A.1 Integrator Element

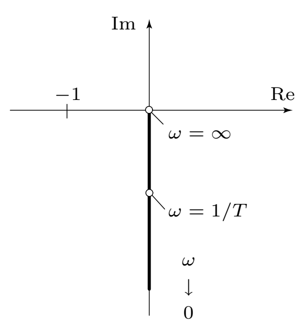
Element Acronym: $\boxed{\mathrm{I}}$

Transfer Function: $\Sigma(s) = \frac{1}{T \cdot s}$

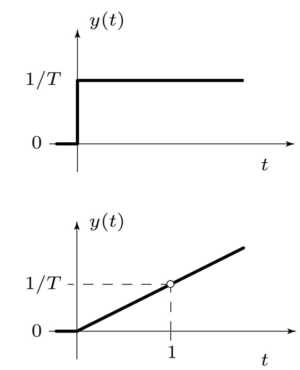
Poles/Zeros: $\pi_1 = 0$, $\zeta_1 = \infty$

Internal Description: $\frac{\mathrm{d}}{\mathrm{d}t}x(t) = \frac{1}{T} \cdot u(t)$
$y(t) = x(t)$

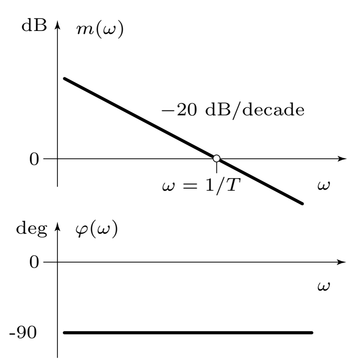
Nyquist Diagram



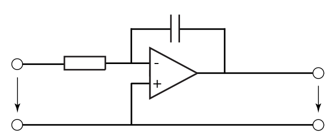
Impulse/Step Response



Bode Diagram



Analog/Digital Realization




```
...
analog_input(e_k);
u_k=u_k_1+T_s/(2*T)*(e_k+e_k_1);
analog_output(u_k);
u_k_1=u_k;
e_k_1=e_k;
...
```

A.2 Differentiator Element

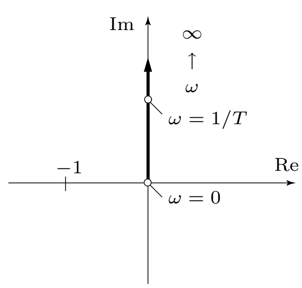
Element Acronym: $\boxed{\mathrm{D}}$

Transfer Function: $\Sigma(s) = T \cdot s$

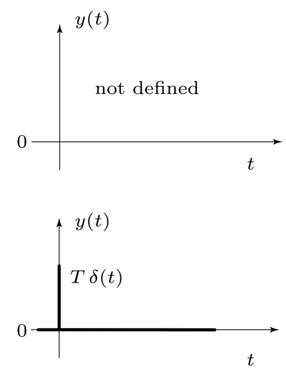
Poles/Zeros: $\pi_1 = \infty$, $\zeta_1 = 0$

Internal Description: $y(t) = T \cdot \frac{\mathrm{d}}{\mathrm{d}t} u(t)$

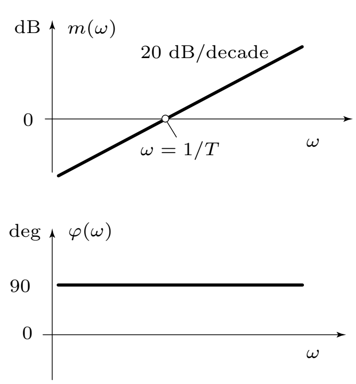
Nyquist Diagram



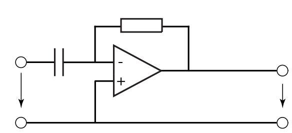
Impulse/Step Response



Bode Diagram



Analog/Digital Realization



```
...
analog_input(e_k);
u_k=2*T*(e_k-e_k_1)/T_s - u_k_1;
analog_output(u_k);
u_k_1=u_k;
e_k_1=e_k;
...
```

