Exercise 8 - Turbocompressors

Turbocompressor systems are well described with the coupling of a compressor and a turbine, as depicted in Figure 1. The processes are modeled using themodynamic Brayton cycles. The cycles are described by

- $1 \to 2$: isentropic compression.
- $2 \to 3$: isobar fuel combustion.
- $3 \to 4$: isentropic expansion.
- $4 \to 1$: isobar heat rejection into atmosphere.

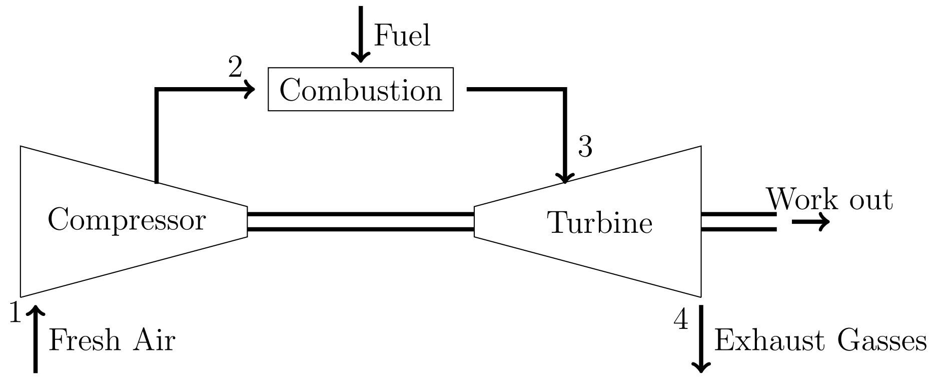

Figure 1: Schematic turbine/compressor system

8.1 Gas Turbines

We model gas turbines as static elements with input variables: pressure p_3 before and pressure p_4 after the turbine, the temperature ϑ_3 at turbine intake and the turbine speed ω_t. Furthermore, if the turbine has a variable nozzle geometry, an additional control input u_{vnt} should be taken into account. Otherwise, the radius of the turbine r_t and the inlet area A_{vnt} are assumed to be constant. Outputs of the system are the temperature after the turbine $\vartheta_t = \vartheta_4$, the gas mass flow $\overset{*}{m}(t)$ and the produced shaft torque T_t. A sketch of the causality diagram of such a system is depicted in Figure 2.

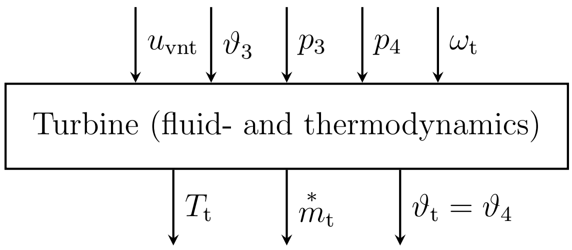

Figure 2: Causality diagram of a gas turbine.

In order to model the system, we want to be able to find a relationship between the outputs and the inputs. The turbine flows are assumed to be similar to the ones of an

isenthalpic valve (see Figure 3). In particular, experimental outcomes are resumed in maps.

Outputs Derivation

Considering the energy conservation for an open system

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \dot{H}_{\text{in}} - \dot{H}_{\text{out}} - \dot{W}_{\text{t}} + \dot{Q}$$

, we consider two simplifications

- The turbine does not store energy over time → $\frac{\mathrm{d}E}{\mathrm{d}t} = 0$.
- The turbine is assumed to be **adiabatic** (no heat transfer)→ $\dot{Q} = 0$.

