



# Exercise Session W09

Computer Science (CSE & CBB & Statistics) – AS 23

# Overview

## Today's Agenda

Follow-up

Feedback on **code** expert

Objectives

Recursion II

Structs

Outro



`n.ethz.ch/~agavranovic`

# 1. Follow-up

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# Follow-up from previous exercise sessions

- Efficiency of "**read\_matrix**":
  - The version with the pre-initialized matrix should generally be more efficient, since it uses less dynamically allocated memory<sup>1</sup>
  - It only really makes a difference for very big matrices, so you do not need to worry about the efficiency for now

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<sup>1</sup>You will learn what this means towards the end of the semester

## 2. Feedback on **code** expert

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# General things regarding **code expert**

A few simplifications for your code<sup>2</sup>

```
if(condition == true){  
    // ...  
}
```

→

```
if(condition){  
    //...  
}
```

```
if(condition){  
    return true;  
} else {  
    return false;  
}
```

→

```
return condition;
```

In case of function that return a `bool`

<sup>2</sup>Remember: simplifications aren't always better for comprehension

# Specific things regarding **code** expert

## E7:T1: "Const and reference types"

- What does **const** mean?
  - Once a **const** variable has been initialized, its value cannot be changed
  - The variable can be used in the program (but "read only")
- When is constness (not) respected?
  - Default: if nothing is declared **const** then constness is respected
  - Otherwise: you must not attempt to modify the value of a **const** variable (no "write access")

# Questions?



## 3. Objectives

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# Objectives

- be able to solve more advanced problems involving recursion
- be able to define and use structs

## 4. Recursion II

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# Exercise Power Set

## **DiskMath Recap**

- A power set is the set of all subsets

# Exercise Power Set

## DiskMath Recap

- A power set is the set of all subsets

- $2^S = \{X \mid X \subseteq S\}$

$\mathcal{P}(S)$

$$S = \{?\} = 2^0 = 1$$

$$\{\{?\}\}$$

$$|2^S| = 2^{|S|}$$

# Exercise Power Set

## DiskMath Recap

- A power set is the set of all subsets
- $2^S := \{X \mid X \subseteq S\}$
- Example:
  - Given the set  $A = \{a, b, c\}$

# Exercise Power Set

## DiskMath Recap

- A power set is the set of all subsets
- $2^S := \{X \mid X \subseteq S\}$
- Example:
  - Given the set  $A = \{a, b, c\}$
  - Its power set is  $2^A = \{\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

# Primer on set.h

- `set` is a self-made type! (a `class`)
- How does it work? See for yourself in `set.h`!

```
template <typename T>
class Set {
public:
    Set(const Set& other);
    // Creates an empty set
    Set();
    // Creates a new set from a set of elements
    Set(const std::set<T>& elements);
    // Creates a new set from a single element
    Set(T element);
    // ...
};
```



# Exercise Power Set

- Open "Power Set" on **code** expert

# Exercise Power Set

- Open "Power Set" on **code expert**
- Think about how you would approach the problem with pen and paper

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- Implement a solution (optionally in groups)

# Exercise Power Set

- Open "Power Set" on **code expert**
- Think about how you would approach the problem with pen and paper
- Implement a solution (optionally in groups)
- You can find the functionalities of the type `set` in the `main.cpp` file

# Solution to "Power Set" (Base case)

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```
SetOfCharSets power_set(const CharSet& set) {  
    // base case: empty set  
    if (set.size() == 0) {  
        return SetOfCharSets(CharSet());  
    }  
}
```

set: {}

{}

{ {} }

# Solution to "Power Set"

```
// set has at least 1 element -> split set into two sets.
```

```
CharSet first_element_subset = CharSet(set.at(0));
```

```
CharSet remaining_subset = set - first_element_subset;
```

```
// get power set for remaining subset
```

```
SetOfCharSets remaining_subset_power_set = power_set(remaining_subset);
```

```
// init result with power set of remaining subset
```

```
SetOfCharSets result = remaining_subset_power_set;
```

{ {}, {a}, ... }

```
// add first element to every set in the powerset
```

```
for (unsigned int i = 0; i < remaining_subset_power_set.size(); ++i) {  
    result.insert(first_element_subset + remaining_subset_power_set.at(i));  
}
```

{a}      {s}

{a, s}

