Algorithms and Data Structures

Exercise Session 13

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Quiz

Exercise 12.3 *Exploring connectivity of MSTs* **(1 point)**.

In this exercise, we explore connectivity properties of the set of spanning trees and MSTs of a graph using only 'local' changes. First we prove whats called the symmetric basis exchange property. Let $G = (V, E)$ be a connected graph and $w : E \to \mathbb{R}_{\geq 0}$ be a weight function.

(a) Let T_1 and T_2 be two different spanning trees of G and let $e \in T_1 \setminus T_2$. Show that there exists an edge $f \in T_2 \setminus T_1$ such that $(T_1 \setminus \{e\}) \cup \{f\}$ and $(T_2 \setminus \{f\}) \cup \{e\}$ are both spanning trees.

Now consider the graph $H = (\mathcal{B}, \mathcal{E})$ where each vertex of H corresponds to a spanning tree of G and we assign an edge between two vertices of H if their corresponding spanning trees differ by exactly two edges.

(b) Show that the graph H is connected.

(c) Consider the subgraph of H, H_{MST}, whose vertices are all MSTs of G and we keep an edge between two vertices if, again, the corresponding MSTs differ by two edges. Show that H_{MST} is connected.

Hint: *Reuse the proof for (b) but also analyze the weights of the new spanning trees produced by (a).*

Exercise 12.5 *Heavy and light edges* **(1 point).**

Let $G = (V, E)$ be a connected, undirected, weighted graph with positive weights $w_e > 0$ for $e \in E$. We say an edge $e \in E$ is *heavy* if there exists a cycle $C \subseteq E$ so that $e \in C$ is the (strictly) heaviest edge in C, i.e.,

$$w_e > w_f \text{ for all } f \in C \text{ with } f \neq e.$$

We say an edge is *light* if there exists a minimum spanning tree $T \subseteq E$ of G which contains e.

(a) Show that a heavy edge cannot be light.

Hint: Assume for a contradiction that $T \subseteq E$ is an MST of G and that T contains a heavy edge e. Say e is the heaviest edge in a cycle $C \subseteq E$. Construct a strictly cheaper spanning tree of G by removing e from T, and replacing it by a different edge $f \in C$.

(b*) Show that an edge which is not heavy, must be light. Conclude that an edge is heavy if and only if it is not light.

Hint: *You* may use without proof that Kruskal's algorithm is correct regardless of the order in which edges of equal weight are processed.

115. Distinct Subsequences

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Given two strings s and t, return *the number of distinct* ***subsequences*** *of* s *which equals* t.

The test cases are generated so that the answer fits on a 32-bit signed integer.

Example 1:

```
Input: s = "rabbbit", t = "rabbit"
Output: 3
Explanation:
As shown below, there are 3 ways you can generate "rabbit" from s.
rabbbit
rabbbit
rabbbit
```

Example 2:

```
Input: s = "babgbag", t = "bag"
Output: 5
Explanation:
As shown below, there are 5 ways you can generate "bag" from s.
babgbag
babgbag
babgbag
babgbag
babgbag
```

Peer Grading

Exercise 12.5