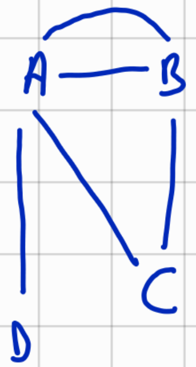
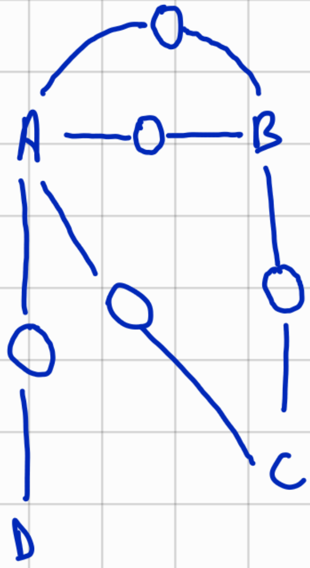
10.1.a



Subdivide all edges →

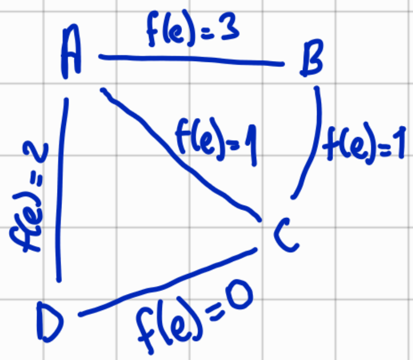


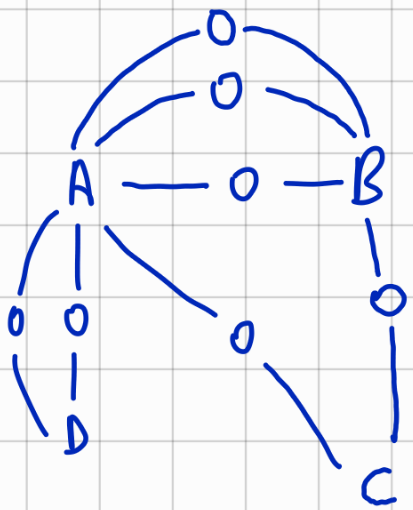
It has no EULERIAN-TOUR.
It has an Eulerian-Path.

It has no Eulerian Tour.
It has an Eulerian Path.

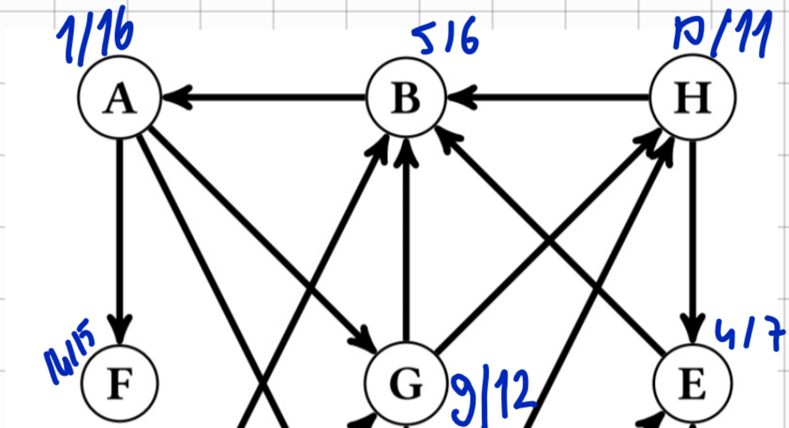
Proof of the iff-statement is required.

