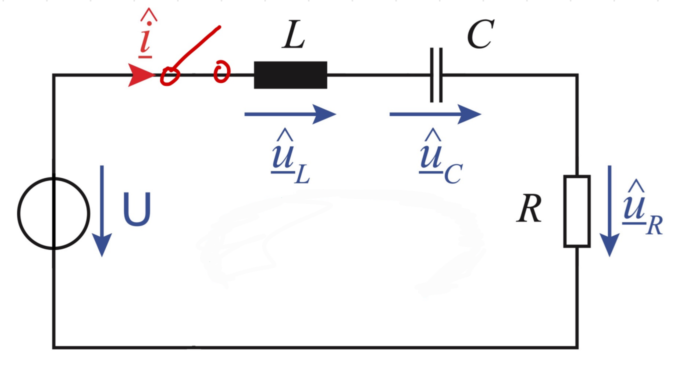
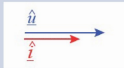
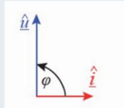
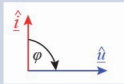
FLASH-EXAM - LW10



ZFS :)

Bauelement	Zeitbereich	Bildbereich	Zeigerdiagramm
Widerstand	$u_R = R \cdot i_R$	$\hat{u}_R = R \cdot \hat{\imath}_R$	
Induktivität	$u_L = L \cdot \frac{di_L}{dt}$	$\hat{u}_L = j\omega L \cdot \hat{\imath}_L$	
Kondensator	$u_C = \frac{1}{C} \cdot \int i_C dt$ $i_C = C \cdot \frac{du_c}{dt}$	$\hat{u}_C = \frac{1}{j\omega C} \cdot \hat{\imath}_C$ $= -\frac{j}{\omega C} \cdot \hat{\imath}_C$	

Anfangsbedingungen : · $i_L(t=0) = 0$
· $u_C(t=0) = 0$

Gesucht : · $u_C(t)$
· $i_L(t)$

Tipps :

▷ Maschenregel aufstellen

↳ $u = u_L(t) + \dots$

▷ Alle Spannungen abhängig von $u_C(t)$ ausdrücken

↳ zB : $u_R(t) = R \cdot i_R(t) = R \cdot i_L(t)$

$i_C(t) = i_L(t) = C \cdot \frac{d}{dt} u_C(t)$

▷ DGL aufstellen : $\ddot{u}_C(t) + a \cdot \dot{u}_C(t) + b\, u_C(t) = g(t)$ hier : $g(t) = \frac{u}{L \cdot C} \in \mathbb{R}$

▷ Partikuläre Lösung bestimmen ($t \to \infty$)

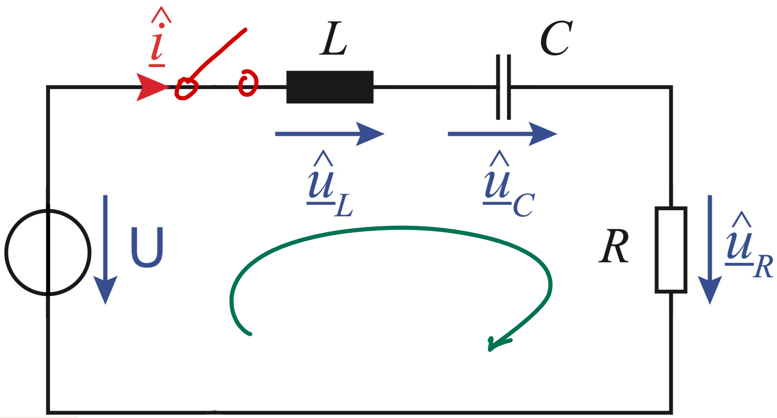
↳ $u_{C,P}(t) =$ ______

▷ Homogene Lösung finden → Exponentialansatz

▷ Anfangsbedingungen verwenden → spezielle Lösung :)

▷ $u_C(t) = u_{C,P}(t) + u_{C,H}(t)$

Differentialgleichung höherer Ordnung Beispiel (2. Ordnung)



- **Berechne $i_L(t)$ und $u_C(t)$**
- **Folgende Anfangswerte sind gegeben**
  - $i_L(t{=}0) = 0$
  - $u_C(0) = 0$

$$: \; U = u_R(t) + u_L(t) + u_C(t)$$

$$= R \cdot i_L(t) + L \cdot \frac{d}{dt} i_L(t) + u_C(t)$$

$$= R \cdot C \cdot \frac{d}{dt} u_C(t) + L \cdot C \cdot \frac{d^2}{dt^2} u_C(t) + u_C(t)$$

$$i_C(t) = i_L(t) = C \cdot \frac{d}{dt} u_C(t)$$

$$\Longleftrightarrow \begin{cases} \ddot{u}_C(t) + \frac{R}{L} \dot{u}_C(t) + \frac{1}{L \cdot C} u_C(t) = \frac{U}{L \cdot C} \\ \text{HT} \quad u_{C,P}(t) = U \qquad (t \rightarrow \infty) \end{cases}$$

Beispiel DGL 2. Ordnung: [Continuation 2]

$$\begin{cases} \ddot{u}_c(t) + \frac{R}{L}\dot{u}_c(t) + \frac{1}{LC}u_c(t) = \frac{U}{LC} \\ \text{mit } u_{c,p}(t) = U \quad (t \to \infty) \end{cases}$$

