Electrical Conductivity of Pb and PbIn (7mol%)

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In this experiment we measure the resistance of a Pb and PbIn (7mol%) sample and determine the resistivity as a function of temperature $\rho(T)$. After carrying out the experiment we have for ρ_{Pb}(273.0K)= $(2.1 \pm 0.1) \times 10^{-7}\Omega$m and $\frac{d\rho}{dT}|_{T=273.0K_{Pb,PbIn}} = (8.0 \pm 0.2), (7.30 \pm 0.01) \times 10^{-10} \frac{\Omega m}{K}$ respectively. A critical temperature was for neither of the two samples reached. We conclude that the experimentally obtained values do not align with the theoretical model and the literature value consequently does not lie within the uncertainty of our results. Therefore the experiment can be improved by conducting an extensive error analysis on the experiment setup as well as improvement of the measuring techniques.

To eliminate any dependences of the resistance on any sample related quantity such as the shape or size, one introduces a quantity characteristic only of the material. For our purposes we will define the resistivity

$$\rho = R\frac{A}{L} \tag{1}$$

where R is the resistance, A the cross-sectional area and L the length of the sample. Furthermore the conductivity can then be written as

$$\sigma = \frac{1}{\rho} = \frac{n_e e^2 \tau}{m_e} \tag{2}$$

which is known as Drude's formula. Here n_e is the number of electrons per unit volume, e the elementary charge, τ the relaxation time and m_e the mass of the electron. The relaxation time τ is an average time between two scattering processes, namely collisions with sample defects and thermal excitations i.e. phonons. Therefore the phonon scattering time τ_{ph} as well as the collision of electrons with static defects τ_e which is essentially elastic are introduced as new relaxation times. The total relaxation time is then given by

$$\frac{1}{\tau} = \frac{1}{\tau_e} + \frac{1}{\tau_{ph}} \tag{3}$$

which leads to the Matthiessen's rule for the total resistivity

$$\rho = \rho_e + \rho_{ph} \tag{4}$$

which serves as a good approximation for the resistivity of metals when the only time dependence is contained in ρ_{ph} which requires that the defects remain static. This implies that $\frac{d\rho}{dT}$ should be independent of the impurity concentration if it is not too high.

Methods

The purpose of this experiment is to measure the electrical resistence of two different samples, once for pure lead and once for a lead-indium alloy with a concentration of 7 mol% as a function of temperature down to a superconducting critical temperature T_c, for all temperatures above T_c it is expected that the resistivity is linear. The resistence then is used to calculate the temperature dependent resistivity ρ for the temperature $T_0 = 273K$ as well as $\frac{d\rho}{dT}$ and $\Delta\rho$ $(= \rho(273K) - \rho(T_{min}))$, where T_{min} is the lowest possible temperature that was reached with the given experimental setup) in order to compare the experimentally obtained values to literature values and therefore verify the Drude model and Matthiessen's rule for the total resistivity. The experimental setup consists of the samples Pb and PbIn (7mol%) which are wires wound up on a plastic core.

Furthermore a continuous flow cryostat is used to cool the samples down to the desired temperatures between 4.2K and 300K in order to perform the measurements. To ensure a constant temperature for each measuring point is retained and therefore reducing possible errors, the cryostat is equipped with a temperature controller which allows for an adjustment of the flow in the cryostat and power dissipation of the heater.

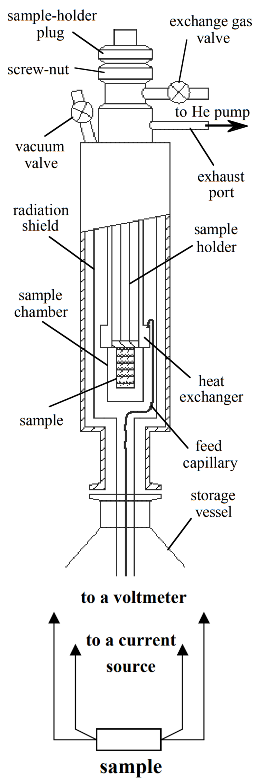

Fig. 1 | Sketch of the continuous flow cryostat used in the experiment after inserting the feed capillary in the storage vessel to allow for temperature exchange with the sample located in the sample chamber. Additionally the wiring for a 4-point measurement is sketched below the continuous flow

cryostat. The experimental setup also includes a heater (not in the sketch) which allows for temperature regulation in order to obtain constant temperatures to carry out accurate measurements at specific temperatures. The sketches were taken from the corresponding lab report manual [1].

