem
Code Example
Programming Exercise
Past Exam Questions
Tips for **code** expert

n.ethz.ch/~agavranovic

Material

Webpage

Mail

# 1. Follow-up

# Follow-up from last session

**Random Access and Pointer Machine Models**

- Nothing that's very relevant for the exam, so no need to worry about it too much...

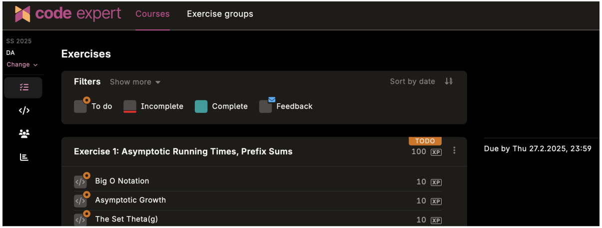
# Follow-up from last session

## Regarding the "one week Deadline for the first"

- I was wrong! because I didn't verify the (provided) information...
- The deadline for the **code** expert Exercise 1 (Asymptotic Running Times, Prefix Sums) is

**Thu, 27.02.2025 at 23:59**

which can by seen on the **code** expert page



# 2. Feedback regarding **code** expert

# General things regarding **code** expert

- *Please* learn some $\LaTeX$ and markdown for the submissions
- Amazing tool for finding the right commands: Detexify
- Also: read the descriptions carefully!

# Any questions regarding **code** expert on your part?

# 3. Learning Objectives

# Learning Objectives

- ☐ Understand what Container are and what benefits they bring
- ☐ Understand what Templates are and what benefits they bring
- ☐ Understand how to do Induction Proofs in this course
- ☐ Be prepared to solve the next **code** expert exercises

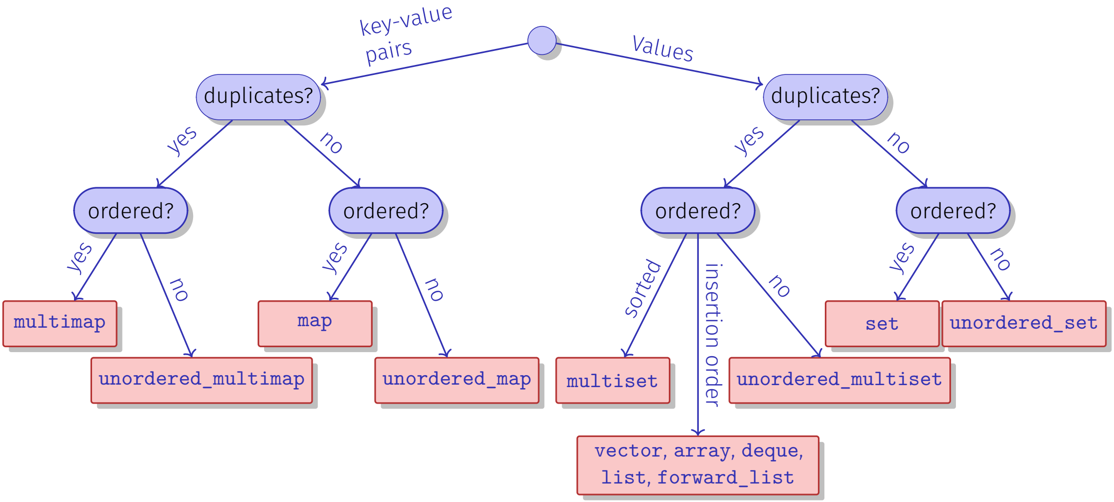
# 4. Summary

# Getting on the same page

- What was covered this week and what would you like to revist?