```
return result;
```

# Solution to "Power Set" (Conceptually)

Given:  $\{a, b, c, d\}$

---

<sup>3</sup>Here is where the *Recursive Leap of Faith* kicks in



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$\{a\}, \quad \{b, c, d\}$

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Given:  $\{a, b, c, d\}$

// set has at least 1 element -> split set into two sets

$\{a\}, \quad \{b, c, d\}$

// get power set for remaining subset<sup>3</sup>

---

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$$\mathcal{P}(\{b, c, d\}) = \{\{\}, \{b\}, \{c\}, \{d\}, \{b, c\}, \dots\}$$

---

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result  $\leftarrow \{\{\}, \{b\}, \{c\}, \{d\}, \{b, c\}, \dots\}$

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// init result with power set of remaining subset

result  $\leftarrow \{\{\}, \{b\}, \{c\}, \{d\}, \{b, c\}, \dots\}$

// add first element to every set in the powerset

---

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result  $\leftarrow \{\{\}, \{b\}, \{c\}, \{d\}, \{b, c\}, \dots\}$

// add first element to every set in the powerset

$\left\{ \begin{array}{l} \{\}, \{b\}, \{c\}, \{d\}, \{b, c\}, \dots, \\ \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \dots, \end{array} \right\}$

---

<sup>3</sup>Here is where the *Recursive Leap of Faith* kicks in



# Questions?

# Towers of Hanoi

# Towers of Hanoi

Everyone: it's a game for kids



Programmers:



# Experiment: The Towers of Hanoi



left

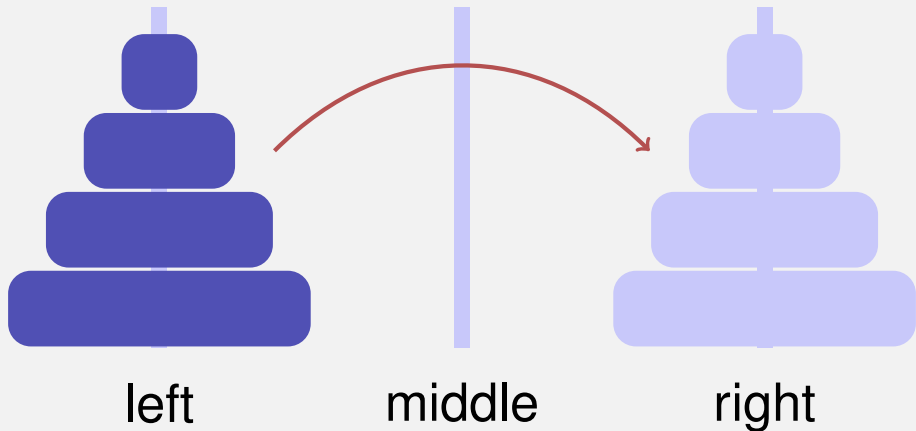


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# Experiment: The Towers of Hanoi



# Die Türme von Hanoi - So gehts!



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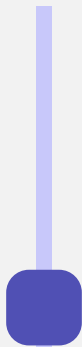


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# Die Türme von Hanoi - So gehts!



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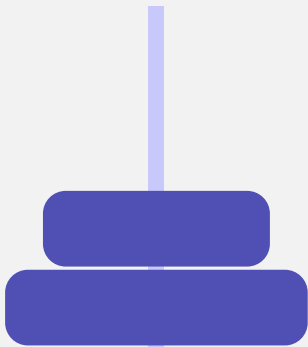


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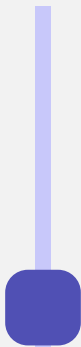


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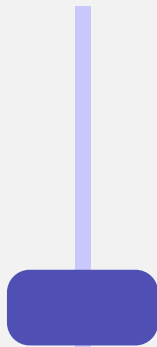
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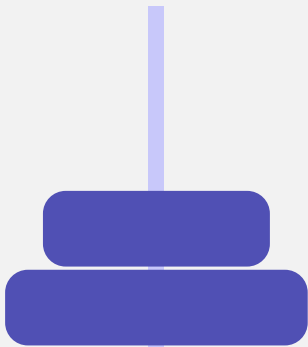
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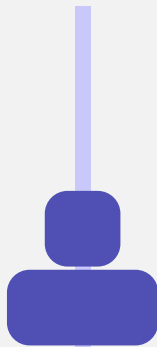
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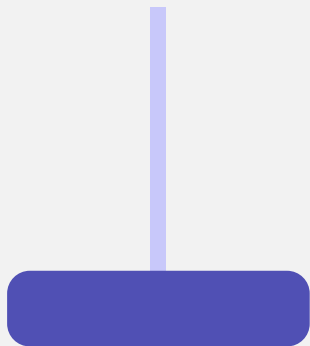


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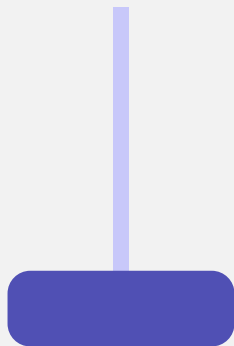


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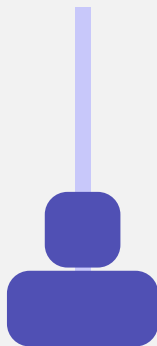
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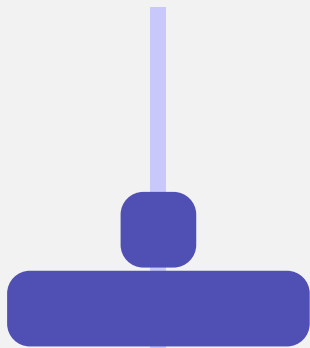


