Datastructures and Algorithms

Minimum Spanning Trees, Max Flow in Flow Networks, Traveling Salesperson

Adel Gavranović — ETH Zürich — 2025

Overview

Learning Objectives
Remark Bonus Exercises
Minimum Spanning Trees
- Recap Theory
- Code-Expert Exercise

MaxFlow
- Recap Theory
- Quiz
- Old Exam Questions

TSP

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Material

Webpage

Mail

1. Feedback regarding **code** expert

General things regarding **code** expert

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  - Some comparisons, if implemented efficiently, **do not necessarily change** the time complexity.
- Time complexity classes are not measured with stopwatches, but through analytical methods.
- The term *"Runtime"* can be ambiguous,
  but the exercise clearly specifies *asymptotic runtime*.

Any questions regarding **code** expert on your part?

2. Learning Objectives

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- Understand and be able to perform the following Algorithms by hand:
  - Kruskal (MST)
  - Jarnik-Prim-Dijkstra (MST)
  - Ford-Fulkerson (Max-Flow)
  - Edmonds-Karp (Max-Flow)
- Understand how UnionFind works and where one can use it
- Be able to implement a “2-approximation” of the (metric) TSP problem using MSTs and a DFS

3. Summary

Getting on the same page

Getting on the same page

- What did you cover in class?
- Is there anything that I need to be aware of?

4. Remark Bonus Exercises

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It was clearly stated in the task description that standard library functions which significantly simplify the task are not allowed. This will be graded with 0 points.

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Bonus 2
Here, the use of all standard library functions is explicitly allowed!

5. Minimum Spanning Trees

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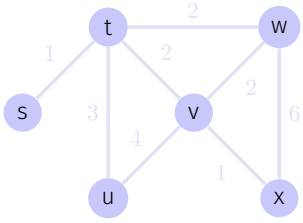
5.1. Recap Theory

Minimum Spanning Trees



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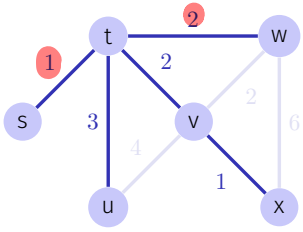

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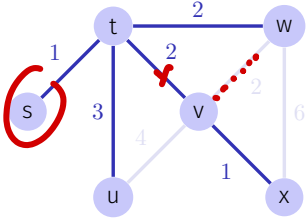
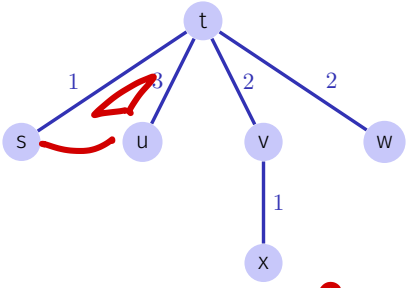
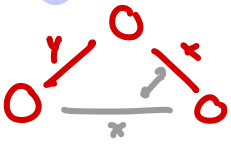




(Solution is not unique.)



Jarnik-Prim-Dijkstra Algorithm

- Finds a minimum spanning tree.
- Starts from a single node and grows.
- Uses a priority queue.
- Evaluates edges, not paths.

Algorithm Jarnik-Prim-Dijkstra(G)

Input: A connected, undirected graph $G = (V, E)$ with weights w
Output: A minimum spanning tree T

Initialize $T = \emptyset$
Choose arbitrary vertex v_0 from V and add v_0 to T
while $V \neq \emptyset$ **do**
    Choose edge (u, v) with smallest weight such that u is in T and v is in $V - T$
    Add v to T

return T

Differences from Dijkstra's Algorithm

- Jarnik-Prim-Dijkstra evaluates edges. Dijkstra evaluates paths.
- Jarnik-Prim-Dijkstra creates a minimum spanning tree. Dijkstra finds the shortest path.
- Jarnik-Prim-Dijkstra can handle negative weights, in contrast to Dijkstra.[1]

[1] The shortest path algorithm by Dijkstra can only deal with negative edge weights if it is known that no negative cycles can occur. And the algorithm can then still have an exponential runtime. It is therefore not practicable, if negative edges occur.