A.3 First-Order Element

Element Acronym: $\boxed{\text{LP-1}}$

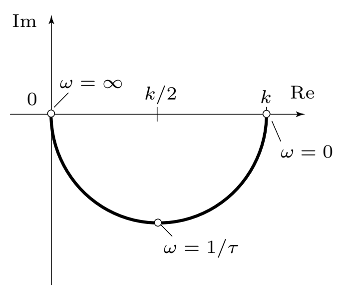
Transfer Function: $\Sigma(s) = \frac{k}{\tau \cdot s+1}$

Poles/Zeros: $\pi_1 = -\frac{1}{\tau}$, $\zeta_1 = \infty$

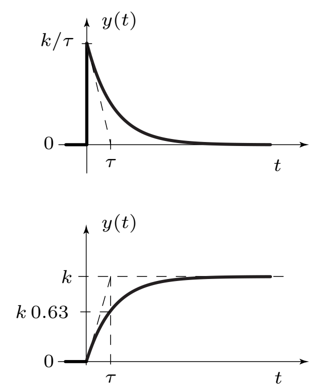
Internal Description: $\frac{\mathrm{d}}{\mathrm{d}t}x(t) = -\frac{1}{\tau} \cdot x(t) + \frac{1}{\tau} \cdot u(t)$

$y(t) = k \cdot x(t)$

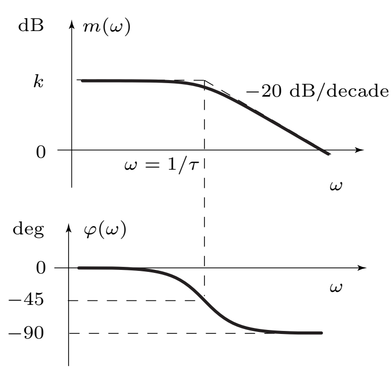
Nyquist Diagram



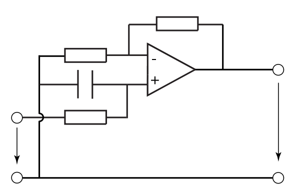
Impulse/Step Response



Bode Diagram



Analog/Digital Realization




```
...
analog_input(e_k);
u_k=u_k_1*(2*tau-T_s)/(2*tau+T_s)+...
    (e_k_1+e_k)*(k*T_s)/(2*tau+T_s);
analog_output(u_k);
u_k_1=u_k;
e_k_1=e_k;
...
```

A.4 Realizable Derivative Element

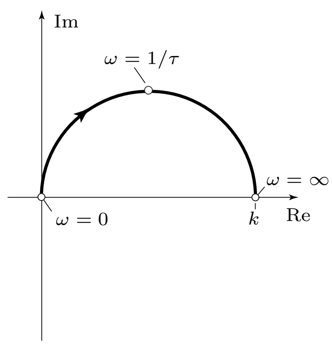
Element Acronym: $\boxed{\text{HP-1}}$

Transfer Function: $\Sigma(s) = k \cdot \frac{\tau \cdot s}{\tau \cdot s + 1} = k \cdot \left(1 - \frac{1}{\tau \cdot s + 1}\right)$

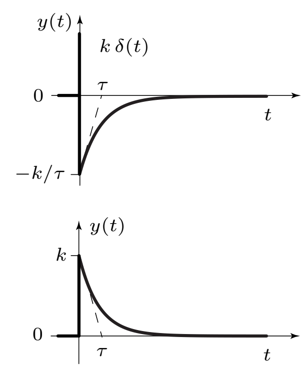
Poles/Zeros: $\pi_1 = -\frac{1}{\tau}$, $\zeta_1 = 0$

Internal Description: $\frac{\mathrm{d}}{\mathrm{d}t} x(t) = -\frac{1}{\tau} \cdot x(t) + \frac{1}{\tau} \cdot u(t)$
$y(t) = -k \cdot x(t) + k \cdot u(t)$