With the simplifications the power produced by the turbine reads

$$P_{\text{t}} = \dot{W}_{\text{t}} = \dot{H}_{\text{in}} - \dot{H}_{\text{out}} = \dot{m}_{\text{t}} \cdot c_{\text{p}} \cdot (\vartheta_3 - \vartheta_4).$$

Using the **isentropic relation** for a turbine

$$\frac{\vartheta_3}{\vartheta_{4,\text{is}}} = \left(\frac{p_3}{p_4}\right)^{\frac{\kappa-1}{\kappa}}.$$

and the **isentropic efficiency** of a turbine

$$\eta_{\text{t}} = \frac{\vartheta_3 - \vartheta_4}{\vartheta_3 - \vartheta_{4,\text{is}}},$$

one can compute, using variables referenced to a nominal case $\vartheta_{3,\text{ref}}$, $p_{3,\text{ref}}$, the normalized mass flow

$$\overset{*}{\mu}_{\text{t}} = \frac{\overset{*}{m}_{\text{t}} \cdot \sqrt{\frac{\vartheta_3}{\vartheta_{3,\text{ref}}}}}{\frac{p_3}{p_{3,\text{ref}}}}.$$

, the produced shaft torque

$$T_{\text{t}} = \frac{\eta_{\text{t}} \cdot \overset{*}{m}_{\text{t}} \cdot c_{\text{p}} \cdot \vartheta_3}{\omega_{\text{t}}} \cdot \left(1 - \Pi_{\text{t}}^{\frac{1-\kappa}{\kappa}}\right)$$

, and, assuming perfect gases, the turbine temperature

$$\vartheta_4 = \vartheta_3 \cdot \left(1 - \eta_{\text{t}} \cdot \left(1 - \Pi_{\text{t}}^{\frac{(1-\kappa)}{\kappa}}\right)\right).$$

The pressure ratio reads

$$\Pi_{\text{t}} = \frac{p_3}{p_4},$$

which is > 1 for the gas flow through the turbine. The speed of the gas resulting from an isentropic expansion reads

$$c_{\text{us}} = \sqrt{2 \cdot c_{\text{p}} \cdot \vartheta_3 \cdot \left(1 - \Pi_{\text{t}}^{\frac{(1-\kappa)}{\kappa}}\right)},$$

where c_{p} is the specific heat at constant pressure and κ is the ratio $\kappa = \frac{c_{\text{p}}}{c_{\text{v}}}$ of the gases flowing through the turbine. The turbine blade-tip speed ratio reads

$$c_{\text{u}} = \frac{r_{\text{t}} \cdot \omega_{\text{t}}}{c_{\text{us}}}.$$

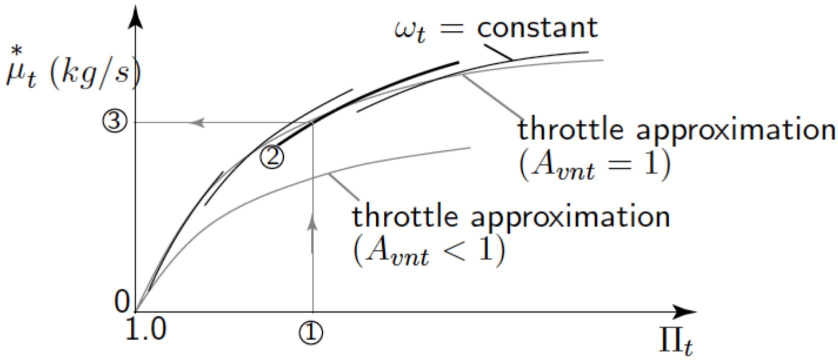

Figure 3: Gas turbine massflow behavior.

8.2 Compressors

We model compressors as static elements with input variables: pressure p_1 before and pressure p_2 after the compressor, the temperature ϑ_1 of the inflowing gases and the compressor speed ω_c. Outputs of the system are the temperature after the compressor $\vartheta_c = \vartheta_2$, the compressor mass flow $\overset{*}{m}_c(t)$ and the torque absorbed by the compressor T_c. A sketch of the causality diagram of such a system is depicted in Figure 4.

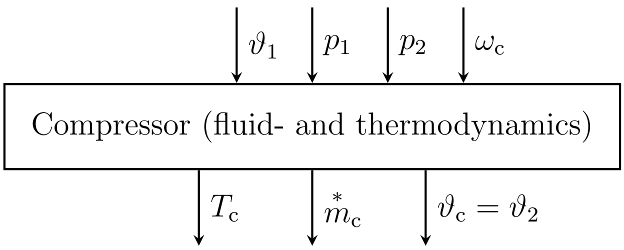

Figure 4: Causality diagram of a gas compressor.