MakeSet, Union, and Find

- Make-Set(i): create a new set represented by i.
- Find(e): name of the set i that contains e .
- Union(i, j): union of the sets with names i and j.

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In MST-Kruskal:

- Make-Set(i): New tree with i as root.
- Find(e): Find root of e
- Union(i, j): Union of the trees i and j.

Algorithm MST-Kruskal(G)

From the Lecture

Input: Undirected weighted Graph $G = (V, E, c)$
Output: MST of G

Sort edges by weight $c(e_1) \leq \ldots \leq c(e_m)$

$M \leftarrow \emptyset$

for $i = 1$ **to** $|V|$ **do**
  `MakeSet`(i)

for $i = 1$ **to** m **do**
  $(u, v) \leftarrow e_i$
  if `Find`$(u) \neq$ `Find`(v) **then**
    `Union`(`Find`(u), `Find`(v))
    $M \leftarrow M \cup \{e_i\}$
  // conceptually: else $D \leftarrow D \cup \{e_i\}$

return (V, M, c)

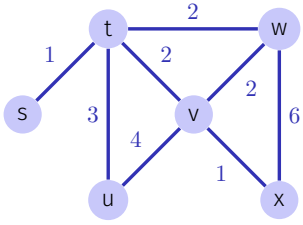
← use std::sort with a lambda function that implements the comparison (see task description)

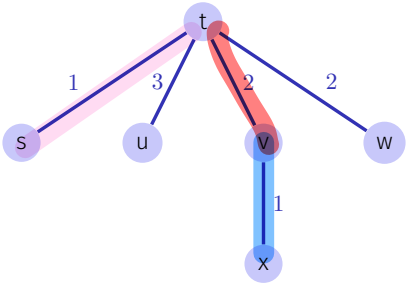
to initialize the

```
[](const Edge& e1, const Edge& e2) { return e1.length < e2.length; }
```

←

Representation as array





Index	s	t	w	v	u	x

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Parent	t	t	t	t	t	v

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From the Lecture

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Tree may degenerate... Running time of `Find`: $\Theta(n)$

←

Optimisation of the runtime for `Find`

From the Lecture

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`MakeSet`(i)

Optimisation of the runtime for `Find`

`MakeSet`(i)	$p[i] \leftarrow i$; $h[i] \leftarrow 1$; **return** i

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`Union`(i, j)

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$\Rightarrow$ Tree depth (and worst-case running time for `Find`) in $\Theta(\log n)$

(Proof: lecture notes)

Alternative improvement

From the Lecture

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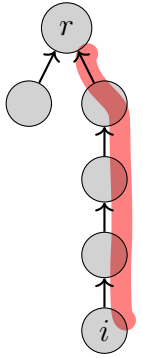
Link all nodes to the root when `Find` is called.

`ImprovedFind`(i):

$r \leftarrow$ `Find`(i)
$v \leftarrow i$
while $(v \neq r)$ **do**
    $p_v \leftarrow p[v]$
    $p[v] \leftarrow r$
    $v \leftarrow p_v$
return r



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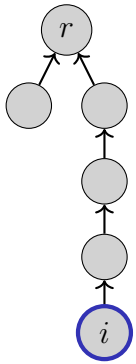

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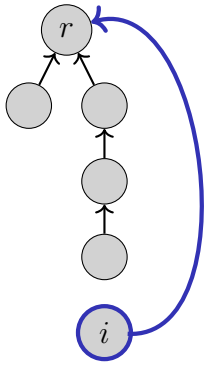

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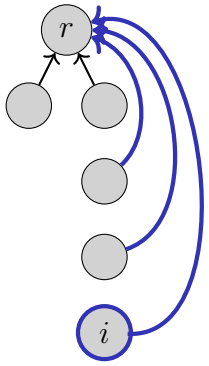

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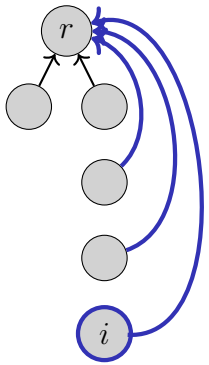