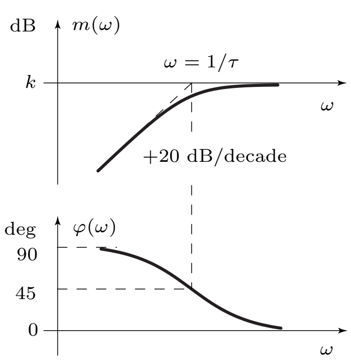
Nyquist Diagram



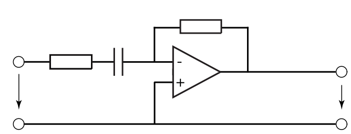
Impulse/Step Response



Bode Diagram



Analog/Digital Realization



```
...
analog_input(e_k);
u_k=1/(T_s+2*tau)*(u_k_1*(2*tau-T_s)+...
                 (e_k-e_k_1)*2*k*tau);
analog_output(u_k);
u_k_1=u_k;
e_k_1=e_k;
...
```

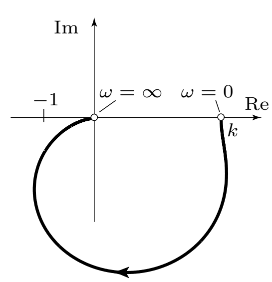
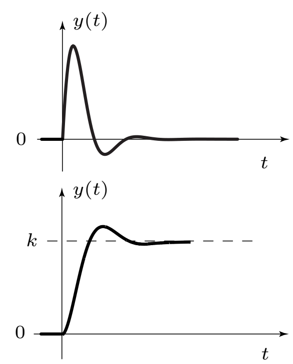
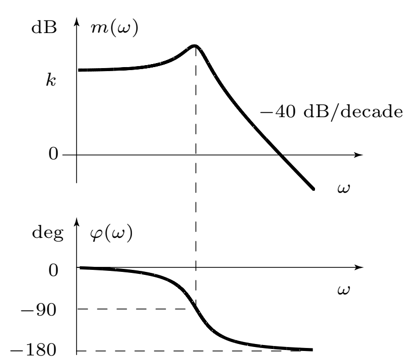
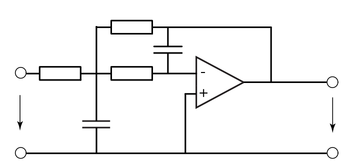
A.5 Second-Order Element

Element Acronym: $\boxed{\text{LP-2}}$

Transfer Function: $\Sigma(s) = k \cdot \frac{\omega_0^2}{s^2+2\cdot\delta\cdot\omega_0\cdot s+\omega_0^2}$

Poles/Zeros: $\pi_{1,2} = -w_0 \cdot \delta \pm w_0\sqrt{\delta^2-1}$, $\zeta_{1,2} = \infty$

Internal Description: $\frac{\mathrm{d}}{\mathrm{d}t}x_1(t) = x_2(t)$,
$\frac{\mathrm{d}}{\mathrm{d}t}x_2(t) = -\omega_0^2 \cdot x_1(t) - 2\cdot\delta\cdot\omega_0\cdot x_2(t) + \omega_0^2 \cdot u(t)$
$y(t) = k \cdot x_1(t)$

<table>
<tr><td>Nyquist Diagram</td><td>Impulse/Step Response</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Bode Diagram</td><td>Analog/Digital Realization</td></tr>
<tr><td>
</td><td>

...
analog_input(e_k);
...
use Matlab's c2dm
...
analog_output(u_k);
u_k_2=u_k_1;
e_k_2=e_k_1;
u_k_1=u_k;
e_k_1=e_k;
...</td></tr>
</table>

A.6 Lag Element

Element Acronym: $\boxed{\text{LG-1}}$

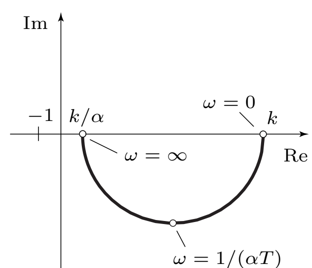
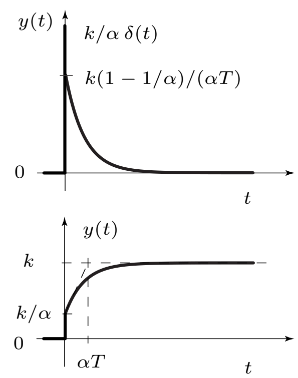
Transfer Function: $\Sigma(s) = k \cdot \frac{T \cdot s+1}{\alpha \cdot T \cdot s+1} = \frac{k}{\alpha} + k \cdot \frac{1-1/\alpha}{\alpha \cdot T \cdot s+1} \qquad 1 < \alpha$