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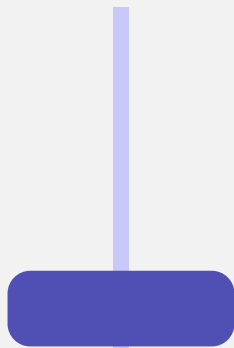


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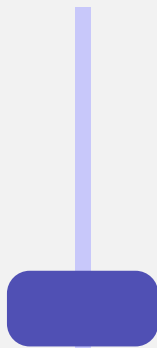
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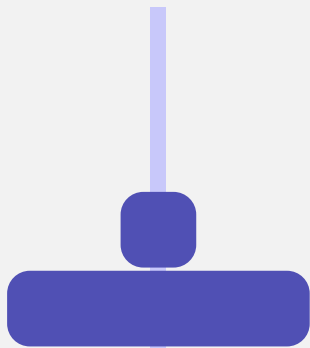


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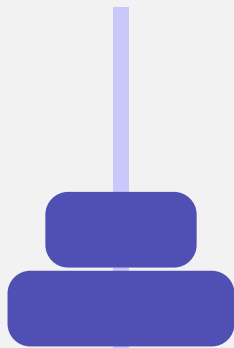


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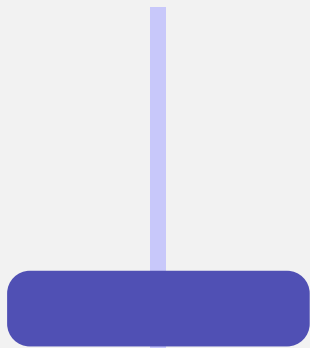


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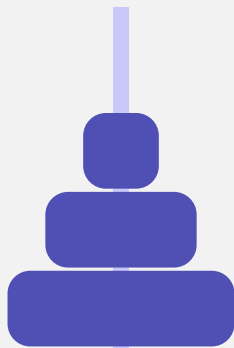


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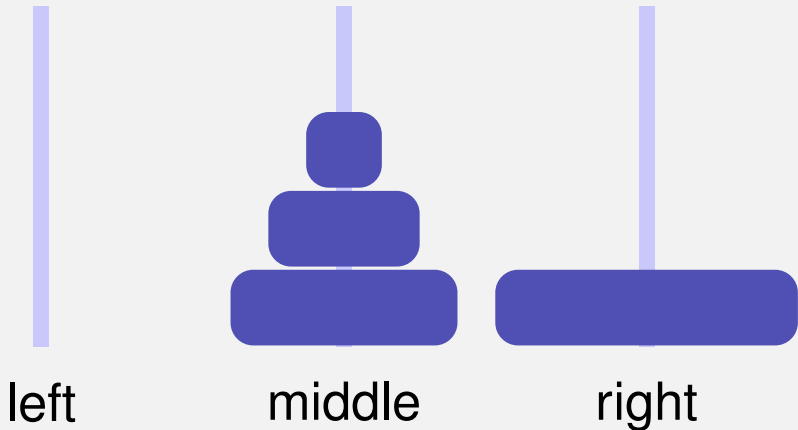


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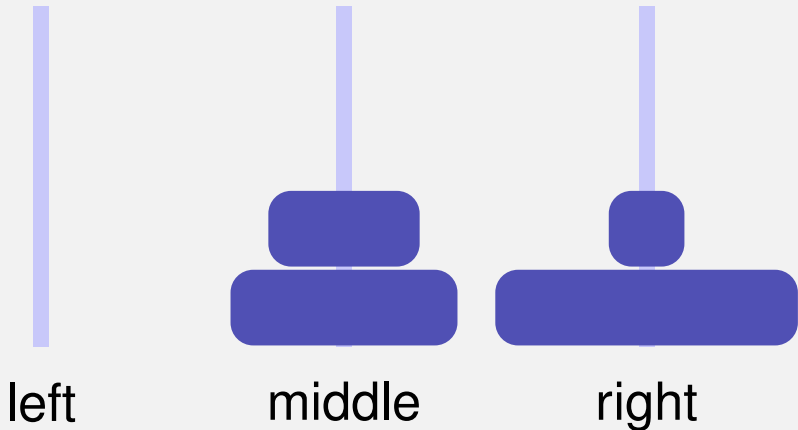


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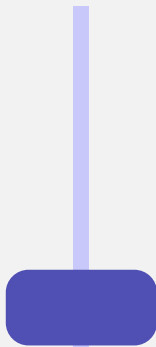
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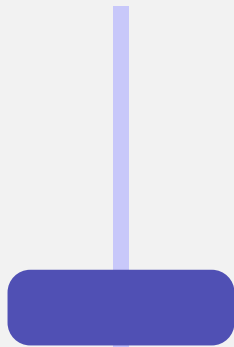
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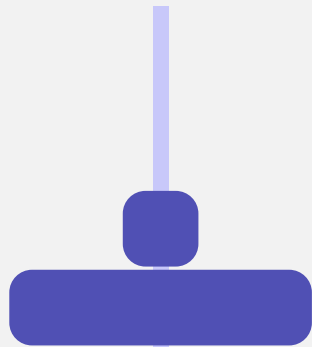
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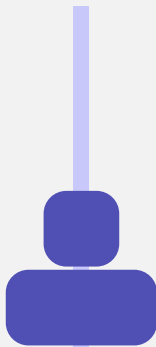
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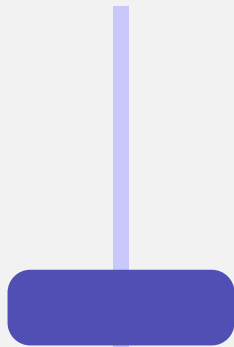
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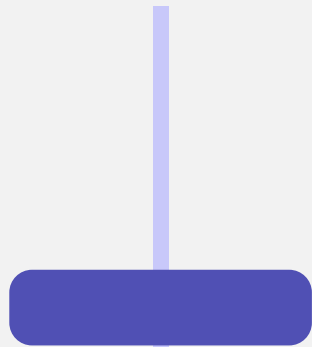
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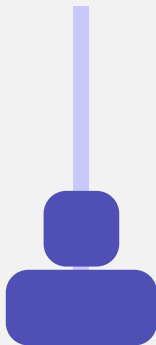


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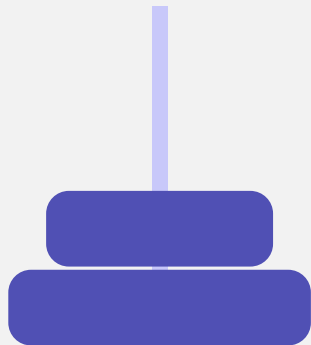
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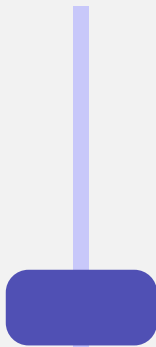


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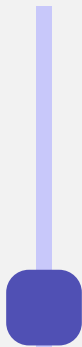


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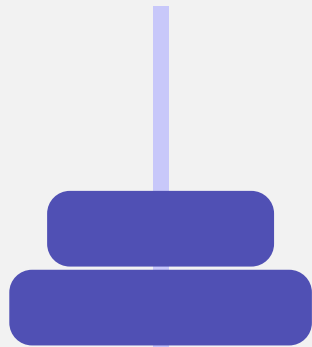
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middle

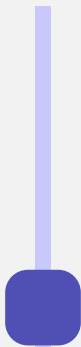


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# Die Türme von Hanoi - So gehts!



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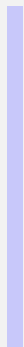


middle



right

# Die Türme von Hanoi - So gehts!



left

In the first call!



SRC  
source



middle

AUX  
auxillary



right

DST  
destination

# Exercise Towers of Hanoi

- Open "Towers of Hanoi" on **code expert**

# Exercise Towers of Hanoi

- Open "Towers of Hanoi" on **code expert**
- Think about how you would approach the problem with pen and paper

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- Open "Towers of Hanoi" on **code expert**
- Think about how you would approach the problem with pen and paper
- Implement a solution (optionally in groups)



# Exercise Towers of Hanoi