Finally the sample mounting includes a 4-point measurement setup in order to eliminate the influence of the contact resistances and therefore measuring the voltage and current through the sample to calculate the resistance and therefore the resistivity using Equation 1. [1] A sketch of the 4-point measurement setup is shown in Figure 1.

To perform the measurement to obtain a data point, we did the following:

1. Open the valve of the He-pump to start the cooling process down to a specific temperature, where we always overshot the desired temperature by a little bit.

2. Close the valve of the He-pump to stop the cooling process.

3. Heat the sample to the desired temperature using the heater (since overshooting).

4. Wait for the temperature to stabilize.

5. Measure the resistance using the 4-point measurement setup.

6. Repeat the process for different temperatures and different samples.

Data Analysis and Results

If not mentioned otherwise, the uncertainties of the values are calculated using Gaussian error propagation. (see Appendix A.1).

Besides the current, the length L and diameter d of the samples as well as the purity content were indicated in the laboratory itself, therefore no further error analysis or measurements have been performed on them. The values are shown in the following table.

Table 1 | Dimensions of the used samples and applied current.

	Pb 99.99%	PbIn (7mol%)
L (m) $\pm$ 0.5 (mm)	0.995	2.687
d $\pm$ 0.001 (mm)	1.00	0.98
I $\pm$ 0.5 (mA)	100	50.0

Given the values from Table 1 and our measurements for the resistance at different temperatures, we can then calculate the resistivity and since a linear behavior is expected for temperatures higher than the critical temperature T_c, we can perform a linear fit.

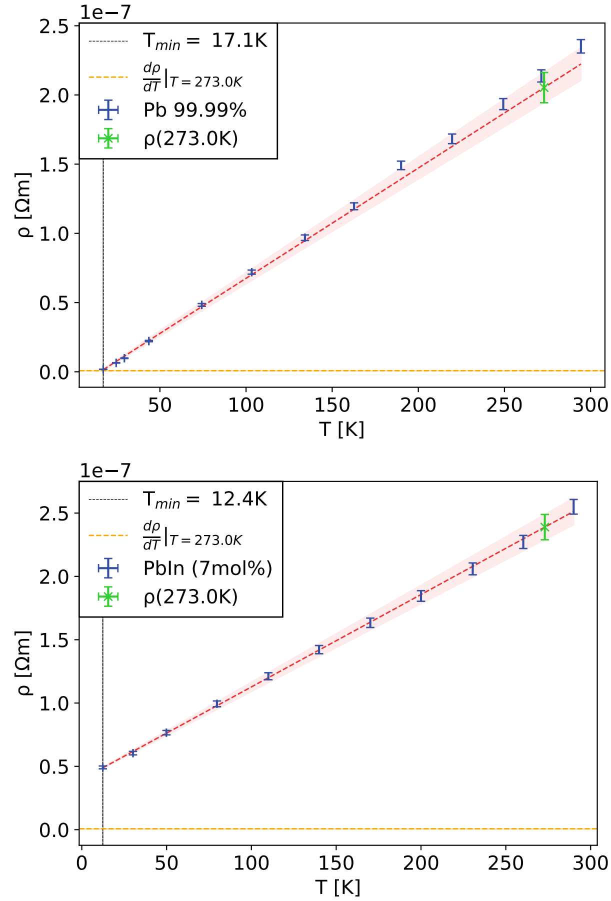

Fig. 2 | Measured resistivity of the samples Pb and PbIn (7mol%) as a function of temperature down to T_{min}. The linear fit is shown in red with the error of the fit indicated by the shaded area. Additionally marked in green is the value for the resistivity at 273.0K which was not measured but calculated using the linear fit performed on the measured dataset for each sample.

Using the results of the fits, we can then calculate the resistivity at $T_0 = 273$K and $\frac{d\rho}{dT}$ as well as $\Delta\rho$ for both samples and compare it to literature values.

Table 2 | Measured and calculated values used for the determination of the resistivity using Equation (1). Since different minimal temperatures T_{min} were reached for each sample and measurement, we define $\Delta\rho := \rho(273.0K) - \rho(T_{min})$. Furthermore the literature values for the critical temperature T_c of lead and lead-indium alloys for concentrations of (5mol%) and (10mol%) is listed.