Cost: amortised *nearly* constant (inverse of the Ackermann-function).[2]

[2] When combined with union by size, cf. Cormen et al, Kap. 21.4

Running time of Kruskal's Algorithm

- Sorting of the edges: $\Theta(|E| \log |E|) = \Theta(|E| \log |V|)$. [3]
- Initialisation of the Union-Find data structure $\Theta(|V|)$
- $|E|\times$ Union(Find(x),Find(y)): $\mathcal{O}(|E| \log |V|)$.

Overall $\Theta(|E| \log |V|)$**.**

[3] because G is connected: $|V| \leq |E| \leq |V|^2$

5.2. Code-Expert Exercise

Code-Example

'Kruskal MST' on Code-Expert

```
5  EdgeSet kruskal(const EdgeSet& edges) {
6    /*...  */
13
14   // check maximum node number used, for kruskal algorithm
15   size_t max_node_number = 0;
16   for (const auto e: edges){
17     auto mn = std::max(e.node1,e.node2);    // e = {node1, node2}
18     max_node_number = std::max(max_node_number,mn);
19   }
20
21   // sort edges by length
22   // Note: The "<" operator for edges does not sort them by length!
23   std::vector<Edge> sorted_edges(edges.begin(), edges.end());
24                                  // ^--misleading name - they're not sorted yet
25   std::sort(sorted_edges.begin(), sorted_edges.end(),
26             [](const Edge& e1, const Edge& e2) { return e1.length < e2.length; });
27   //          ^^^ - - - - - fear not - this is just a lambda function - - - - - ^^^
28
29   // initialize union find datastructure
30   UnionFind uf(max_node_number+1);  // create the array for the unionfind data structure
31
32   // build mst by greedily adding edges from short to long.
33   EdgeSet mst_edges;
34   for (auto e : sorted_edges) {
35     // unify the two nodes connected by this edge
36     if (uf.unify(e.node1, e.node2)) { // unify() returns false IF node1 node2 are
37                                       // already in the same set/union
38       // edge start and end nodes where not in same component already => add edge to mst
39       mst_edges.insert(e);
40     }
41   }
42
43   return mst_edges;
44 }
```

6. MaxFlow

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6.1. Recap Theory

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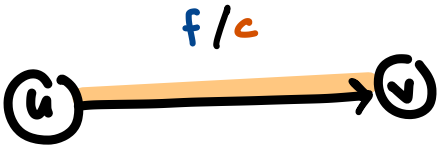

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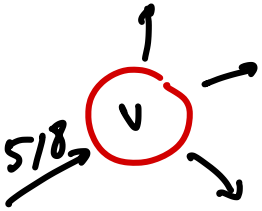

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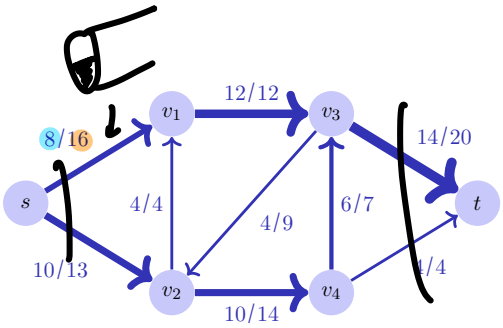

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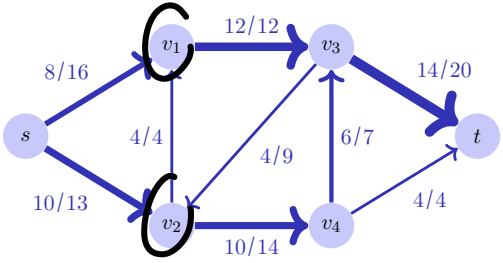

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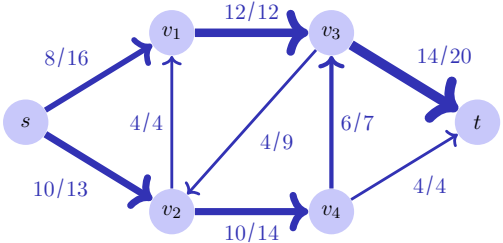

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Here $|f| = 18$.