# 5. C++ Container Library

# C++ Containers



# Sequence-Container

| `vector` | `array` | `deque` | `list` | `forward_list` |
|---|---|---|---|---|
| contiguous dynamic memory | contiguous static memory | Non-contig. dynamic memory | Non-contig. dynamic memory | Non-contig. dynamic memory |
| random access | random access | random access | | |
| fast push/pop back | | fast push/pop front/back | fast push/pop front/back | fast push/pop front |
| bidirectional iteration | bidirectional iteration | bidirectional iteration | bidirectional iteration | forward iteration |

**dynamic**: size can change during runtime, **static**: size fixed at compile-time, **random acccess**: direct, immediate access to any element by its *index* (e.g. `vec[42]`), **bidirectional**: backward and forwards iterable

# Sets and Multisets

- `std::set<E>` contains unique elements
- `std::multiset<E>` allows duplicate elements
  - Iteration yields all elements in ascending order (in non-deterministic order if `unordered_multiset`)
  - `std::multiset<E>::count(elem)` returns the number of occurences of a given element

**Example of `std::multiset`**

```
Content: Xanten Xenon Xenon Xenon Xerografie Xerophil Xylose
count("Xenon") = 3
count("Xylose") = 1
```

# Maps and Multimaps

- `std::map<K,V>` contains pairs (key, value), where a key maps to at most one value
- `std::multimap<K,V>` allows duplicate pairs
  - Iteration yields all pairs in ascending key order (in non-deterministic order, if `unordered_multimap`)
  - `std::multimap<K,V>::count(key)` returns the number of occurrences of a given key
  - `std::multimap<K,V>::equal_range(key)` returns all values (in non-det. order) for a given key

**Example of `std::multimap<K,V>`**

```
Content: {2, er} {2, du} {2, es} {3, Axt} {3, sie} {4, Igel}
count(2) = 3
Values for key 2: er du es
```

# 6. Templates Recap

# Motivational Example

From the Lecture

Example goal: generic class and functionality for matrices (and vectors), without duplicating code!

```
class Matrix { ... };
auto m1 = Matrix<int>(5,3);
auto m2 = Vector<std::string> {"Zurich", "Locarno"};
auto m3 = Matrix<Complex>(...);

m1(1,2) = 10;
auto sum = m1 + m2 + m3;
std::cout << m3.max();
```

# Parametric Polymorphism

From the Lecture

**Types as template parameters**

1. In the concrete implementation of a class replace the type that should become generic (in our example: `dint`) by a representative element, e.g. `T`.
2. Put in front of the class the construct `template<typename T>` Replace `T` by the representative name).

The construct `template<typename T>` can be understood as **"for all types `T`"**.

# Integer Matrix

```
class Matrix {
    unsigned sizeR, sizeC;
    std::vector<int> data;
public:
    Matrix(unsigned R, unsigned C): sizeR(R), sizeC(C), data(R*C) {}

    int& operator() (unsigned r, unsigned c){
      assert ( r < sizeR && c < sizeC);
      return data[r*sizeC + c];
    }

    const int& operator() (unsigned r, unsigned c) const { .. }
    unsigned rows() const { return sizeR; }
  ...
};
```

# Generic Matrix

```
template <typename T>
class Matrix {
    unsigned sizeR, sizeC;
    std::vector<T> data;
public:
    Matrix(unsigned R, unsigned C): sizeR(R), sizeC(C), data(R*C) {}

    T& operator() (unsigned r, unsigned c){
      assert ( r < sizeR && c < sizeC);
      return data[r*sizeC + c];
    }
    const T& operator() (unsigned r, unsigned c) const { .. }
    unsigned rows() const { return sizeR; }
    ...
};
```

# Parametric Polymorphism II

From the Lecture

Algorithms and functions can also be parameterised with a type:

**Function Templates**

1. To make a concrete implementation generic, replace the specific type (e.g. int) with a name, e.g. `T`,
2. Put in front of the function the construct `template<typename T>` (Replace `T` by the chosen name)

# Examples

- For free functions

```
template <typename T>
void swap(T& x, T& y) {
  T temp = x;
  x = y;
  y = temp;
}
```

```
template <typename Iter>
void is_sorted(Iter begin, Iter end){
  ...
}
```

- For operators

```
template <typename T>
ostream& operator<<(ostream& out, const Node<T> root) {
  ...
}
```

