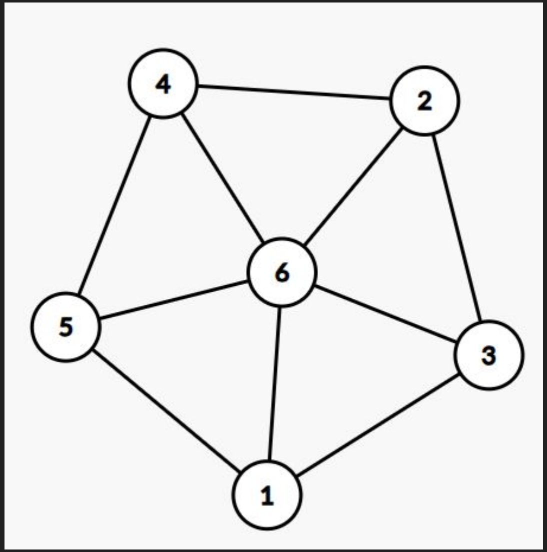
Algorithms and Probability

Exercise Session 2

https://n.ethz.ch/~ahmala/anw

Dirac's theorem: every graph $G = (V, E)$ with $|V| \geq 3$ and minimal degree $\geq |V|/2$ contains a Hamiltonian cycle.



PROVE

For any two arbitrary nodes x and y in V, where $x \neq y$, there exists an x-y path of length at most two.

(Assume $|V| \geq 3$ and minimal degree $\geq |V|/2$)

What is the sum of all integers from 1 to 100 that are multiples of 2 or 3?

What is the sum of all integers from 1 to 100 that are multiples of 2 or 3 or 5?

Inclusion–Exclusion Principle

Satz 1.35. *(Siebformel, Prinzip der Inklusion/Exklusion)* Für endliche Mengen $A_1, \ldots, A_n$ $(n \geq 2)$ gilt:

$$\begin{aligned}\left|\bigcup_{i=1}^{n} A_i\right| &= \sum_{l=1}^{n} (-1)^{l+1} \sum_{1 \leq i_1 < \cdots < i_l \leq n} |A_{i_1} \cap \cdots \cap A_{i_l}| \\ &= \sum_{i=1}^{n} |A_i| - \sum_{1 \leq i_1 < i_2 \leq n} |A_{i_1} \cap A_{i_2}| + \sum_{1 \leq i_1 < i_2 < i_3 \leq n} |A_{i_1} \cap A_{i_2} \cap A_{i_3}| \\ &\quad - \cdots + (-1)^{n+1} \cdot |A_1 \cap \cdots \cap A_n|.\end{aligned}$$

In-Class Exercise

https://n.ethz.ch/~ahmala/anw/material/In_Class_Exercise.pdf

Matchings

A matching M is maximal (inklusions-maximal) if and only if no edge in $E \setminus M$ can be added to the matching without violating the definition of a matching.

A matching M is maximum (kardinalitäts-maximal) if and only if there is no matching in the graph that contains more edges

A matching M is called perfect (perfekt) if and only if $|M| = |V| / 2$ i. e. every vertex of the graph is incident to exactly one edge of the matching.

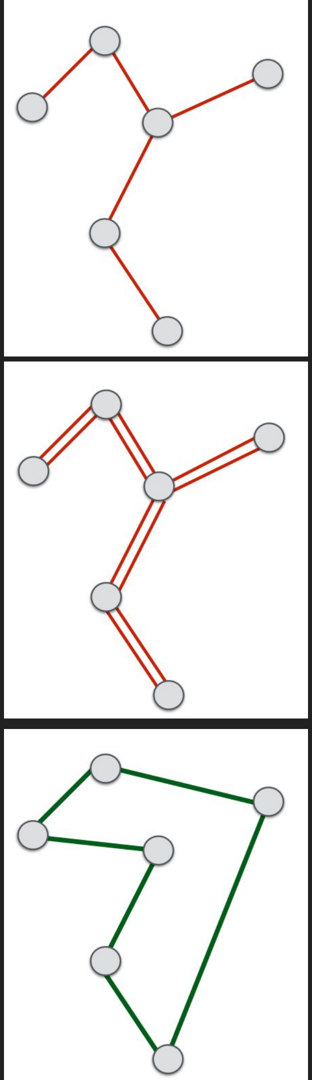
2-Approximation Algorithm for the Metric TSP

1. Compute an MST in the graph in O(n^2).

2. Create a multi-graph by doubling every edge in the MST.

3. Find an Eulerian Tour in that multi-graph

4. Translate this tour in the multi-graph to a cycle in the original graph: Use the same sequence of vertices but whenever a vertex is visited for the second time, instead go directly to the next unvisited vertex in the tour.



