Slides "Regelungstechnik II"

Part 1: Advanced SISO Methods

L. Guzzella, Spring 2020

Lecture I

Advanced PID Design Methods

Plant model $P(s)$ known, fixed controller transfer function

$$C(s) = k_p \cdot (1 + \frac{1}{T_i \cdot s} + T_d \cdot s) \tag{1}$$

Many extensions known to the basic design methods introduced in RT I.

Three advanced approaches discussed here:

- Aström/Hägglund rules
- analytical crossover specification
- predictive control for plants with large delays

Vast topic! For more information see [3].

Aström-Hägglund Rules

Extension of classical Ziegler-Nichols and Chien-Rhones-Reswick rules. Here just one set of rules using the “critical loop-gain method” [3].

Pro memoria:

1. Set gains of PID controller to $k_p =$ “small,” $T_i = \infty$ and $T_d = 0$.
2. Increase k_p until closed-loop system reaches its stability limits.
3. The parameter k_p^* is this critical gain and the period T^* ist the oscillation period observed in this critical condition.
4. Choose the controller parameters following the rules given below.

Aström-Hägglund tuning rules (rules adapted from [3], symbols of RT I):

$$x = \alpha_0 \cdot e^{\alpha_1 \cdot \kappa + \alpha_2 \cdot \kappa^2} \tag{2}$$

where $\kappa^{-1} = |P(0)| \cdot k_p^*$ and x any of the (normalized) controller parameters

$$x = \left\{ k_p/k_p^*,\ T_i/T^*,\ T_d/T^*,\ a \right\} \tag{3}$$

The parameter $P(0)$ indicates the plant's DC gain (static gain).

The coefficients α_i are defined in the two table below (one for PI and one for PID controllers). The parameter a will be introduced in Lecture IV.

The parameter μ is the minimum return difference that the loop gain should achieve (no guarantee!), i.e.

$$\mu = \min_{\omega} |1 + L(j\,\omega)|$$

Aström-Hägglund parameters for PI controllers:

	$\mu_{\min} = 0.7$			$\mu_{\min} = 0.5$		
x	α_0	α_1	α_2	α_0	α_1	α_2
k_p/k_p^*	0.053	2.9000	-2.6	0.13	1.9	-1.30
T_i/T^*	0.900	-4.4000	2.7	0.90	-4.4	2.70
a	1.100	-0.0061	1.8	0.48	0.4	-0.17

Aström-Hägglund parameters for PID controllers:

	$\mu_{\min} = 0.7$			$\mu_{\min} = 0.5$		
x	α_0	α_1	α_2	α_0	α_1	α_2
k_p/k_p^*	0.33	-0.31	-1.00	0.72	-1.60	1.20
T_i/T^*	0.76	-1.60	-0.36	0.59	-1.30	0.38
T_d/T^*	0.17	-0.46	-2.10	0.15	-1.40	0.56
a	0.58	-1.30	3.50	0.25	0.56	-1.20

The Aström-Hägglund method is a generalization of the older Ziegler-Nichols ideas and often yields better overall performance. Example