```
std::string text = "text";
```

- Open "Towers of Hanoi" on **code expert**
- Think about how you would approach the problem with pen and paper
- Implement a solution (optionally in groups)

# The Towers of Hanoi – Recursive Approach



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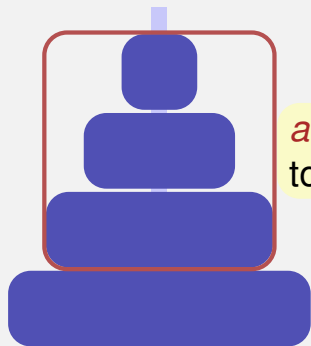


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# The Towers of Hanoi – Recursive Approach



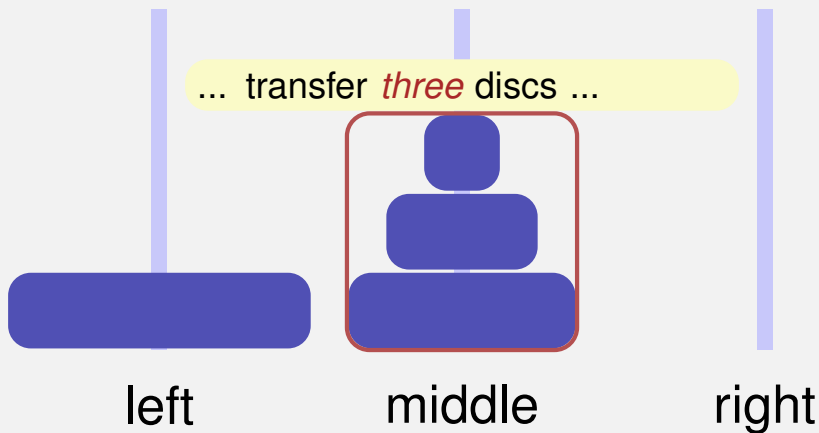
*assume* we knew how  
to ...

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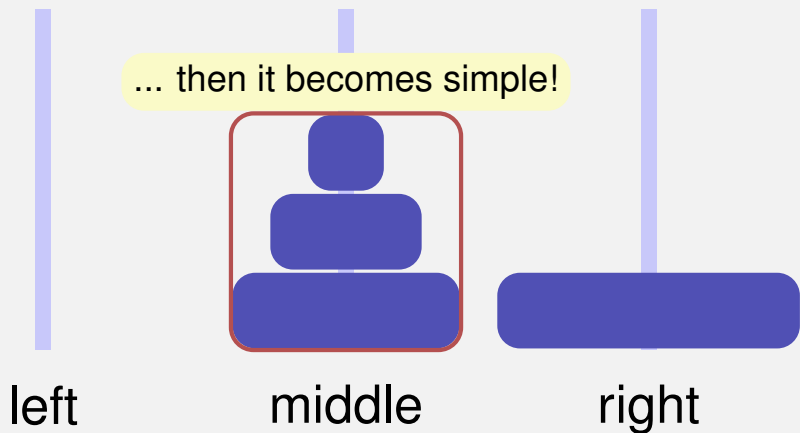
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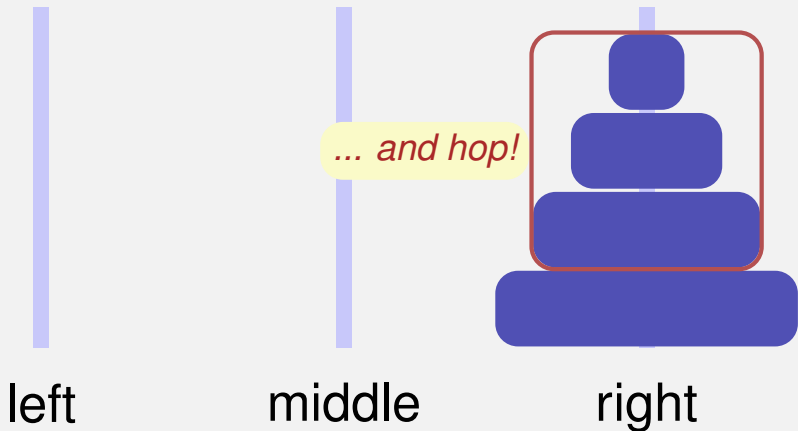
