Exercise 3.1

(a) (1) True

(2) False

(3) True

(4) True

(b)

(1)

- $(\sin(n)+2)\,n \leq 3n$
- $n \leq (\sin(n)+2)\,n$

(2) $\sum_{i=1}^{n} \sum_{J=1}^{i} J = \Theta(n^3)$

- $\sum_{i=1}^{n} \sum_{J=1}^{i} J \leq \sum_{i=1}^{n} \sum_{J=1}^{n} J \leq \sum_{i=1}^{n} \sum_{J=1}^{n} n = n \cdot n \cdot n = n^3$
- $\sum_{i=1}^{n} \sum_{J=1}^{i} J \geq \sum_{i=\lceil \frac{n}{2} \rceil}^{n} \sum_{J=1}^{i} J \geq \sum_{i=\lceil \frac{n}{2} \rceil}^{n} \sum_{J=1}^{\lceil \frac{n}{2} \rceil} J \geq \sum_{\lceil \frac{n}{2} \rceil}^{n} \frac{\left(\frac{n}{2}\right) \cdot \left(\frac{n}{2}-1\right)}{2}$

$$\geq \frac{n}{2} \cdot \frac{n}{2} \cdot \frac{1}{2} \cdot \left(\frac{n-2}{2}\right) = \frac{n^2}{4} \cdot \frac{(n-2)}{2} \geq \Omega(n^3)$$

(3)

- $\log(n^4+n^3+n^2) \geq \Omega(\log(n^3+n^2+n))$, as log is monotone
- $\log(n^4+n^3+n^2) \leq \log(n^4+n^4+n^4) = \log(3n^4)$

$\log(\ldots) = \log(\ldots) = \log(3n^4)$

$$= \log(3) + \log(n^4) = \log(3) + 4\log(n) \leq 4\left(\log(3) + \log(n)\right) =$$

$$4 \cdot \left(\log(3) + \log(n)\right) \leq 4 \cdot \log(3n) = 4 \cdot \log(n+n+n) \leq$$

$$4 \cdot \log(n + n^2 + n^3) \leq O\left(\log(n + n^2 + n^3)\right)$$

(4)

$$\sum_{i=1}^{n} \sqrt{i} = \Theta\left(n\sqrt{n}\right)$$

- $\sum_{i=1}^{n} \sqrt{i} \leq \sum_{i=1}^{n} \sqrt{n} = n\sqrt{n}$
- $\sum_{i=1}^{n} \sqrt{i} \geq \sum_{i=\lceil \frac{n}{2} \rceil}^{n} \sqrt{i} \geq \sum_{i=\lceil \frac{n}{2} \rceil}^{n} \sqrt{\lceil \frac{n}{2} \rceil} \geq \sum_{i=\lceil \frac{n}{2} \rceil}^{n} \sqrt{\frac{n}{2}}$

$$\geq \frac{n}{2} \cdot \sqrt{\frac{n}{2}} = \frac{n}{2\sqrt{2}} \cdot \sqrt{n} = \frac{1}{2\sqrt{2}} \cdot n\sqrt{n}$$

Exercise 3.2

(a)

```
count = 0
for start_index 0 to n-1 :
    for size 1 to (n - start_index) :
        sum = 0
        string = ""
        for curr_index start_index to (start_index + size - 1) :
            if S[curr_index] == "1"
                sum = sum + 1
            string = string + S[curr_index]
        if sum == k
```

```
    count = count + 1
return count
```

Runtime and correctness analysis should be added

(b)

```
T ← int[n+1] with all elements equal to 0
count ← 0
for i ← 0,...,n-1 do
    if S[i] = "1"
        count ← count + 1
    T[count] ← T[count] + 1
```

(c)

Example : String $S = \underline{0111}\,\underline{010001}$

m

Suffix Prefix

Suffixtable of {0...m} = [0, 1,1,2,0]
Prefixtable of {(m+1)...(n-1)} = [1,4,1,0,0,0,0]

```
SF ← Suffixtable of {0,...m}
PF ← Prefixtable of {(m+1)...(n-1)}
count ← 0
for i ← 0,...,k do :
    if (i ≤ m) && ((k-i) ≤ (n-1) - (m+1))
        count ← SF[i] * PF[k-i]
```

Complexity is $O(k)$, since $k \le n$, we have $O(n)$.

(d) see official solution

Exercise 3.3

(a) $\Theta(n)$

(b) $\sum_{i=1}^{n} \sum_{j=1}^{i^3} 1 = \sum_{i=1}^{n} i^3 = \frac{n^2 (n+1)^2}{4} = \Theta(n^4)$

Exercise 3.4

$f_0 = 0, \quad f_1 = 1, \quad f_{n+1} = f_n + f_{n-1}$

a) B.C.: $f_1 = 1 \geq 0{,}5 = \frac{1}{3} \cdot 1{,}5^1$; $f_2 = 1 \geq 0{,}75 = \frac{1}{3} \cdot (1{,}5)^2$

I.H.: There exists $m \in \mathbb{Z}^+$ such that $f_k \geq \frac{1}{3} \cdot 1{,}5^k$ for all $k \in \mathbb{Z}^+$ satisfying $1 \leq k \leq m$

Induction Step: $f_{k+1} = f_k + f_{k-1} \overset{\text{I.H.}}{\geq} \frac{1}{3} \left(1{,}5^k + 1{,}5^{k-1} \right)$

$= \frac{1}{3}\left(\left(\frac{3}{2}\right)^k + \left(\frac{3}{2}\right)^{k-1}\right) = \frac{1}{3} \cdot \left(\frac{3}{2}\right)^k + \frac{1}{3}\left(\frac{3}{2}\right)^{k-1} = \frac{1}{3} \cdot \left(\frac{3}{2}\right)^k + \frac{2}{9} \cdot \left(\frac{3}{2}\right)^k$

$= \left(\frac{3}{9} + \frac{2}{9}\right) \cdot 1{,}5^k = \frac{1}{2} \cdot (1{,}5^k) = \frac{1}{3} \cdot \frac{3}{2} \cdot 1{,}5^k = \frac{1}{3}\left(1{,}5^{(k+1)}\right)$

(b) look at official solution

(c)

```
F ← int[]
F[0] ← 0
F[1] ← 1
```

```
i ← 1
while ( F[i] ≤ k) do:
    F[i+1] ← F[i] + F[i-1]
    i ← i+1
return F[i-1]
```

We are inside the while loop when $k \geq f_i$ holds. Hence by $k \geq f_i \geq \frac{1}{3} \cdot (1,5)^i$ we get $k \geq \frac{1}{3} \cdot (1,5)^i$.

$$\Rightarrow \quad 3k \geq (1,5)^i \quad \Rightarrow \quad \log_{1,5}(3k) \geq i$$

$$\Rightarrow \quad i \leq \log_{1,5}(3k) \leq O(\log(k))$$

$$\Rightarrow \quad i \leq O(\log(k))$$

Exercise 3.5

Check official solutions.