Partikuläre Lösung:

$$u_{c,p}(t) = U \qquad (t \to \infty,\ Z_L \to 0,\ Z_C \to \infty)$$

Homogene Lösung:

$$\ddot{u}_c(t) + \frac{R}{L}\dot{u}_c(t) + \frac{1}{LC}u_c(t) = 0$$

$$\text{charP}(\lambda) = \lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} \overset{!}{=} 0$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$$

$$= \boxed{\frac{1}{2L}\left[-R \pm \sqrt{\frac{R^2C - 4L}{C}}\right]}$$

$$\longrightarrow u_{c,h} = C_1 e^{\lambda_1 \cdot t} + C_2 e^{\lambda_2 t}$$

$$= C_1 \exp\left[\frac{1}{2L}\left[-R + \sqrt{\frac{R^2C-4L}{C}}\right] \cdot t\right] + C_2 \exp\left[\frac{1}{2L}\left[-R - \sqrt{\frac{R^2C-4L}{C}}\right] \cdot t\right]$$

Allgemeine Lösung $u_c(t)$:

$$u_c(t) = \underbrace{U}_{\text{Partikuläre Lösung}} + \underbrace{C_1 \exp\left[\frac{1}{2L}\left[-R + \sqrt{\frac{R^2C-4L}{C}}\right] \cdot t\right] + C_2 \exp\left[\frac{1}{2L}\left[-R - \sqrt{\frac{R^2C-4L}{C}}\right] \cdot t\right]}_{\text{Homogene Lösung}}$$

Anfangswerte:

(I) $$u_c(t=0) = 0 = U + C_1 \exp\left[\frac{1}{2L}\left[-R + \sqrt{\frac{R^2C-4L}{C}}\right] \cdot 0\right] + C_2 \exp\left[\frac{1}{2L}\left[-R - \sqrt{\frac{R^2C-4L}{C}}\right] \cdot 0\right]$$

$$\Longleftrightarrow 0 = U + C_1 + C_2$$

(II) $$i_c(t=0) = 0 = C \cdot \frac{d}{dt} u_c(t=0)$$

$$\Longleftrightarrow 0 = \frac{d}{dt} u_c(t=0) = C_1 \frac{1}{2L}\left[-R + \sqrt{\frac{R^2C-4L}{C}}\right] \exp\left[\frac{1}{2L}\left[-R + \sqrt{\frac{R^2C-4L}{C}}\right] \cdot 0\right] + C_2 \frac{1}{2L}\left[-R - \sqrt{\frac{R^2C-4L}{C}}\right] \exp\left[\frac{1}{2L}\left[-R - \sqrt{\frac{R^2C-4L}{C}}\right] \cdot 0\right]$$

$$= C_1 \cdot \lambda_1 + C_2 \lambda_2$$

Beispiel DGL 2. Ordnung : [Continuation 3]

Anfangsbedingungen

$$\mathrm{I}: \quad 0 = u + C_1 + C_2$$

$$\mathrm{II}: \quad 0 = C_1 \cdot \lambda_1 + C_2 \lambda_2$$

- Substitution
- Matrix-Form → Gauss, ...
- ...

$$\mathrm{I}: \quad C_1 = -u - C_2$$

$$\rightarrow \mathrm{II}: \quad 0 = (-u - C_2)\lambda_1 + C_2 \lambda_2$$

$$\Longleftrightarrow \quad \boxed{\frac{u\lambda_1}{(-\lambda_1 + \lambda_2)}} = C_2$$

$$\rightarrow \mathrm{I}: \quad C_1 = \boxed{-u - \frac{u\lambda_1}{(-\lambda_1 + \lambda_2)}}$$

→ Einsetzen :

Spezielle Lösung (mit Anfangsbedingungen)

$$u_c(t) = u_{c,p}(t) + u_{c,h}(t)$$

$$= u + \boxed{\left[-u - \frac{u\lambda_1}{(-\lambda_1+\lambda_2)}\right]} \exp\left[\boxed{\frac{1}{2L}\cdot\left[-R + \sqrt{\frac{R^2C - 4L}{C}}\right]}\cdot t\right] + \boxed{\left[\frac{u\lambda_1}{(-\lambda_1+\lambda_2)}\right]} \exp\left(\boxed{\frac{1}{2L}\cdot\left[-R - \sqrt{\frac{R^2C - 4L}{C}}\right]}\cdot t\right) \sigma(t)$$

$$i_c(t) = C \cdot \frac{d}{dt} u_c(t)$$

$$= C \cdot \left[\boxed{\left[-u - \frac{u\lambda_1}{(-\lambda_1+\lambda_2)}\right]} \lambda_1 \exp\left[\boxed{\frac{1}{2L}\cdot\left[-R + \sqrt{\frac{R^2C - 4L}{C}}\right]}\cdot t\right] + \boxed{\left[\frac{u\lambda_1}{(-\lambda_1+\lambda_2)}\right]} \cdot \lambda_2 \exp\left(\boxed{\frac{1}{2L}\cdot\left[-R - \sqrt{\frac{R^2C - 4L}{C}}\right]}\cdot t\right) \sigma(t) \right]$$