2-Approximation Algorithm for the Metric TSP

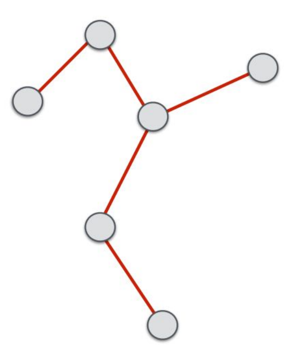
1. Compute an MST in the graph in O(n^2). $\ell(T) \leq \mathrm{opt}(K_n, \ell)$

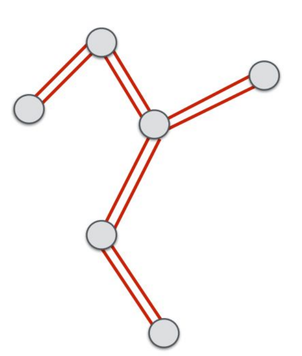
2. Create a multi-graph by doubling every edge in the MST. $2\ell(T) \leq 2\mathrm{opt}(K_n, \ell)$

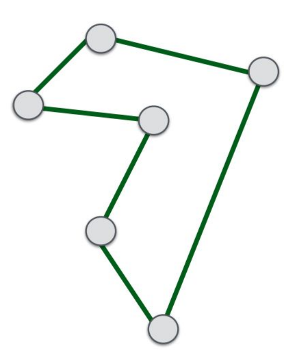
3. Find an Eulerian Tour W in that multi-graph $\ell(W) = 2\ell(T) \leq 2\mathrm{opt}(K_n, \ell)$

4. Translate this tour in the multi-graph to a cycle C in the original graph: Use the same sequence of vertices but whenever a vertex is visited for the second time, instead go directly to the next unvisited vertex in the tour.

$\ell(C) \leq \ell(W) = 2\ell(T) \leq 2\mathrm{opt}(K_n, \ell)$





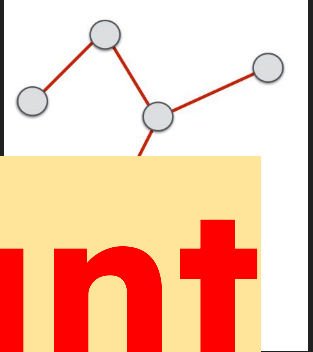


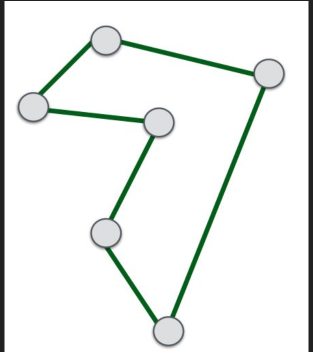
2-Approximation Algorithm for the Metric TSP

What if we want 1.5-Approximation?

4. Translate this tour in the mu… e C in the original graph: Use the sam… tices but whenever a vertex is visite… me, instead go directly to the next … the tour.

$$\ell(C) \leq \ell(W) = 2\ell(T) \leq 2\mathrm{opt}(K_n, \ell)$$





1.5-Approximation Algorithm for the Metric TSP

Satz 1.51. Für das METRISCHE TRAVELLING SALESMAN PROBLEM gibt es einen 3/2-Approximationsalgorithmus mit Laufzeit $O(n^3)$.

Algorithmen und Wahrscheinlichkeit

Info Material Links **Challenges**

In some weeks, I will publish challenge tasks.
I guarantee that there exist elegant and short correct solutions for these tasks.
The first five correct solutions emailed to me will win a chocolate.
No super formal proofs required, just a clear explanation, which makes sense.
You must probably use an algorithm from the lecture as a subroutine, possibly with a little twist, to solve the problem.
First challenge task is going to be published in the evening of February 29th and you will have two weeks of time to send me an e-mail.