Residual Network

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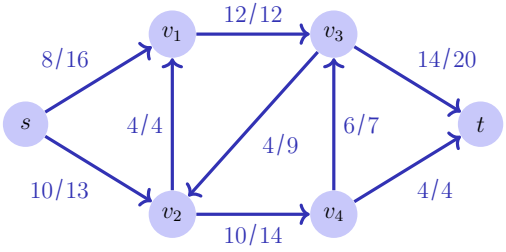

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reconstructing orig. FN from RN.

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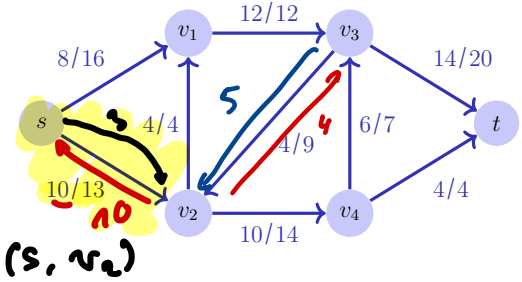

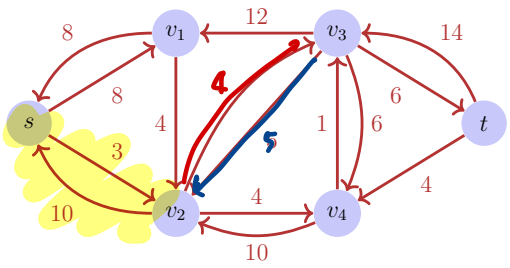

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Augmenting Paths

Augmenting Path p: simple path from s to t in the residual network G_f.

Residual Capacity $c_f(p)$: the least capacity along the ~~expansion~~ augmenting path p

$$c_f(p) = \min\{c_f(u, v) : (u, v) \text{ edge in } p\}$$

Note: There can be multiple augmenting paths!

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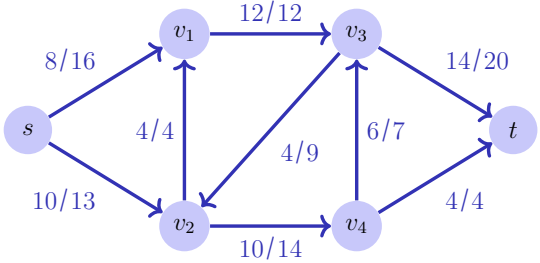

Note: There can be multiple augmenting paths!

Augmenting Paths

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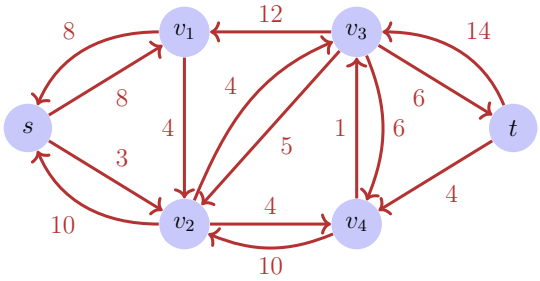

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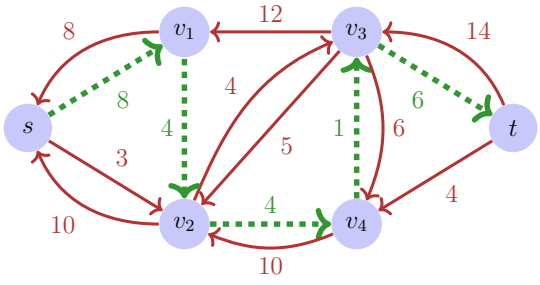