In order to estimate the compressor massflow and the compressor efficiency, we need experiments and measurements. We use the known pressure ratio

$$\Pi_c = \frac{p_2}{p_1},$$

and the known compressor speed ω_c, one can interpolate the massflow and the efficiency η_c from the mapped measurements. An example of such a map is depicted in Figure 5.

Outputs Derivation

Considering the energy conservation for an open system

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \dot{H}_{\mathrm{in}} - \dot{H}_{\mathrm{out}} - \dot{W}_c + \dot{Q}$$

, we consider two simplifications

- The compressor does not store energy over time $\rightarrow \frac{\mathrm{d}E}{\mathrm{d}t} = 0$.

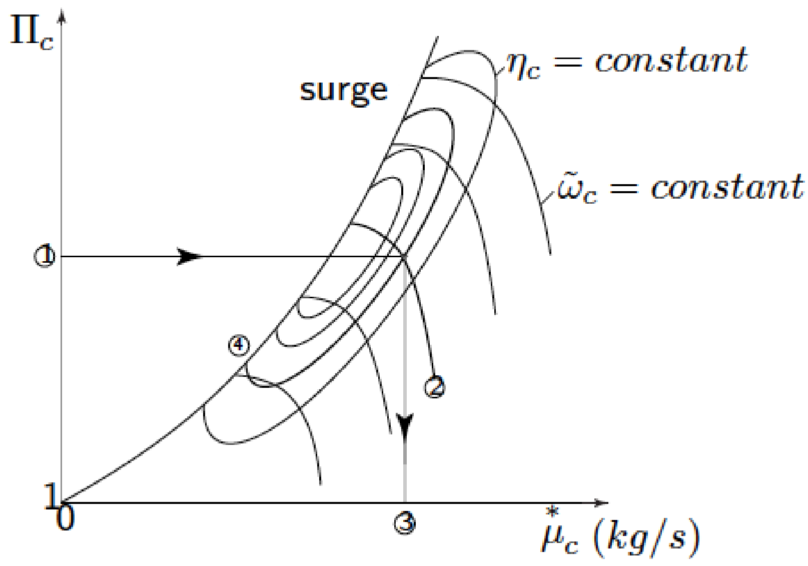

Figure 5: Compressor map for mass flow and efficiency.

- The compressor is assumed to be **adiabatic** (no heat transfer)→ $\dot{Q} = 0$.

With the simplifications the power produced by the turbine reads

$$P_\text{c} = \dot{W}_\text{c} = \dot{H}_\text{in} - \dot{H}_\text{out} = \dot{m}_\text{c} \cdot c_\text{p} \cdot (\vartheta_2 - \vartheta_1).$$

Using the **isentropic relation** for a compressor

$$\frac{\vartheta_{2,\text{is}}}{\vartheta_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}.$$

and the **isentropic efficiency** of a compressor

$$\eta_\text{t} = \frac{\vartheta_{2,\text{is}} - \vartheta_1}{\vartheta_2 - \vartheta_1},$$

one can compute, using variables referenced to a nominal case $\vartheta_{1,\text{ref}}$, $p_{1,\text{ref}}$, the normalized mass flow

$$\overset{*}{\mu}_\text{c} = \frac{\overset{*}{m}_\text{c} \cdot \sqrt{\frac{\vartheta_1}{\vartheta_{1,\text{ref}}}}}{\frac{p_1}{p_{1,\text{ref}}}},$$

the torque absorbed by the compressor

$$T_\text{c} = \frac{\overset{*}{m}_\text{c} \cdot c_\text{p} \cdot \vartheta_1}{\eta_\text{c} \cdot \omega_\text{c}} \cdot \left(\Pi_\text{c}^{\frac{\kappa-1}{\kappa}} - 1\right),$$

where c_p is the specific heat at constant pressure and κ is the ratio $\kappa = \frac{c_\text{p}}{c_\text{v}}$ of the gases flowing through the compressor. Assuming perfect gases, the compressor outlet temperature reads

$$\vartheta_2 = \vartheta_1 \cdot \left[1 + \frac{1}{\eta_\text{c}} \cdot \left(\Pi_\text{c}^{\frac{\kappa-1}{\kappa}} - 1\right)\right].$$