	Pb 99.99%	PbIn (7mol%)
$T_{min} \pm 0.05$ (K)	17.1	12.4
$\rho(273.0K)$ $(10^{-7}\Omega m)$	2.1 ± 0.1	2.4 ± 0.1
$\rho(T_{min})$ $(10^{-7}\Omega m)$	0.017 ± 0.001	0.49 ± 0.01
$\frac{d\rho}{dT}\|_{T=273.0K}$ $(10^{-10}\frac{\Omega m}{K})$	8.0 ± 0.2	7.30 ± 0.01
$\Delta\rho$ $(10^{-7}\Omega m)$	2.0 ± 0.1	1.9 ± 0.1

Literature Values for T_c [2][3]
$T_{c,Pb} = 7.19 \pm 0.02$K
$T_{c,PbIn(5mol\%)} = 7.19 \pm 0.02$K
$T_{c,PbIn(10mol\%)} = 7.05 \pm 0.02$K

Discussion of Results

As the values in Table 2 indicate, the critical temperature T_c was for neither of the two samples reached using the given experimental setup. Since for both samples we have $T_c < T_{min}$ it is not expected and we were not able to investigate any superconducting behavior. This might be due to the fact that the vacuum created in the cryostat might not have been sufficient nor stable enough to reach lower temperatures, which might be a consequence of the given experimental setup i.e. the vacuum pump or shielding of the sample chamber.

Furthermore the literature value for the resistivity of lead at 273K is $\rho(273K) = (1.92 \pm 0.005) \times 10^{-7}\Omega$m [4]. This value does not lie within the error of our experimentally obtained value for the lead sample and therefore does not align with the predictions of the theoretical model. A possible reasoning for this observed discrepancy might be the fact that this particular value was calculated using a linear fit gained from only a few measurements instead of aiming to perform one single measurement at 273K or many more datapoints. This results in a error of the true value acquired through the single measurements at other temperatures if the sample was not in thermal equilibrium such that the resistance fluctuated too much. Additionally since no error analysis on the purity content or possible oxidation of the sample was performed this might also contribute to a lower error than effectively present.

Regarding $\frac{d\rho}{dT}|_{T=273.0K}$ which is using Mathiessen's rule (Equation (4)) expected to be the same value for both samples if the impurity content is not too high. Our measurements indicate that it might be that either the impurity content is too high for one or both of the samples or that the above mentioned lack of datapoints leads to a non expected behavior.

Finally the values for $\Delta\rho$ coincide for the two samples, also for PbIn when using 17.1K as the minimal temperature one acquires the same value including the error as for 12.4K.

Conclusion

In this experiment we determine the temperature dependant resistivity for two samples, pure lead and a lead-indium alloy with a concentration of 7 mol%. The literature values and theoretical predictions do not lie within the uncertainty of our experimental results.

We expect this is the consequence of systematic errors included in the experiment setup and acquired during the measurement process as well as only performing the measurement once which also comes with random error and therefore falsifies our final results.

Therefore there are many ways one might improve this experiment. For example performing an extensive error analysis on the purity of the samples as well as ensuring that a proper vacuum is present at all times during the experiment. Furthermore an attempt at a stable thermal equilibrium such that the resistance fluctuations are minimized. To conclude, error analysis on the experiment setup to reduce systematic errors as well as performing many measurements will lead to more accurate results.

Acknowledgments

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A Appendix

A.1 Calculation of Uncertainties Using Gaussian Error Propagation

For uncorrelated variables $x_1, x_2, ..., x_n$ with uncertainties $\sigma_{x_1}, \sigma_{x_2}, ..., \sigma_{x_n}$ and a function $f(x_1, x_2, ..., x_n)$, the uncertainty of f is given by

$$\sigma_f = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2} \tag{5}$$

A.2 Possible Security Risks

Operating a continuous-flow cryostat with liquid helium poses several security risks. Liquid helium is extremely cold, and direct contact can cause severe frostbite or cold burns. Additionally, helium gas can displace oxygen in poorly ventilated areas. Improper venting or blockages in the system can lead to dangerous pressure build-up, potentially resulting in explosions or equipment failure. Helium leaks may also damage equipment or displace air, leading to safety hazards. Proper training, ventilation, pressure relief mechanisms, and regular equipment maintenance are essential for mitigating these hazards.