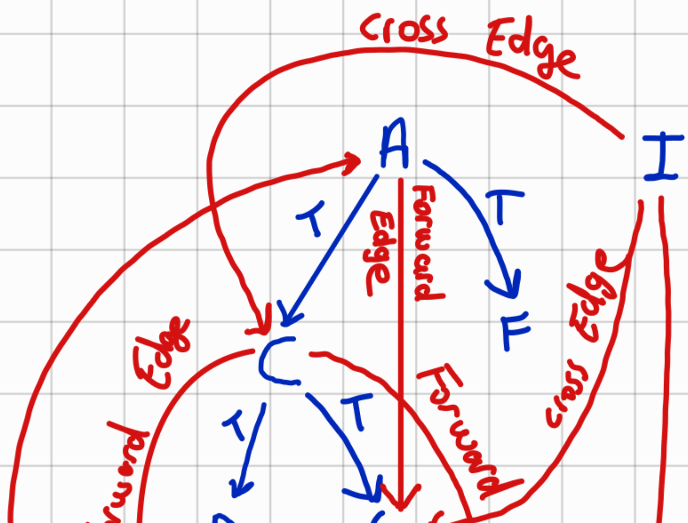
10.1.b

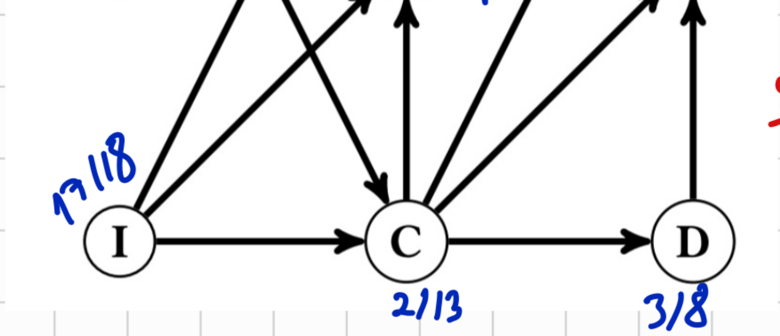


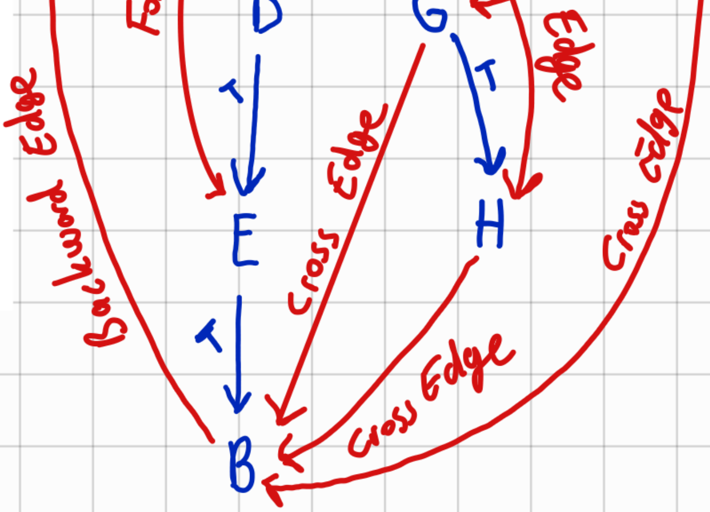


10.2





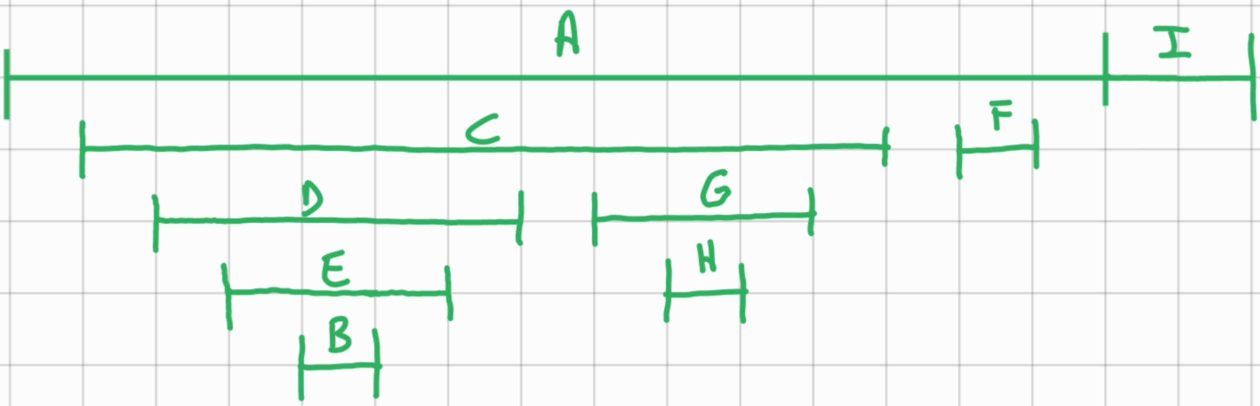




Pre-Ordering: A-C-D-E-B-G-H-F-I

Post-Ordering: B-E-D-H-G-C-F-A-I

There is a backward edge, so the graph is cyclic; it has no topological ordering.



$I_u \subset I_v \iff$ u is visited during the call of visit(v)

An edge from F to I is a cross edge.

The edge from B to A is the only backward edge in the graph. If we remove it, the graph becomes acyclic and there exists a topological ordering.

Reverse post ordering is the topological ordering. In pre-ordering "cross edges" cause a problem and it is not the topological ordering.

