Algorithms and Probability

Exercise Session 13



https://n.ethz.ch/~ahmala/anw

Categories

- easy 🖈
- medium $\bigstar \bigstar$
- hard $\bigstar \bigstar \bigstar$
- challenging $\bigstar \bigstar \bigstar \bigstar$

perception of the difficulties may not be exactly spot on

Let G = (V, E).

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How can you make G Eulerian adding only 1 vertex and O(|V|) edges?

Let G be a graph. The edge-graph of G that we denote by G' is a graph whose vertices are the edges of G.

Two vertices of G' are connected by an edge if the corresponding edges in G share a vertex.

Show that if G has Euler cycle, then G ' has a Hamilton cycle and also an Euler cycle.

If $G = A \cup B$ is bipartite and the degree of every vertex in A is $\ge k$ and the degree of every vertex in B is $\le k$, **then there exists a matching of size** |A|.

Let G be a graph on n vertices v_1, \ldots, v_n . Assume that for every i, the vertex v_i has at most 5 neighbors from among v_1, \ldots, v_{i-1} (but may have more neighbors in G).

Find an upper bound for the chromatic number of G.

Show that if G does not contain a cycle of even length, then no two cycles in G may share an edge.

It is known that T is a tree with 10 vertices of degree 10 and all other vertices are leaves. How many vertices does T have?

Rachel gives one-on-one snowboarding lessons. For the first six days of each week, Rachel's student chooses one of her six available snowboards to use, at random. Then, on the last day of each week, she sharpens every snowboard that was used at least once that week.

What is the expected number of snowboards she needs to sharpen at the end of each week?

7.ii

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What is the expected number of snowboards she needs to sharpen at the end of each week? **Use a different indicator random variable than 7.i**

17.15 - 18.00 in ML D28