ETH zürich

Informatik

Übungsstunde Woche 5

Repetition

1. Perform the following steps:
   1.1 Convert the integer numbers $a = 4$ and $b = 7$ into their binary representation.
   1.2 Add the binary representations.
   1.3 Convert the result into decimal.
2. Evaluate the following expressions:
   2.1 `5 < 4 < 1`
   2.2 `true > false`

Repetition

Solutions

1.
   - 1.1 $4_{10} = 100_2$ and $7_{10} = 111_2$
   - 1.2 $100_2 + 111_2 = 1011_2$
   - 1.3 $1011_2 = 11_{10}$
2.
   - 2.1

```
5 < 4 < 1
(5 < 4) < 1
false < 1
0 < 1
true
```

   - 2.2

```
true > false
true > 0
1 > 0
true
```

Kommazahlen konvertieren

Compute the binary expansions of the following decimal numbers.

1. 0.25
2. 11.1

Kommazahlen konvertieren

Solutions:

1. $0.25_{10} = 0.01_2$

x	b_i	$x - b_i$	$2 \cdot (x - b_i)$
0.25	0	0.25	0.5
0.5	0	0.5	1
1	1	0	0

2. $11.1_{10} = 1011.0\overline{0011}_2$

   11.1_{10} is first split into $11_{10} + 0.1_{10}$. The binary representation of 0.1_{10} is derived as follows:

x	b_i	$x - b_i$	$2 \cdot (x - b_i)$
0.1	0	0.1	0.2
0.2	0	0.2	0.4
0.4	0	0.4	0.8
0.8	0	0.8	1.6
1.6	1	0.6	1.2
1.2	1	0.2	0.4
0.4	0	0.4	0.8

Fliesskommazahlen

State the following numbers in $F^*(2, 4, -2, 2)$:

1. the largest number;
2. the smallest number;
3. the smallest non-negative number.

Compute how many numbers are in the set $F^*(2, 4, -2, 2)$.

Fliesskommazahlen

Solutions:

1. The largest number is $1.111 \cdot 2^2$ which is 7.5 in decimal.
2. The smallest number is $-1.111 \cdot 2^2$ which is -7.5 in decimal.
3. The smallest non-negative number is $1.000 \cdot 2^{-2}$ which is 0.25 in decimal.

The set has 80 numbers in it. This can be seen as follows. For a fixed exponent there are three digits we can vary freely, and for each number also the negative number is in the set, thus resulting in $2 \cdot 2^3 = 16$ numbers per exponent. On the other hand, there are 5 possible exponents, thus resulting in $5 \cdot 16 = 80$ numbers. Notice that in normalized number systems we cannot "count some numbers twice" as we've seen in the lecture that the representation of a number is unique.

Fliesskommazahlen (Prüfungsaufgabe)

Beantworten Sie die folgenden Fragen zum normalisierten Flieskommasystem F*.

Answer the following questions regarding the normalized floating point system F*.

$F^*(\beta = 2, p = 3, e_{\min} = -1, e_{\max} = 4)$

Erinnerung: Bei F* schliesst die Präzision (Stelligkeit) das führende Bit mit ein.

Reminder: For F*, the precision (number of digits) includes the leading bit.

Wahr	Falsch	
○	○	3.25 ist im Fliesskommazahlensystem F* exakt repräsentierbar. 3.25 can be represented exactly in the floating point system F*.
○	○	Es gibt keine Zahl $Z \in F^*$ für die gilt 0.0625 < Z < 0.25. There is no number $Z \in F^*$ such that 0.0625 < Z < 0.25.
○	○	1.25 ist im Fliesskommasystem F* exakt repräsentierbar: 1.25 can be represented exactly in the floating point system F*

Fliesskommazahlen (Prüfungsaufgabe)

Beantworten Sie die folgenden Fragen zum normalisierten Flieskommasystem F*.

Answer the following questions regarding the normalized floating point system F*.

$F^*(\beta = 2, p = 3, e_{\min} = -1, e_{\max} = 4)$

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Wahr	Falsch		
✗	☑	3.25 ist im Fliesskommazahlensystem F* exakt repräsentierbar. 3.25 can be represented exactly in the floating point system F*.	✔
☑	✗	Es gibt keine Zahl $Z \in F^*$ für die gilt 0.0625 < Z < 0.25. There is no number $Z \in F^*$ such that 0.0625 < Z < 0.25.	✔
☑	✗	1.25 ist im Fliesskommasystem F* exakt repräsentierbar: 1.25 can be represented exactly in the floating point system F*	✔