8.3 Example

Inspired by Mr. Balerna's lecture, you aim to reduce the consumptions and increase the power of the car of SpaghETH. Among others, the power unit of your vehicles should include an turbocharger. However, this makes the operation of your vehicle more challenging. In order not to overcharge your drivers with additional tasks, you decide to design a control system to take over the task of managing the energy in the system.
The system consists of a internal combustion engine, a compressor, and a turbine, mechanically connected through a shaft of inertia Θ. Air flows from the ambient to the intake manifold (volume V_{IM} and constant temperature ϑ_{IM}) through the compressor and the intercooler (ensuring a constant temperature of the manifold). Then, air flows to the engine of a throttle of opening surface $A(t)$ to the engine. After the combustion, the warm mixture flows to the exhaust manifold (volume V_{EM} and isolated) through the turbine back to the ambient. The following assumptions are made:

- The mass-flow and efficiency of the compressor and the turbine are to find in maps:

$$\begin{aligned} \dot{m}_{\mathrm{c}} &= \mathcal{M}_{c,1}(\Pi_{\mathrm{c}}, \omega_{\mathrm{c}}), & \dot{m}_{\mathrm{t}} &= \mathcal{M}_{t,1}(\Pi_{\mathrm{t}}, \omega_{\mathrm{t}}), \\ \eta_{\mathrm{c}} &= \mathcal{M}_{c,2}(\Pi_{\mathrm{c}}, \omega_{\mathrm{c}}), & \eta_{\mathrm{t}} &= \mathcal{M}_{t,2}(\Pi_{\mathrm{t}}, \omega_{\mathrm{t}}). \end{aligned}$$

- The temperature after the engine is a known function of the fuel injected in the engine. That is, $\vartheta_{\mathrm{e}} = \Phi(\dot{m}_{\mathrm{f}})$.
- The throttle has discharge coefficient c_{d} and, as the pressure before the engine a known function of $p_{\mathrm{IM}}(t)$, the function Ψ is known and reads $\Psi(p_1(t))$.
- Assume that air does not accumulate before the engine, i.e., $\dot{m}_{\mathrm{cyl}} = \dot{m}_{\mathrm{th}}$. Moreover, the fuel burns stoichiometrically, i.e., $\dot{m}_{\mathrm{f}} = \dot{m}_{\mathrm{cyl}}/\sigma$.
- Both air the and air-fuel mixture have constant c_p, c_v, and κ.
- The engine power is given by $P_{\mathrm{e}} = K_1 \cdot \dot{m}_{\mathrm{f}} + K_2 \cdot (p_{\mathrm{IM}} - p_{\mathrm{EM}})$.

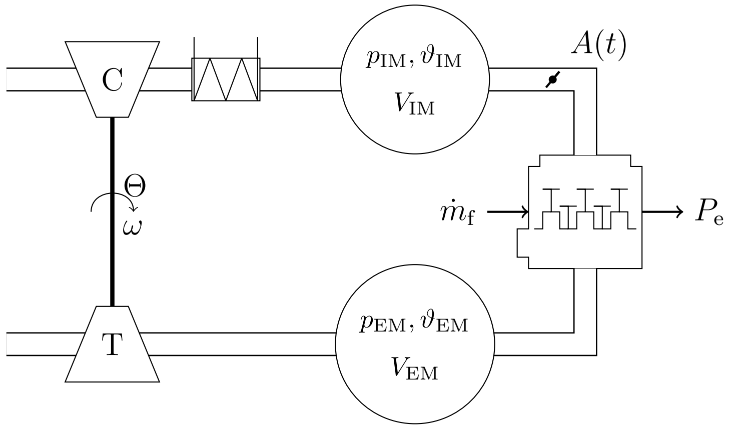

Figure 6: Sketch of the system.