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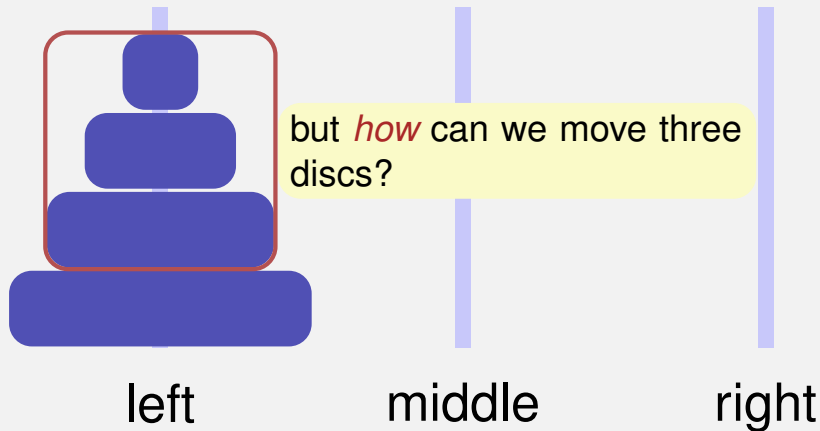


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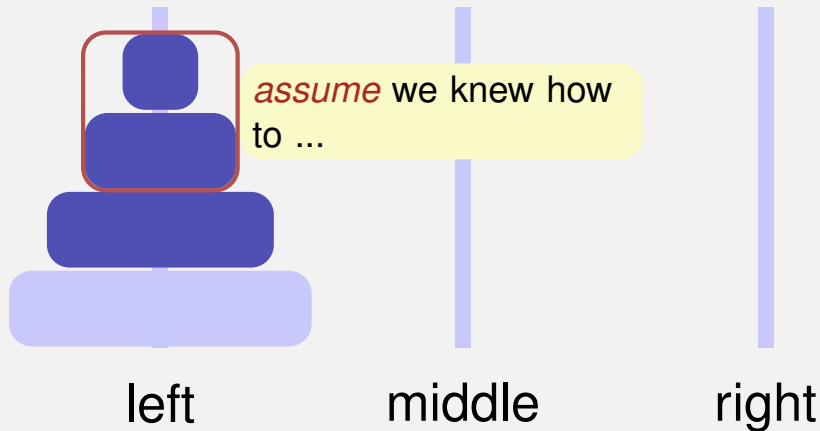
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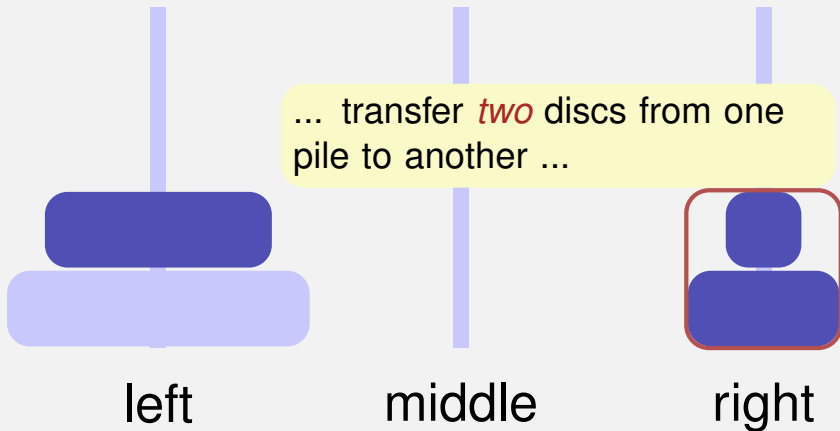




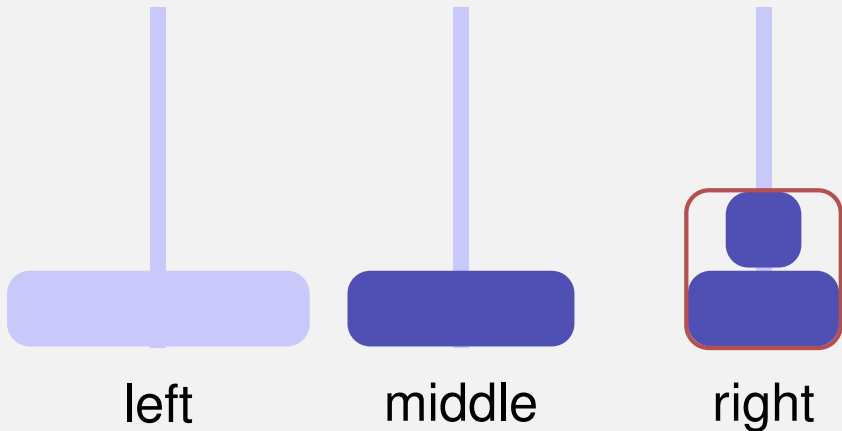
# The Towers of Hanoi – Recursive Approach



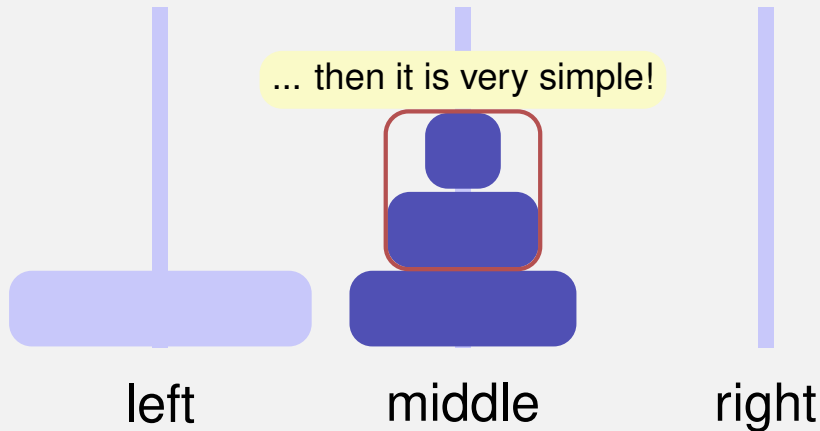
# The Towers of Hanoi – Recursive Approach



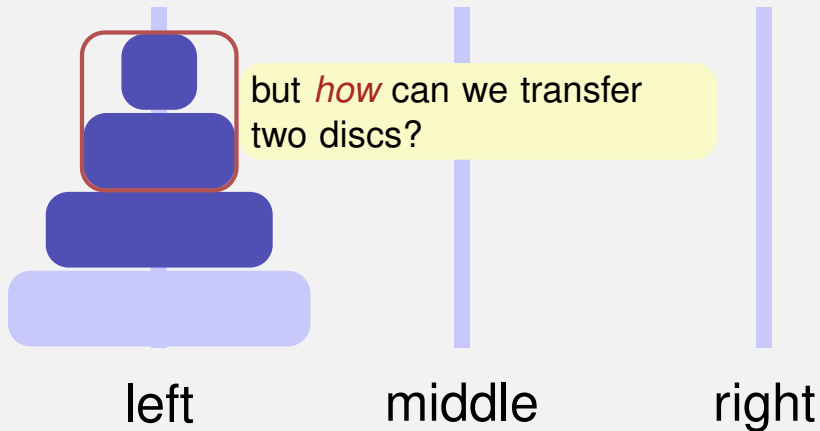
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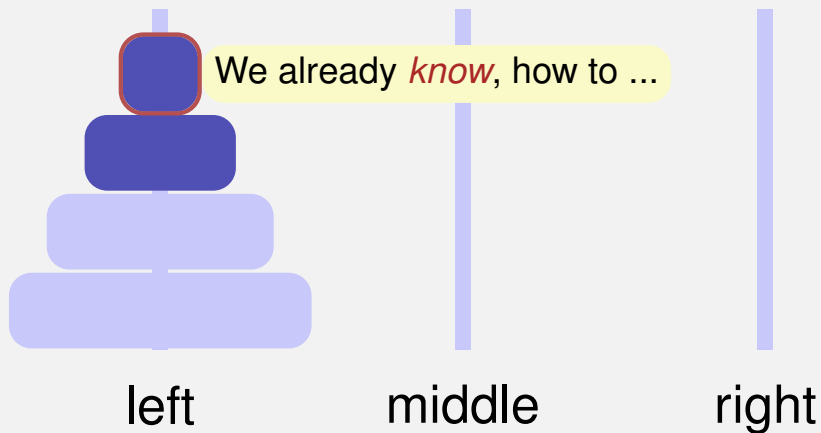