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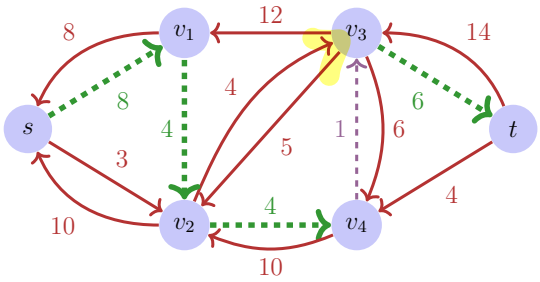


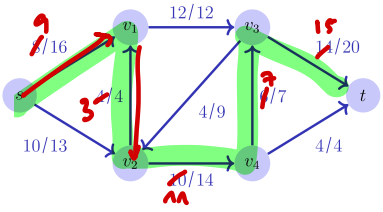
Note: There can be multiple augmenting paths!

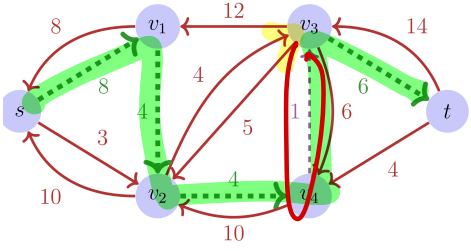
Algorithm Ford-Fulkerson(G, s, t)

Input: Flow network $G = (V, E, c)$
Output: Maximal flow f.

for $(u, v) \in E$ **do**
  $f(u, v) \leftarrow 0$

while Exists path $p : s \rightsquigarrow t$ in residual network G_f **do**
  $c_f(p) \leftarrow \min\{c_f(u, v) : (u, v) \in p\}$
  foreach $(u, v) \in p$ **do**
    if $(u, v) \in E$ **then**
      $f(u, v) \leftarrow f(u, v) + c_f(p)$
    else
      $f(v, u) \leftarrow f(u, v) - c_f(p)$





Edmonds-Karp Algorithm

Choose in the Ford-Fulkerson-Method for finding a path in G_f the expansion path of shortest possible length (e.g. with BFS)

Theorem 1

When the Edmonds-Karp algorithm is applied to some integer valued flow network $G = (V, E)$ with source s and sink t then the number of flow increases applied by the algorithm is in $\mathcal{O}(|V| \cdot |E|)$

$\Rightarrow$ ***Overall asymptotic runtime:*** $\mathcal{O}(|V| \cdot |E|^2)$

Max-Flow Min-Cut Theorem

Theorem 2

Let f be a flow in a flow network $G = (V, E, c)$ with source s and sink t. The following statements are equivalent:

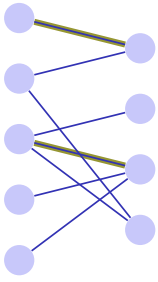
1. *f is a maximal flow in G*
2. *The residual network G_f does not provide any expansion paths*
3. *It holds that $|f| = c(S, T)$ for a cut (S, T) of G.*

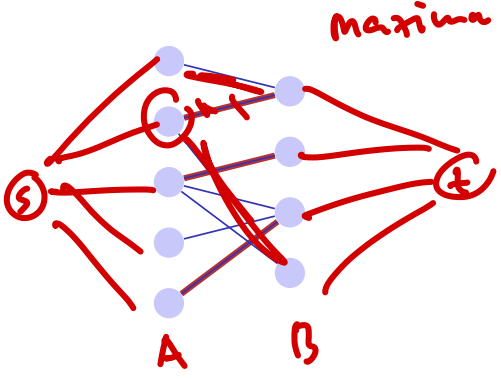
Application: maximal bipartite matching

Given: bipartite undirected graph $G = (V, E)$.

Matching M: $M \subseteq E$ such that $|\{m \in M : v \in m\}| \leq 1$ for all $v \in V$.

Maximal Matching M: Matching M, such that $|M| \geq |M'|$ for each matching M'.