1. What are the input(s) and the output(s)?
2. What are the reservoirs and the corresponding level variables?
3. Draw a causality diagram of the system.
4. Formulate the algebraic and differential equations describing your system.
5. Use your model the two main advantages and main disadvantage of turbochargers.

Solution.

1. The input is the throttle position. The output is the engine power.
2. The reservoirs are
   - Mass in the intake manifold with level variable $p_{\text{IM}}(t)$;
   - Mass in the exhaust manifold with level variable $m_{\text{EM}}(t)$;
   - Energy in the exhaust manifold with level variable $\vartheta_{\text{EM}}(t)$;
   - Energy of the turbocharger with level variable $\omega(t)$.
3. The causality diagram is shown in Figure 7.
4. The dynamics of the intake manifold is described by

$$\frac{\mathrm{d}}{\mathrm{d}t}p_{\text{IM}}(t) = \frac{1}{V_{\text{IM}}}(\dot{m}_{\text{c}} - \dot{m}_{\text{th}}) \cdot R \cdot \vartheta_{\text{IM}},$$

where

$$\begin{aligned}\dot{m}_{\text{c}} &= \mathcal{M}_{\text{c},1}(p_{\text{IM}}(t)/p_\infty, \omega(t)),\\ \dot{m}_{\text{th}} &= c_{\text{d}} \cdot A(t) \cdot \frac{p_{\text{IM}}(t)}{\sqrt{R \cdot \vartheta_{\text{IM}}}} \cdot \Psi(p_{\text{IM}}(t)).\end{aligned}$$

The dynamics of the exhaust manifold is described by

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}m_{\text{EM}}(t) &= \dot{m}_{\text{th}} + \dot{m}_{\text{f}} - \dot{m}_{\text{t}}\\ &= \left(1 + \frac{1}{\sigma}\right) \cdot \dot{m}_{\text{th}} - \dot{m}_{\text{t}}.\\ \frac{\mathrm{d}}{\mathrm{d}t}(m_{\text{EM}}(t) \cdot c_v \cdot \vartheta_{\text{EM}}(t)) &= (\dot{m}_{\text{th}} + \dot{m}_{\text{f}}) \cdot c_p \cdot \vartheta_{\text{e}} - \dot{m}_{\text{t}} \cdot c_p \cdot \vartheta_{\text{EM}}\\ &= \left(1 + \frac{1}{\sigma}\right) \cdot \dot{m}_{\text{th}} \cdot c_p \cdot \vartheta_{\text{e}} - \dot{m}_{\text{t}} \cdot c_p \cdot \vartheta_{\text{EM}}\end{aligned}$$

where $\vartheta_{\text{e}} = \Phi(\dot{m}_{\text{f}})$. For the turbocharger, conservation of mechanical energy gives

$$\Theta \cdot \frac{\mathrm{d}}{\mathrm{d}t}\omega(t) = T_{\text{t}} - T_{\text{c}},$$

where

$$\begin{aligned}T_{\text{t}} &= \frac{\dot{m}_{\text{t}}(t) \cdot c_p \cdot \vartheta_{\text{EM}}(t)}{\omega(t)} \cdot \left(1 - \left(\frac{p_{\text{EM}}(t)}{p_\infty}\right)^{\frac{1-\kappa}{\kappa}}\right) \cdot \eta_{\text{t}},\\ T_{\text{c}} &= \frac{\dot{m}_{\text{c}}(t) \cdot c_p \cdot \vartheta_\infty}{\omega(t)} \cdot \left(\left(\frac{p_{\text{IM}}(t)}{p_\infty}\right)^{\frac{\kappa-1}{\kappa}} - 1\right) \cdot \frac{1}{\eta_{\text{c}}},\end{aligned}$$

with $\eta_{\text{c}} = \mathcal{M}_{c,2}(p_{\text{IM}}(t)/p_\infty, \omega(t))$ and $\eta_{\text{t}} = \mathcal{M}_{t,2}(p_{\text{EM}}(t)/p_\infty, \omega(t))$.