# The Towers of Hanoi – Recursive Approach



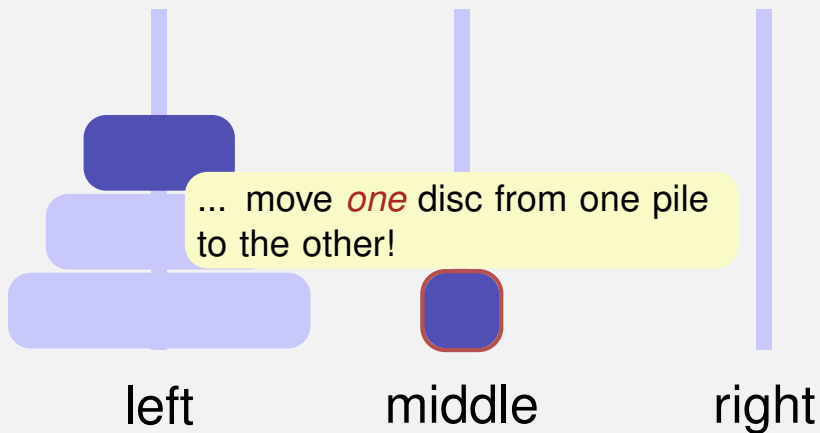
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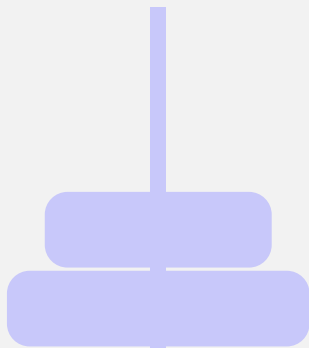
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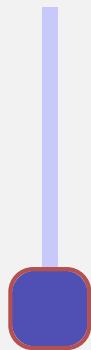
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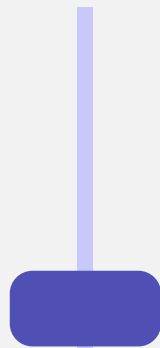
# The Towers of Hanoi – Recursive Approach



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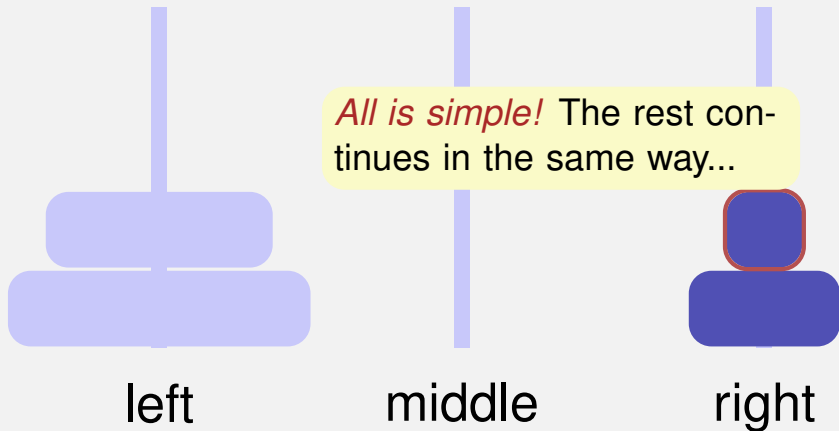
middle



right



# The Towers of Hanoi – Recursive Approach



# The Towers of Hanoi – Code



left

middle

right

Move 4 discs from left to right with auxiliary staple middle:

```
move(4, "left", "middle", "right")
```

# The Towers of Hanoi – Code



`move(n, src, aux, dst)`  $\Rightarrow$

- 1 Move the top  $n - 1$  discs from *src* to *aux* with auxiliary staple *dst*:

`move(n - 1, src, dst, aux);`

- 2 Move 1 disc from *src* to *dst*

`move(1, src, aux, dst);`


- 3 Move the top  $n - 1$  discs from *aux* to *dst* with auxiliary staple *src*:

`move(n - 1, aux, src, dst);`



# The Towers of Hanoi – Code

```
void move(int n, const string &src, const string &aux, const string &dst){  
    if (n == 1) {  
        // base case ('move' the disc)  
        std::cout << src << " --> " << dst << std::endl;  
    } else {  
        // recursive case  
    }  
}
```



# The Towers of Hanoi – Code

```
void move(int n, const string &src, const string &aux, const string &dst){  
    if (n == 1) {  
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    } else {  
        // recursive case  
        move(n-1, src, dst, aux);  
  
    }  
}
```

# The Towers of Hanoi – Code

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    if (n == 1) {
        // base case ('move' the disc)
        std::cout << src << " --> " << dst << std::endl;
    } else {
        // recursive case
        move(n-1, src, dst, aux);
        move(1, src, aux, dst);
    }
}
```