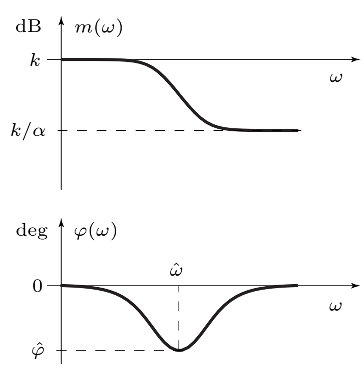
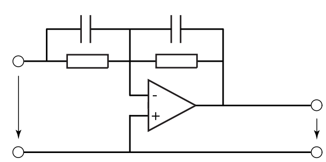
Poles/Zeros: $\pi_1 = -\frac{1}{\alpha \cdot T}$, $\zeta_1 = -\frac{1}{T}$

Internal Description: $\frac{\mathrm{d}}{\mathrm{d}t}x(t) = -\frac{1}{\alpha \cdot T} \cdot x(t) + \frac{1}{\alpha \cdot T} \cdot u(t)$

$y(t) = \frac{k \cdot (\alpha-1)}{\alpha} \cdot x(t) + \frac{k}{\alpha} \cdot u(t)$

Phase minimum: $\hat{\varphi} = \arctan(1/\sqrt{\alpha}) - \arctan(\sqrt{\alpha})$ at $\hat{\omega} = (T \cdot \sqrt{\alpha})^{-1}$

Nyquist Diagram	Impulse/Step Response
	

Bode Diagram	Analog/Digital Realization
	


```
...
analog_input(e_k);
u_k=u_k_1*(2*T*T_s*alpha)/ ...
    (T_s+2*T*alpha)+...
    e_k*k(T_s+2*T*alpha)+...
    e_k_1*k(T_s-2*T)/(T_s+2*T*alpha);
analog_output(u_k);
u_k_1=u_k;
e_k_1=e_k;
...
```

A.7 Lead Element

Element Acronym: $\boxed{\text{LD-1}}$

Transfer Function: $\Sigma(s) = k \cdot \frac{T \cdot s+1}{\alpha \cdot T \cdot s+1} = \frac{k}{\alpha} + k \cdot \frac{1-1/\alpha}{\alpha \cdot T \cdot s+1} \qquad 0 < \alpha < 1$

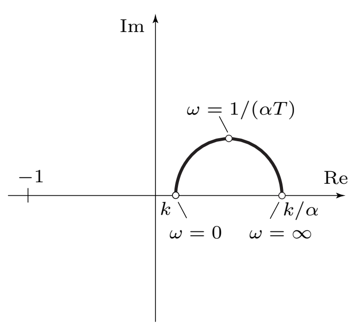
Poles/Zeros: $\pi_1 = -\frac{1}{\alpha \cdot T},\ \zeta_1 = -\frac{1}{T}$

Internal Description: $\frac{\mathrm{d}}{\mathrm{d}t}x(t) = -\frac{1}{\alpha \cdot T} \cdot x(t) + \frac{1}{\alpha \cdot T} \cdot u(t)$

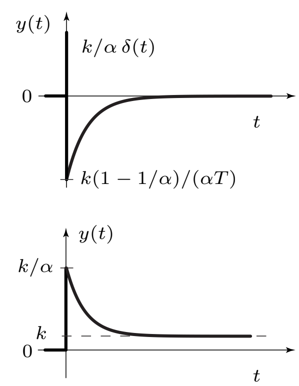
$y(t) = \frac{k \cdot (\alpha - 1)}{\alpha} \cdot x(t) + \frac{k}{\alpha} \cdot u(t)$

Phase maximum: $\hat{\varphi} = \arctan(1/\sqrt{\alpha}) - \arctan(\sqrt{\alpha})$ at $\hat{\omega} = (T \cdot \sqrt{\alpha})^{-1}$