# Semantics (Code-Generation)

For each template instance, the compiler creates a corresponding instantiated class (or function) → static code generation

```
Matrix<int> m1 = ...;
Matrix<std::string> m2 = ...;
Matrix<Student> m3 = ...;
```

m1

```
class Matrix_int {
  ...
  std::vector<int> data
  ...
};
```

m2

```
class Matrix_string {
  ...
  std::vector<std::string>
    data
  ...
};
```

m3

```
class Matrix_student {
  ...
  std::vector<Student> data
  ...
};
```

# Semantics (Code-Generation)

For each template instance, the compiler creates a corresponding instantiated class (or function) $\rightarrow$ static code generation

**Question: what does this imply for seperate compilation?**

- Should templates be spilt up into .h (declarations) and .cpp (definitions) files?
- Is it possible to ship the compiled implementation (binary file compiled from .cpp) alongside the header file?

# No Separate Compilation

- Code is usually separated into `.h` (header) and `.cpp` (source) files for modularity and faster re-compilation. The header contains declarations, while the source file holds definitions/implementations, enabling better code organization and independent compilation.
- Templates can **not** be split into `.h` and `.cpp` files because the entire definition (not just the declaration) must be visible to the compiler.
- It is not possible to ship the compiled binary from the `.cpp` file alongside the header because templates are instantiated at compile time for the specific types used. The compiler doesn't know the types in advance and cannot pre-compile all in a binary file.

# Integer Matrix

```
class Int_Matrix {
  ...
  int& operator() (unsigned r, unsigned c);
};
```
matrix.h

```
int& Int_Matrix::operator() (unsigned r, unsigned c){
  return data[r*sizeC + c];
}
```
matrix.cpp

```
#include <matrix.h>
...
Int_Matrix m(10,10);
m(3,3) = 5; // ok
```
main.cpp

# Generic Matrix?

```
template <typename T>
class Matrix {
  ...
  T& operator() (unsigned r, unsigned c);
};
```

matrix.h

```
template <typename T>
T& Matrix<T>::operator() (unsigned r, unsigned c){
  return data[r*sizeC + c];
}
```

matrix.cpp

```
#include <matrix.h>
...
Matrix<int> m(10,10);
m(3,3) = 5; // error:  undefined reference
```

main.cpp

# Generalizing Code using Templates

```
class Vector {
public:
  Vector() {...}
  float& operator [](int i) { return data[i]; }
private:
  float data[3];
};

float scalar_product(Vector a, Vector b) {
    float result = 0;
    for (int i=0; i<3; ++i)
       result += a[i] * b[i];
    return result;
}
```

# Type testing

- Templates: syntactic checks
- Instances: checks as usual

```
template <typename T>
T abs(T v) {
  return 0 <= v ? v : -v;
}
// main
abs(8); // OK
```

```
template <typename T>
T abs(T v) {
  return 0 <= v ? v : -v; // Error
}
// main
abs("hi"); // Error
```

```
template <typename T>
void swap(T& x, T& y) {
  ...
}
// main
double a = 1.0;
double b = 7;
swap(a, b); // OK
```

```
template <typename T>
void swap(T& x, T& y) {
  ...
}
// main
double a = 1.0;
string b = "seven";
swap(a, b); // Error
```

# Other Languages

All languages try to foster code reuse but chose different solutions.

- C++, Rust:
  - static code generation
  - no runtime overhead
  - difficult to integrate into OOP
- C#, Scala (, Java)
  - type parameters are turned into runtime values
  - well-suited for OOP
  - minor runtime overhead
- Python, JavaScript:
  - dynamic typing (duck typing)
  - no syntactic overhead
  - potentially significant runtime overhead

# 7. Auto vs Templates

# auto

- Placeholder type specifier
  - Must be uniquely determined by direct context: initialiser code, or returns
  - User could write type themself, but leave it to the compiler

```
std::vector<int> vec = ...;
auto it = vec.cbegin();
// placeholder for td::vector<int>::const_iterator
```

- Failing examples:

```
auto x; // x has no initializer
x = 0.0;
auto first_or_else(std::vector<int> data, unsigned int or_else) {
  if (data.size() == 0) return or_else;
  else return data[0];
}
```