# The Towers of Hanoi – Code

```
void move(int n, const string &src, const string &aux, const string &dst){  
    if (n == 1) {  
        // base case ('move' the disc)  
        std::cout << src << " --> " << dst << std::endl;  
    } else {  
        // recursive case  
        move(n-1, src, dst, aux);  
        move(1, src, aux, dst);  
        move(n-1, aux, src, dst);  
    }  
}
```

# The Towers of Hanoi – Code

```
void move(int n, const string &src, const string &aux, const string &dst){
    if (n == 1) {
        // base case ('move' the disc)
        std::cout << src << " --> " << dst << std::endl;
    } else {
        // recursive case
        move(n-1, src, dst, aux);
        move(1, src, aux, dst);
        move(n-1, aux, src, dst);
    }
}

int main() {
    move(4, "left", "middle", "right");
    return 0;
}
```



# The Towers of Hanoi – Code Alternative

```
void move(int n, const string &src, const string &aux, const string &dst){  
    // base case  
    if (n == 0) return;  
  
    // recursive case  
    move(n-1, src, dst, aux);  
    std::cout << src << " --> " << dst << "\n";  
    move(n-1, aux, src, dst);  
}
```

```
int main() {  
    move(4, "left ", "middle", "right ");  
    return 0;  
}
```

# Questions?

## 5. Structs

---

# Structs

# Structs

**A `struct` is a bundle of stuff**

# Structs

## A **struct** is a bundle of stuff

- That could be variables, functions, other structs, and much more ("members")
- The types do not have to be the same
- Offer us a way to define new "objects", e.g. your own number type or mathematical objects such as lines, squares, circles, etc.
- Important: Do not forget the ; at the end of the definition

# Structure of struct

```
struct Person {  
    → unsigned int age;  
    std::string field;  
    std::vector<int> lucky_nums;  
};
```

Definition

void say(...shy word);

std::vector<int> v = {...}

```
int main () {  
    Person Adel = {26, "Computer Science", {42, 161}};  
    Person Deli = Adel;  
    Person Jules = {25, "Linguistics", {13, 12}};  
    Person Lily = {19, "Computational Science", {9, 19}};  
    std::cout << "Adel's " << Adel.age <<  
        " years old\n" << std::endl;  
    return 0;  
}
```

Adel.say("hi");

# Questions?

```
// Optional Subtask 6: overloading the << operators-----
std::ostream& operator<<(std::ostream& os, const vec& v) {
    os << "(" << v.x << "," << v.y << "," << v.z << ")";
    return os;
}
std::ostream& operator<<(std::ostream& os, const line& l) {
    os << l.start << " <-> " << l.end;
    return os;
}
//-----
```



# Exercise "Geometry Exercise"

- Open "Geometry Exercise" on **code** expert

# Exercise "Geometry Exercise"

- Open "Geometry Exercise" on **code expert**
- Think about how you would approach the problem with pen and paper

# Exercise "Geometry Exercise"

- Open "Geometry Exercise" on **code expert**
- Think about how you would approach the problem with pen and paper
- Implement a solution (optionally in groups)

# Solution to "Geometry Exercise"

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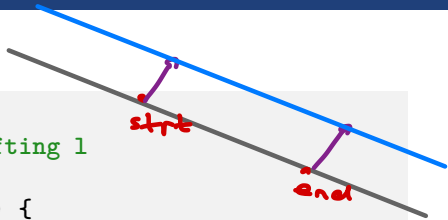
```
// Subtask 1: adding vectors
// POST: returns the sum of a and b
vec sum(const vec& a, const vec& b) {
    // version 1: compact, used for the rest of the example
    return {a.x + b.x, a.y + b.y, a.z + b.z};

    // version 2: longer but maybe easier to understand
    // vec tmp;
    // tmp.x = a.x + b.x;
    // tmp.y = a.y + b.y;
    // tmp.z = a.z + b.z;
    // return tmp;
}
```

# Solution to "Geometry Exercise"

```
// Subtask 2: defining a line in 3D
struct line {
    vec start;
    vec end; // INV: start != end
};
// helper function to print a vector
void print_line(const line& l) {
    print_vec(l.start);
    std::cout << " <-> ";
    print_vec(l.end);
}
```

# Solution to "Geometry Exercise"



```
// Subtask 3: shifting line by a vector  
// POST: returns a new line obtained by shifting l  
// by v.
```

```
line shift_line(const line& l, const vec& v) {  
    return {sum(l.start, v), sum(l.end, v)};  
}
```

```
// Subtask 4: overloading the + operator for vectors  
vec operator+(const vec& a, const vec& b) {  
    return sum(a, b);  
}
```

# Solution to "Geometry Exercise"

```
// Subtask 5: overloading the + operator for lines
// version 1: use the shift_line function
line operator+(const line& l, const vec& v) {
    return shift_line(l, v);
}

// version 2: make use of the overloaded + operator for vectors
line operator+(const line& l, const vec& v) {
    return {l.start + v, l.end + v};
}
```



# Questions?

## 6. Outro

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# General Questions?

Till next time!

Cheers!