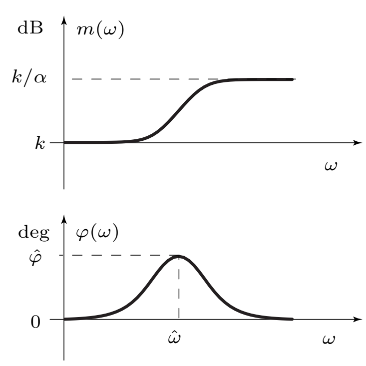
Nyquist Diagram



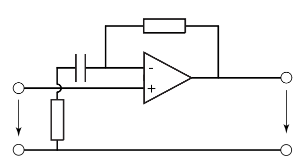
Impulse/Step Response



Bode Diagram



Analog/Digital Realization



```
...
analog_input(e_k);
u_k=u_k_1*(2*T*T_s*alpha)/ ...
     (T_s+2*T*alpha)+...
     e_k*k(T_s+2*T)/(T_s+2*T*alpha)+...
     e_k_1*k*(T_s-2*T)/(T_s+2*T*alpha);
analog_output(u_k);
u_k_1=u_k;
e_k_1=e_k;
...
```

A.8 PID Element

Element Acronym: $\boxed{\text{PID}}$

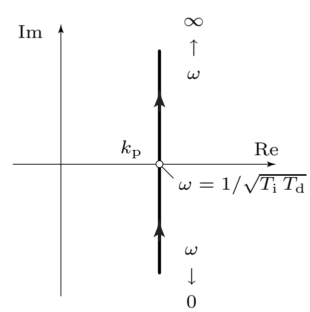
Transfer Function: $\Sigma(s) = k_\mathrm{p} \cdot \frac{T_\mathrm{d} \cdot T_\mathrm{i} \cdot s^2 + T_\mathrm{i} \cdot s + 1}{T_\mathrm{i} \cdot s} = k_\mathrm{p} \cdot (1 + \frac{1}{T_\mathrm{i} \cdot s} + T_\mathrm{d} \cdot s)$

Poles/Zeros: $\pi_1 = 0,\ \pi_2 = \infty,\ \zeta_{1,2} = -\frac{1}{2 \cdot T_\mathrm{d}} \pm \sqrt{\frac{1}{4 \cdot T_\mathrm{d}^2} - \frac{1}{T_\mathrm{i} \cdot T_\mathrm{d}}}$

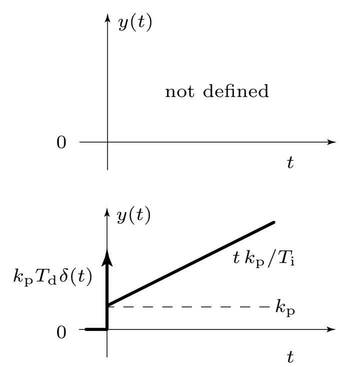
Internal Description: $\frac{\mathrm{d}}{\mathrm{d}t} x_1(t) = \frac{1}{T_i} \cdot u(t)$

$y(t) = k_\mathrm{p} \cdot \big(u(t) + x_1(t) + T_\mathrm{d} \cdot \frac{\mathrm{d}}{\mathrm{d}t} u(t)\big)$

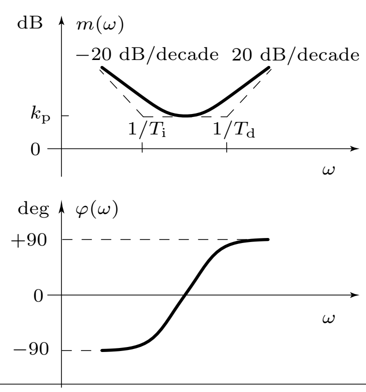
Nyquist Diagram



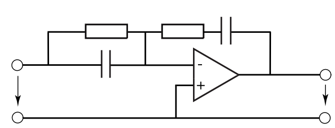
Impulse/Step Response



Bode Diagram



Analog/Digital Realization




```
...
analog_input(e_k);
u_k=e_k*kp*(1+T_s/(2*T_i)+2*T_d/T_s)+ ...
    e_k_1*kp*(T_s/T_i-4*T_d/T_s)+...
    e_k_2*kp*(-1+T_s/(2*T_i)+2*T_d/T_s)+...
    u_k_2;
analog_output(u_k);
u_k_1=u_k;
e_k_1=e_k;
...
```