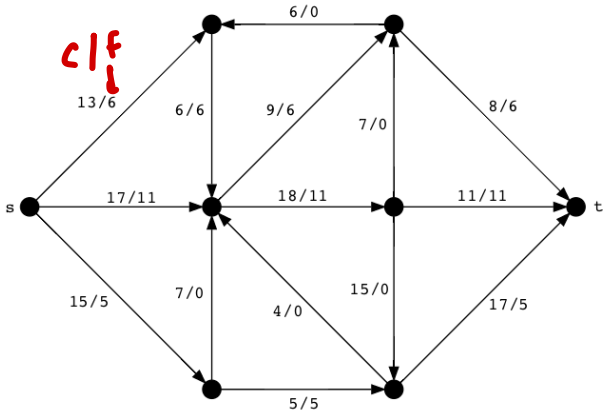


6. MaxFlow

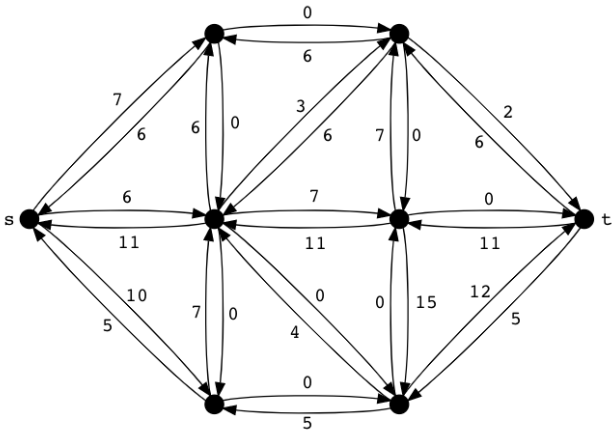
6.2. Quiz

Manual Max Flow Exercise

This graph shows a flow chart that is not at maximum capacity. Run one iteration of the Ford-Fulkerson algorithm to find the max flow.

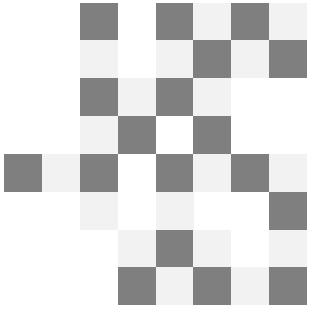


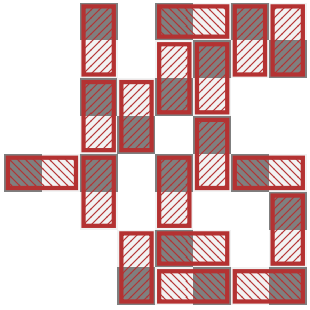
Manual Max Flow Solution



update not shown since it is not unique!

Max Flow Question





Let an $n \times n$ chessboard be given without some squares. Describe an efficient algorithm to find out if the board can be completely covered with dominoes. Then model the problem as a flow problem.

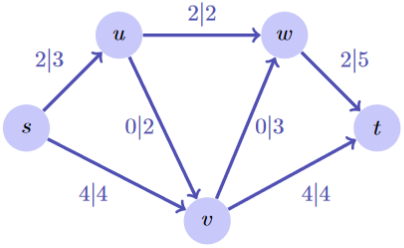
6. MaxFlow

6.3. Old Exam Questions

Exam Question Example

Gegeben ist das folgende Flussnetzwerk G mit Quelle s und Senke t. Die einzelnen Kapazitäten c_i und Flüsse ϕ_i sind an den Kanten angegeben als $\phi_i|c_i$. Geben Sie den Wert des Flusses f an.

Provided in the following is a flow network G with source s and sink t. Capacities c_i and flows ϕ_i are provided at the edges as $\phi_i|c_i$. Provide the value of the flow f.



$|f| =$

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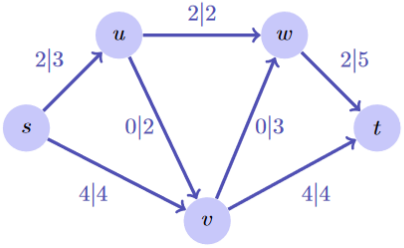
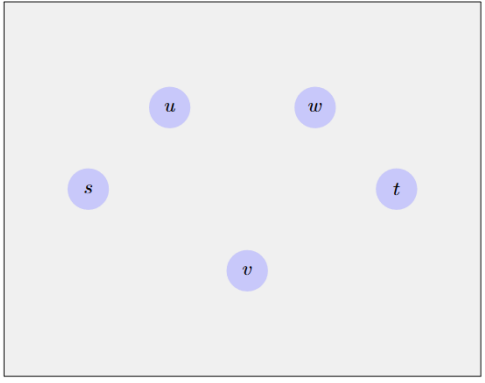