10.3

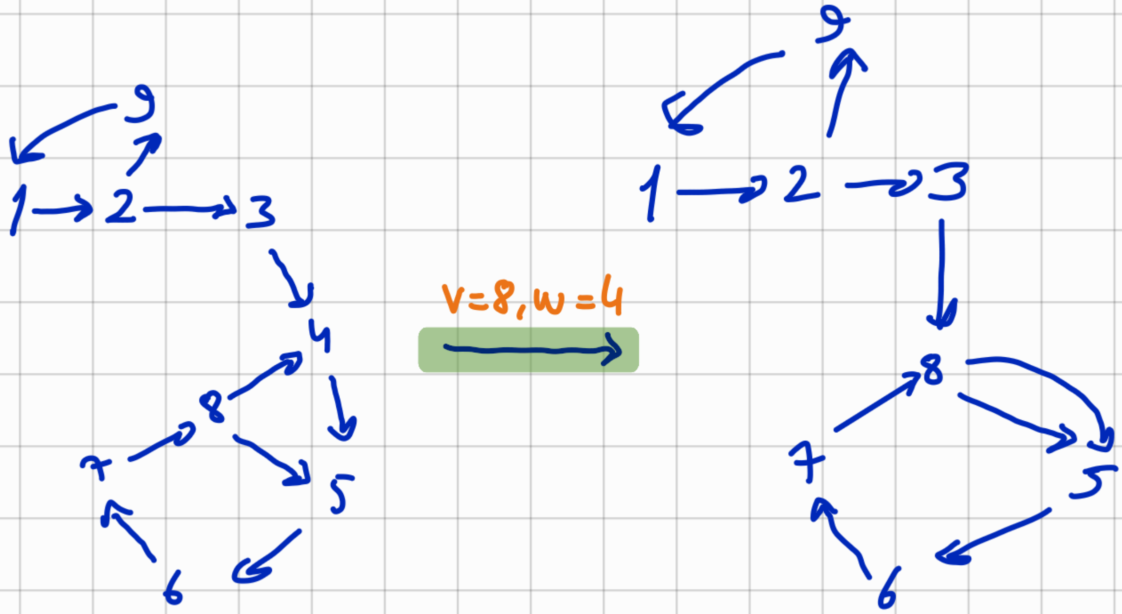
a) This part appears almost in every exam. Check the master solution.

b) Choose an arbitrary vertex v. Execute DFS starting from v.
Reverse all edges. Execute DFS starting from v.

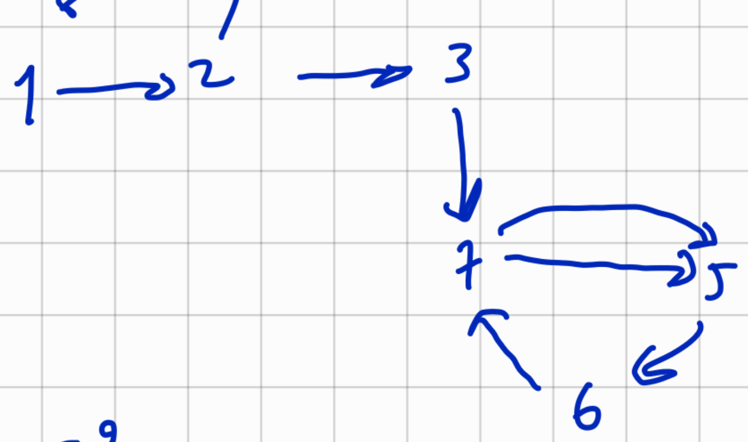
If we visit all vertices in both first and last DFS, it means that every city is reachable from every city.

These two DFSes guarantee that for $\forall x,y \in V$, we can reach x from y going over v. This needs to be proven!