A.9 First-Order All-Pass Element

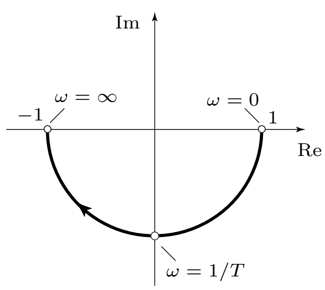
Element Acronym: $\boxed{\text{AP-1}}$

Transfer Function: $\Sigma(s) = \frac{-T \cdot s+1}{T \cdot s+1} = -1 + \frac{2}{T \cdot s+1}$

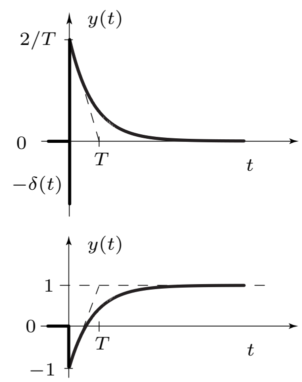
Poles/Zeros: $\pi_1 = -\frac{1}{T}$, $\zeta_1 = \frac{1}{T}$

Internal Description: $\frac{\mathrm{d}}{\mathrm{d}t}x(t) = -\frac{1}{T} \cdot x(t) + \frac{1}{T} \cdot u(t)$
$y(t) = 2 \cdot x(t) - u(t)$

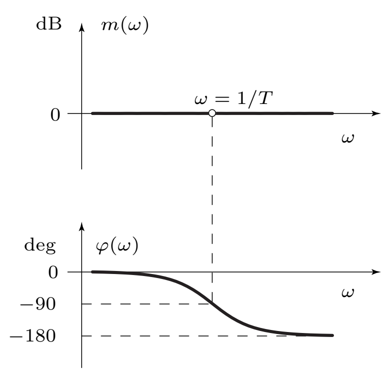
Nyquist Diagram



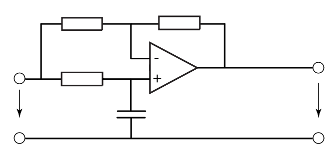
Impulse/Step Response



Bode Diagram



Analog/Digital Realization




```
...
analog_input(e_k);
u_k=(e_k-u_k_1)*(T_s-2*T)/(T_s+2*T)+e_k_1;
analog_output(u_k);
u_k_1=u_k;
e_k_1=e_k;
...
```

A.10 Delay Element

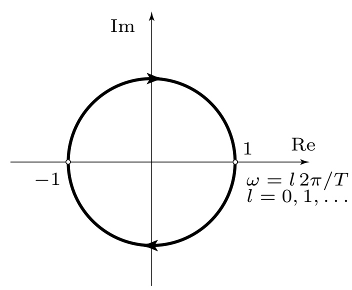
Element Acronym: $\boxed{-}$

Transfer Function: $\Sigma(s) = \mathrm{e}^{-s \cdot T}$

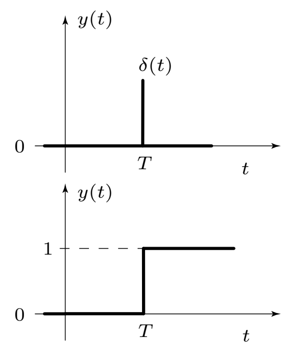
Poles/Zeros: not a real-rational element

Internal Description: $y(t) = u(t - T)$

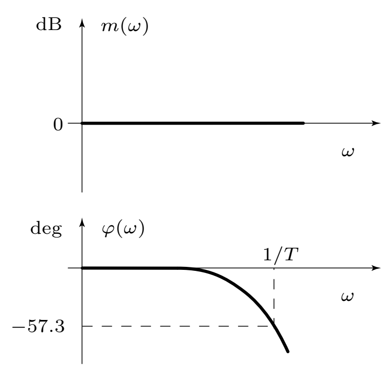
Nyquist Diagram



Impulse/Step Response



Bode Diagram



Analog/Digital Realization

Analog: use Padé elements (allpass elements) as approximation

```
KTZ=integer(T/T_s);
...
analog_input(e_k);
u_k=e_alt(KTZ);
analog_output(u_k);
for i=1:KTZ-1
        e_alt(i+1)=e_alt(i);
end;
e_alt(1)=e_k;
...
```