# Templates

- Parameters are unknown until instantiated

```
template <typename N>
char sign(N v) {
  if (0 <= v) return '+';
  else return '-';
}

template <typename T1, typename T2>
struct Pair {
  T1 fst;
  T2 snd;
};
```

- Instantiation may happen anywhere

```
Pair<int, double> p1 = Pair{1, 0.1};
auto p2 = Pair<std::string, bool>{"Brazil", true};
```

# Combining templates and auto

`auto` inside template must be determined after instantiation

```
template <typename C>
void print(C container) {
  for (auto& e : container)
  std::cout << e << ' ';
}

std::vector<int> numbers = {1, 2, 3};
print(numbers); // now auto can be determined

std::vector<std::string> airports = {"LAX", "LDN", "ZHR"};
print(airports); // now auto can be determined
```

# Combining templates and auto

`auto` inside template must be determined after instantiation

```
template <typename C>
void print(C container) {
  for (auto& e : container)
  std::cout << e << ' ';
}
```

**Question**: Is it possible to not use `auto` here?

**Answer**: Yes, for example by replacing auto with an additional template parameter `E`

# From auto to templates

- Before C++20 auto function parameters are forbidden

```
void print(auto x) {...} // Compiler error
```

  **Question**: Why do you think that is?
  **Answer**: Cannot determine type from context

- Since C++20 auto function parameters are allowed

```
void print(auto x) {...} // ok
```

  Clearly, it is still not possible to determine what auto stands for.
  **Question**: What could be the meaning of auto in this case?
  **Answer**: It is a shorthand for a template parameter!

```
template <typename T>
void Print(T x){ ... }
```

# 8. Repetition theory: Induction

# Induction: what is required?

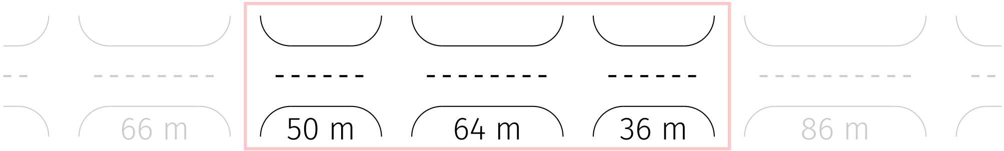
- Prove statements, for example $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- Base clause:
  - The given (in)equality holds for one or more base cases.
  - e.g. $\sum_{i=1}^{1} i = 1 = \frac{1(1+1)}{2}$.
- Induction hypothesis: we assume that the statement holds for some $n$
- Induction step ($n \to n+1$):
  - From the validity of the statement for $n$ (induction hypothesis) it follows the one for $n+1$.
  - e.g.: $\sum_{i=1}^{n+1} i = n+1+\sum_{i=1}^{n} i = n+1+\frac{n(n+1)}{2} = \frac{(n+2)(n+1)}{2}$.

# 9. Subarray Sum Problem

Naïve Solution, prefix sums, binary search, Sliding Window

# Street section of a given length

**Given:** distances between all crossroads on a street



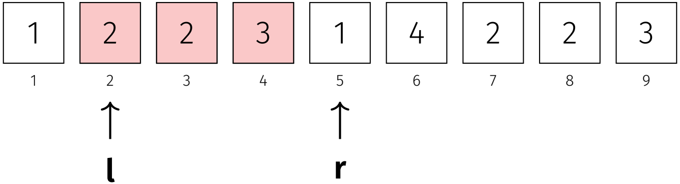

**Wanted:** street section of length $150$ meters between crossroads

# Subarray Sum Problem

**Given:** an array $A = (A[1], \ldots, A[n])$ of non-negative integers

**Wanted:** a subarray with sum $k$:
pair $(l, r)$ with $1 \leq l \leq r \leq n$ such that $\sum_{i=l}^{r-1} A[i] = k$

**Example:** $n = 9, k = 7$ **Solution:** $l = 2, r = 5$.



# Strategies?

**Given:** an array $A = (A[1], \ldots, A[n])$ of non-negative integers
**Wanted:** a subarray with sum $k$:
pair $(l, r)$ with $1 \leq l \leq r \leq n$ such that $\sum_{i=l}^{r-1} A[i] = k$