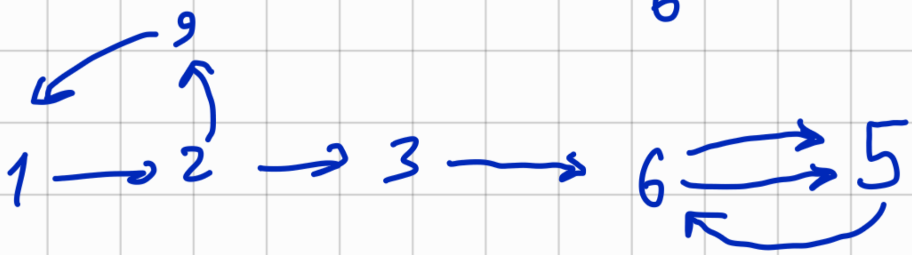
10.4.a



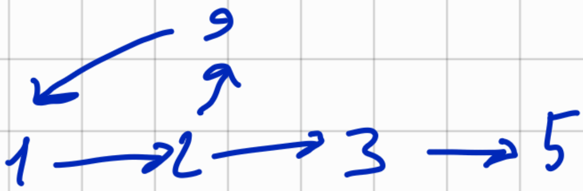
v = 7, w = 8 $\longrightarrow$



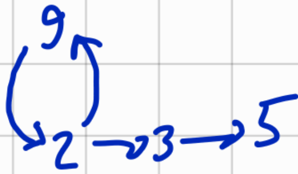
v = 6, w = 7 $\longrightarrow$



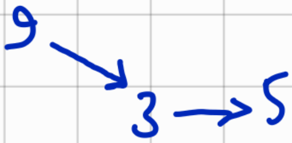
v = 5, w = 6 $\longrightarrow$



v = 9, w = 1 $\longrightarrow$



v = 9, w = 2 $\longrightarrow$

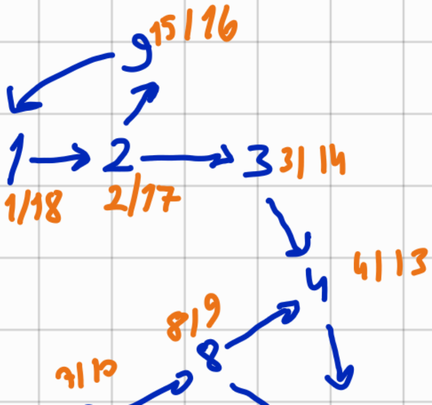


L_9 : 1, 2, 9

L_3 : 3

L_5 : 4, 5, 6, 7, 8

10.4.b



L = [1, 2, 9, 3, 4, 5, 6, 7, 8]

Three Cases if there is a path

- $\exists$ path from 3 to 8.
  1) not sc
  2) 3 appears before 8

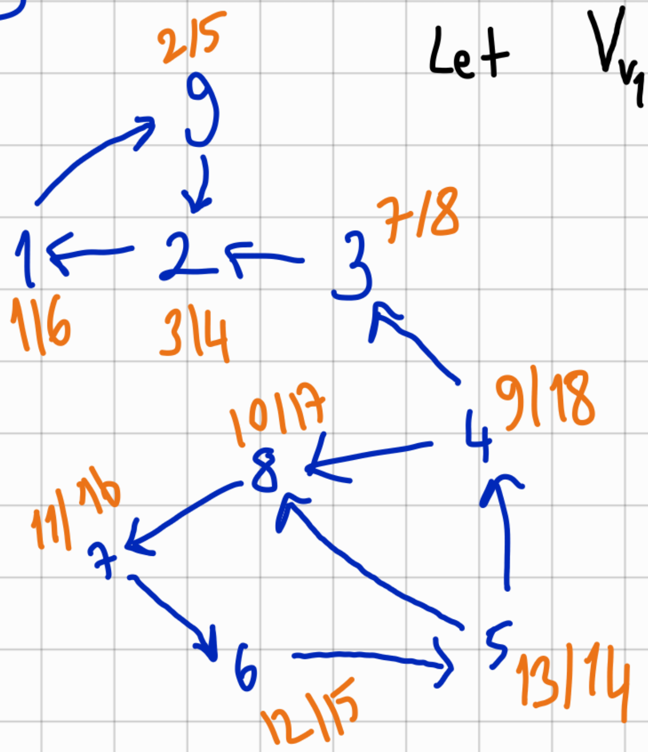
} if not in the same scc, then v must appear before w

- $\exists$ path from 8 to 5
  1) sc
  2) 8 doesn't appear before 5

we have a back-edge here

- $\exists$ path from 5 to 8
  1) sc
  2) 5 appears before 8

10.4.c)



Let V_{v_1} be the scc of v_1 belongs.

For any $w \in V_{v_1}$ $\exists$ path from w to v_1 in G. But then $\exists$ path from v_1 to w in $\vec{G}$. So, $w \in W$. So, $V_{v_1} \subseteq W$.

Let $w \in W$. $\exists$ path from v_1 to w in $\vec{G}$. Then $\exists$-path from w to v_1 in G. Since v_1 is the first element of L, v_1 and w are sc. So, $w \in V_{v_1}$. So, $W \subseteq V_{v_1}$.

10.4.d

Check master solution

10.5

Check Master Solution