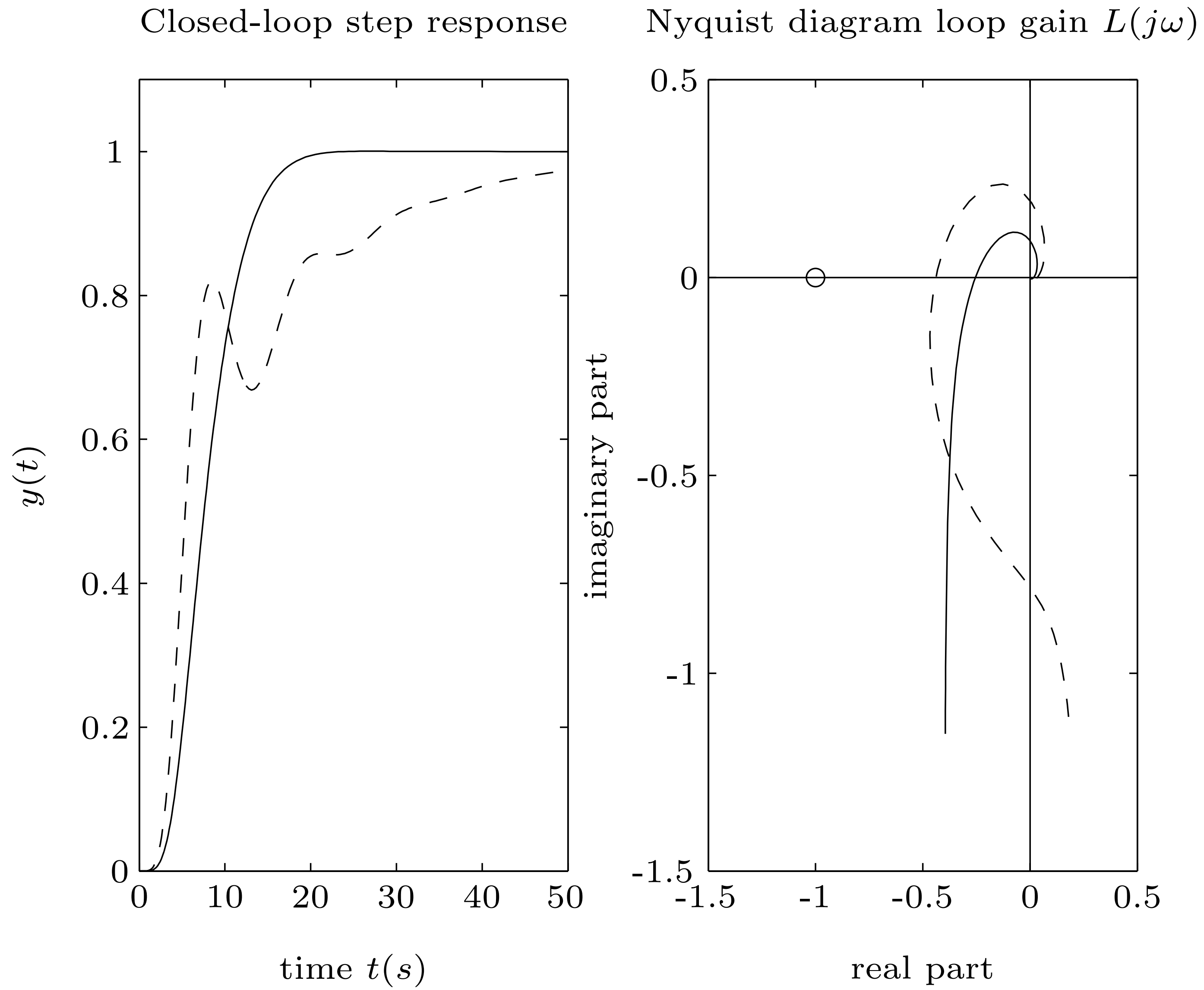
$$P(s) = \frac{1}{(s^2+s+1)\cdot(s^4+4s^3+6s^2+4s+1)}$$

Plant of high-order but "benign." Critical gain $k^* = 1.75$ and period $T^* = 9.7$.

Results of Ziegler-Nichols and Aström-Hägglund rules

$$\text{ZN}: k_p = 0.7,\ T_i = 7.76 \quad \text{AH}: k_p = 0.208,\ T_i = 1.706 \tag{4}$$