**Strategies**

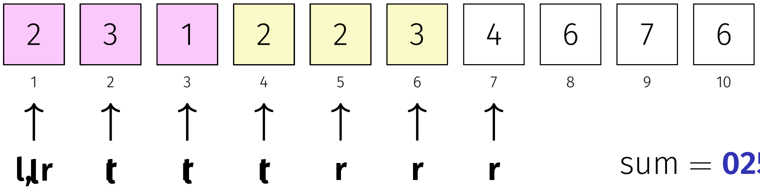
| | |
|---|---|
| $\Theta(n^3)$ | Three loops |
| $\Theta(n^2)$ | Prefix Sums |
| $\Theta(n \log n)$ | Binary Search |
| $\Theta(n)$ | Sliding Window |

# Subarray Sum Problem: Sliding Window

Sliding Window Idea

- start with left and right pointer at 1
- repeat until the end of the sequence:
  - window **too small** (sum $< k$) $\Rightarrow$ increment right pointer
  - window **too large** (sum $> k$) $\Rightarrow$ increment left pointer
  - window **as desired** (sum $= k$) $\Rightarrow$ done!

**Example:** $k = 7$



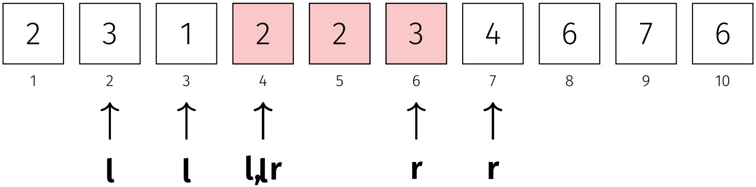

sum $=$ 0256868587

# Subarray Sum Problem: Sliding Window Analysis

- in each step: either $l$ or $r$ is increased
  $\Rightarrow$ algorithm terminates after a maximum of $2n$ steps

**target window:** lexicographically smallest (left-most) window with sum $k$

- if $r$ reaches the end before $l$ reaches the start
  $\Rightarrow$ sum too large $\Rightarrow$ $l$ is increased until it reaches the start of the window
- if $l$ reaches the start before $r$ reaches the end
  $\Rightarrow$ sum too small $\Rightarrow$ $r$ is increased until it reaches the end of the window



# Analysis

We consider the lexicographically smallest (left-most) window with sum $k$, called *target window*

- In each step of the algorithm either $l$ or $r$ is increased. The algorithm terminates after a maximum of $2n$ steps.
- Assume $r$ reaches the end of the target window before $l$ reaches the start of the target window, then $l$ keeps increasing until it reaches the start of the window.
- Assume $l$ reaches the start of the target window before $r$ reaches the end of the target window, then $r$ keeps increasing until it reaches the end of the window.

Exercise: window with sum closest to $k$

# 10. Code Example

# 10. Code Example

Subarray Sum Problem ⟶ CodeExpert

# 11. Programming Exercise

Preparing remarks for the homework (Prefix Sum in 2D)

# Sum in Subarray (naive algorithm)

**Input**: A sequence of $n$ numbers $(a_1, \dots, a_n)$ and a sub-interval $I = [l, r]$
**Output**: $\sum_{i=l}^{r} a_i$.

$\mathcal{S} \leftarrow 0$
**for** $i \in \{l, \dots, r\}$ **do**
  $\mathcal{S} \leftarrow \mathcal{S} + a_i$
**return** $\mathcal{S}$

### Idea of the exercise

- Use the prefix sum to compute the sum of arbitrary sub-intervals with constant running time
- **Generalize** to two dimensions.

# 12. Past Exam Questions

# Altklausur 2020: Aufgabe 2a)

## Aufgabe 2: Asymptotik (16P)

(a) Geben Sie für die untenstehenden Funktionen eine Reihenfolge an, so dass folgendes gilt: Wenn eine Funktion $f$ links von einer Funktion $g$ steht, dann gilt $f \in \mathcal{O}(g)$.
Beispiel: die drei Funktionen $n^3$, $n^5$ und $n^7$ sind bereits in der entsprechenden Reihenfolge, da $n^3 \in \mathcal{O}(n^5)$ und $n^5 \in \mathcal{O}(n^7)$.