5. See causality diagram.

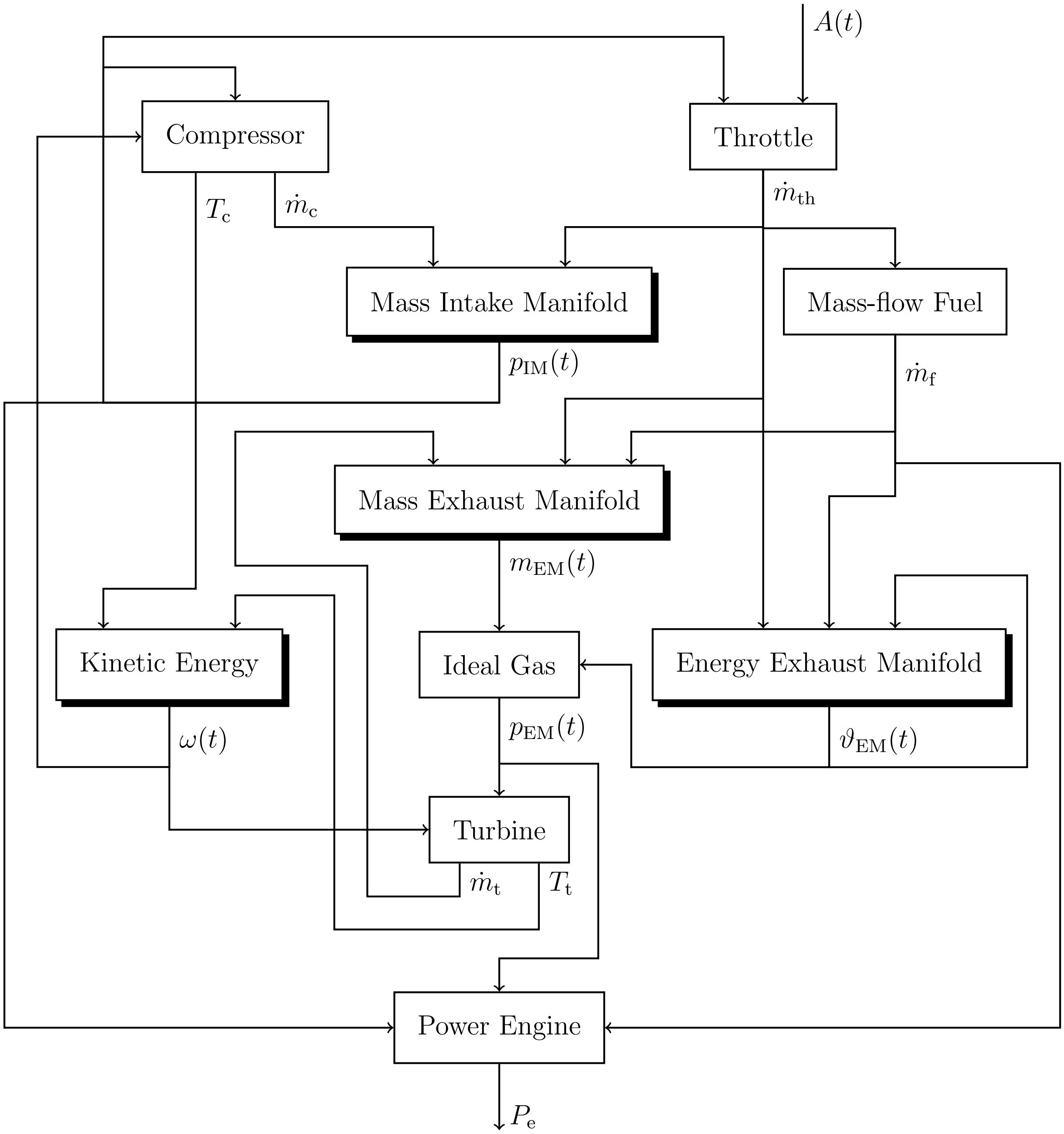

Figure 7: Causality diagram.