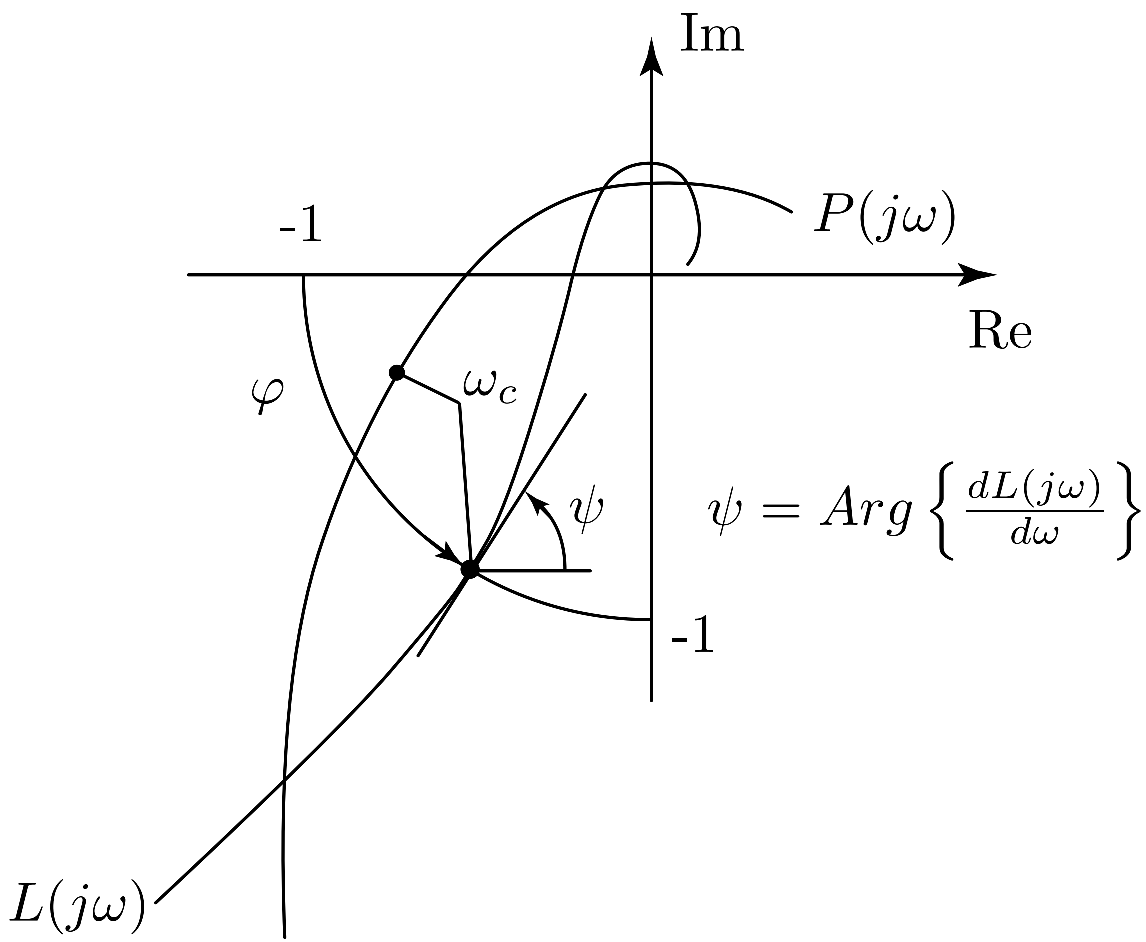
Plots see next slide (dashed Ziegler-Nichols and solid Aström-Hägglund results).



Closed-loop step response
Nyquist diagram loop gain $L(j\omega)$
$y(t)$
0
0.2
0.4
0.6
0.8
1
0
10
20
30
40
50
time $t(s)$
imaginary part
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
real part

Analytical Crossover Specifications

Main idea: select $\{k_p, T_i, T_d\}$ such that the loop gain $L(j\,\omega) = P(j\,\omega) \cdot C(j\,\omega)$ has desired crossover frequency ω_c, phase margin φ and slope ψ (don't forget to check that these specs are feasible!)