*Provide an order for the following functions such that the following holds: If a function $f$ is left of a function $g$ then it holds that $f \in \mathcal{O}(g)$.*
*Example: the functions $n^3$, $n^5$ and $n^7$ are already in the respective order because $n^3 \in \mathcal{O}(n^5)$ and $n^5 \in \mathcal{O}(n^7)$.*

/3P

$$\log \sqrt{n}\ ,\ 2^{\log_4 n}\ ,\ \sqrt{\log n}\ ,\ n!\ ,\ \sum_{i=1}^{n} 2^i\ ,\ n^{1/3}\ ,\ n\sqrt{n}$$

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |

# Altklausur 2020: Aufgabe 2a) — Solution

## Aufgabe 2: Asymptotik (16P)

(a) Geben Sie für die untenstehenden Funktionen eine Reihenfolge an, so dass folgendes gilt: Wenn eine Funktion $f$ links von einer Funktion $g$ steht, dann gilt $f \in \mathcal{O}(g)$.
Beispiel: die drei Funktionen $n^3$, $n^5$ und $n^7$ sind bereits in der entsprechenden Reihenfolge, da $n^3 \in \mathcal{O}(n^5)$ und $n^5 \in \mathcal{O}(n^7)$.

*Provide an order for the following functions such that the following holds: If a function $f$ is left of a function $g$ then it holds that $f \in \mathcal{O}(g)$.*
*Example: the functions $n^3$, $n^5$ and $n^7$ are already in the respective order because $n^3 \in \mathcal{O}(n^5)$ and $n^5 \in \mathcal{O}(n^7)$.*

/3P

$$\log \sqrt{n}\ ,\ 2^{\log_4 n}\ ,\ \sqrt{\log n}\ ,\ n!\ ,\ \sum_{i=1}^{n} 2^i\ ,\ n^{1/3}\ ,\ n\sqrt{n}$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\sqrt{\log n}$ | $\log \sqrt{n}$ | $n^{1/3}$ | $2^{\log_4 n}$ | $n\sqrt{n}$ | $\sum_{i=1}^{n} 2^i$ | $n!$ |

# Altklausur 2020: Aufgabe 2b), 2c)

(c)

```
void g(int n){
  for (int i = 0; i < n; ++i){
    for (int j = i; j < n; ++j){
        f();
    }
  }
}
```

| Anzahl Aufrufe von $f$ / *Number of calls of $f$* |
|---|
| |

/1P

(d)

```
void g(int n){
    f();
    if (n>1){
        f();g(n/2);
    }
    if (n>2) {
        f();g(n/2);
    }
}
```

| Anzahl Aufrufe von $f$ / *Number of calls of $f$* |
|---|
| |

/3P

# Altklausur 2020: Aufgabe 2b), 2c) — Solution

(c)

```
void g(int n){
  for (int i = 0; i < n; ++i){
    for (int j = i; j < n; ++j){
        f();
    }
  }
}
```

/1P

Anzahl Aufrufe von $f$ / *Number of calls of $f$*

$\Theta(n^2)$

(d)

```
void g(int n){
    f();
    if (n>1){
        f();g(n/2);
    }
    if (n>2) {
        f();g(n/2);
    }
}
```

/3P

Anzahl Aufrufe von $f$ / *Number of calls of $f$*

$\Theta(n)$

# 13. Tips for **code** expert

# Tips for code expert Exercise 2

**Task "Prefix Sum in 2D"**

- Study the Prefix Sum in 1D[1] well and go from there
- Make sketches!

**Task "Sliding Window"**

- Sketches!

**Task "Proofs by Induction"**

- The binomial formula will be useful for the second one
- Please format it well or just scan a PDF and upload it

**Task "Karatsuba Ofman"**

- Just translate the math into code

[1] There's an implementation in the code examples on **code** expert

# 14. Outro

# General Questions?

# See you next time!

Have a nice week!