$|f| =$ 6

Exam Question Example

Zeichnen Sie nun das Restnetzwerk G_f zu obigem Fluss und markieren Sie darin einen Erweiterungspfad p. Geben Sie den Wert $c_f(p)$ der Restkapazität des Erweiterungspfades p im Restnetzwerk G_f an.

Draw the residual network G_f to the flow above and mark an augmenting path p. Provide the rest capacity $c_f(p)$ of the path p in the rest network G_f.



$|c_f(p)| =$

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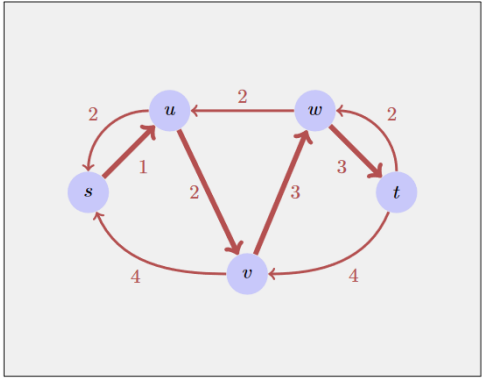

$|c_f(p)| =$ 1

Exam Question Example

Woran erkennen Sie, ob Sie den maximalen Fluss gefunden haben?

How do you see if you have found the maximum flow?

Exam Question Example

Woran erkennen Sie, ob Sie den maximalen Fluss gefunden haben?

How do you see if you have found the maximum flow?

Found the maximum flow if:
The residual network does not have any more augmenting path.
Alternative: Identify a cut with $|c(S,T)| = |f|$.

7. TSP

Travelling Salesperson Problem

Problem

Given a map and list of cities, what is the shortest possible route that visits each city once and returns at the original city?

Travelling Salesperson Problem

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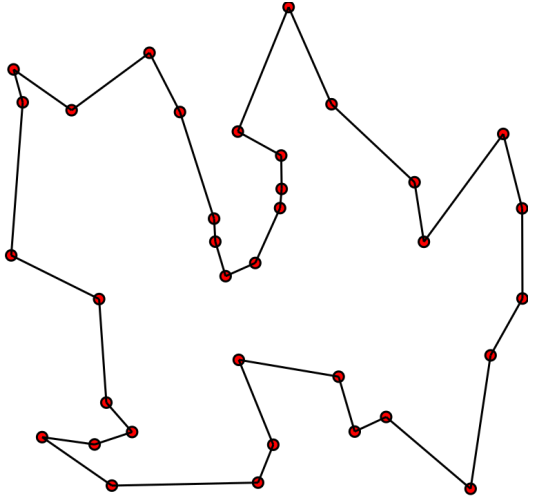
Given a map and list of cities, what is the shortest possible route that visits each city once and returns at the original city?

Mathematical model

On an undirected, weighted graph G, which cycle containing all of G's vertices has the lowest weight sum?

Travelling Salesperson Problem



Travelling Salesperson Problem

- The problem has no known polynomial-time solution.
- Many heuristic algorithms exists. They do not always return the optimal solution.

Travelling Salesperson Problem

- The heuristic algorithm that you are asked to implement on CodeExpert (*The Travelling Student*) on CodeExpert uses an MST:
  1. Compute the minimum spanning tree M
  2. Make a depth first search on M
- The algorithm is 2-approximate, meaning that the solution it generates has at most twice the cost of the optimal solution.
- The algorithm assumes a complete graph $G = (V, E, c)$ that satisfies the triangle inequality: $\forall v, w, x \in V : c(v, w) \leq c(v, x) + c(x, w)$

8. Outro

General Questions?

See you next time!

Have a nice week!