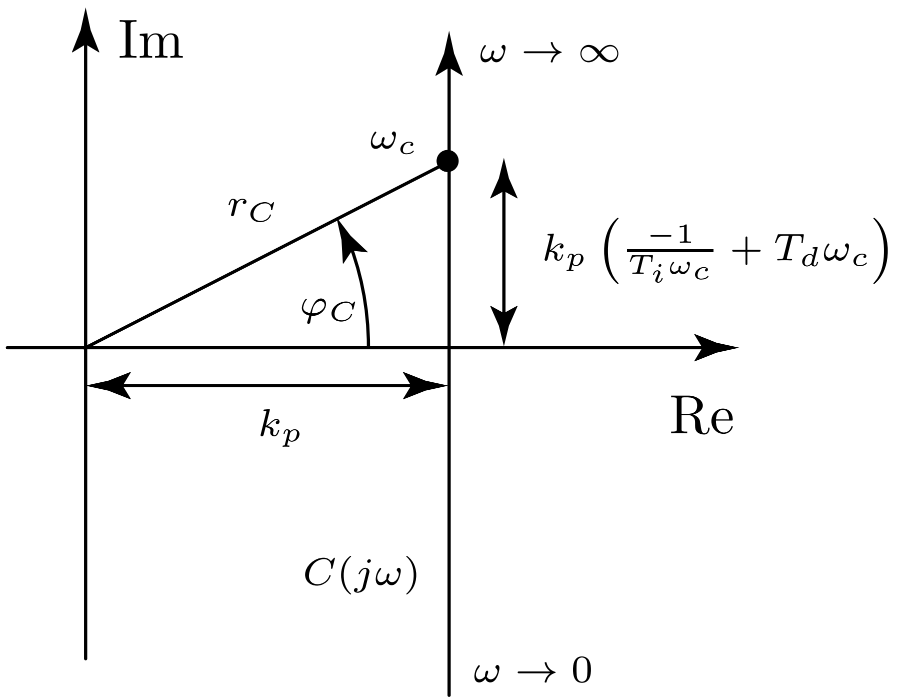


Definitions:

$$P(j\omega_c) = r_P \cdot e^{j\cdot\varphi_P}, \quad C(j\omega_c) = r_C \cdot e^{j\cdot\varphi_C}, \quad L(j\omega_c) = r_L \cdot e^{j\cdot\varphi_L} \tag{5}$$

For the controller

$$\tan(\varphi_C) = \frac{-1}{T_i \cdot \omega_c} + T_d \cdot \omega_c, \quad r_C = \frac{k_p}{\cos(\varphi_C)} \tag{6}$$



Therefore

$$r_L = r_P \cdot r_C \overset{!}{=} 1 \quad \Rightarrow \quad k_p = \frac{1}{r_P} \cdot \cos(\varphi_C) \tag{7}$$

and

$$\varphi_L = \varphi_P + \varphi_C \overset{!}{=} -\pi + \varphi \quad \Rightarrow \quad \varphi_C = \varphi - \pi - \varphi_P \tag{8}$$

The second equation is equivalent to

$$\tan(\varphi_C) = \frac{-1}{T_i \cdot \omega_c} + T_d \cdot \omega_c = \tan(\varphi - \pi - \varphi_P) = \tan(\varphi - \varphi_P) \tag{9}$$

and, using (8), equation (7) can be written in the explicit form

$$k_p = \frac{1}{r_P} \cdot \cos(\varphi - \pi - \varphi_P) = \frac{-1}{r_P} \cdot \cos(\varphi - \varphi_P) \tag{10}$$

So far, the parameters $\{k_p, T_i, T_d\}$ have been expressed as function of the specifications ω_c and φ and of the plant parameters r_P and φ_P. The last missing equation is obtained by using the requirement on ψ. Messy!

Result: Using the known specifications

$$\omega_c = \text{crossover frequency},\ \varphi = \text{phase margin},\ \psi = \text{slope at crossover}$$

and the known parameters of the plant transfer function at ω_c

$r_P =$ magnitude, $\varphi_P =$ phase, $r_P^{'} =$ derivative of r_P, $\varphi_P^{'} =$ derivative of φ_P

the PID controller parameters $\{k_p, T_i, T_d\}$ are found using

$$k_p = \frac{-1}{r_P} \cdot \cos(\varphi - \varphi_P)$$

$$T_d = \frac{1}{2} \cdot \left[\tan(\psi - \varphi_P) \left(\frac{r_P^{'}}{r_P} - \varphi_P^{'} \tan(\varphi - \varphi_P) \right) + \tan(\varphi - \varphi_P) \left(\frac{1}{\omega_c} - \frac{r_P^{'}}{r_P} \right) - \varphi_P^{'} \right]$$

$$T_i = \left[T_d \cdot \omega_c^2 - \tan(\varphi - \varphi_P) \cdot \omega_c \right]^{-1}$$

Implementation tips:

- For many plants, the derivatives $r_P^{'}$ and $\varphi_P^{'}$ may be approximated using finite differences.

- Solutions are not guaranteed to exist for arbitrary specs and plants.

- Similarly, stability is not guaranteed and has to be checked a posteriori.

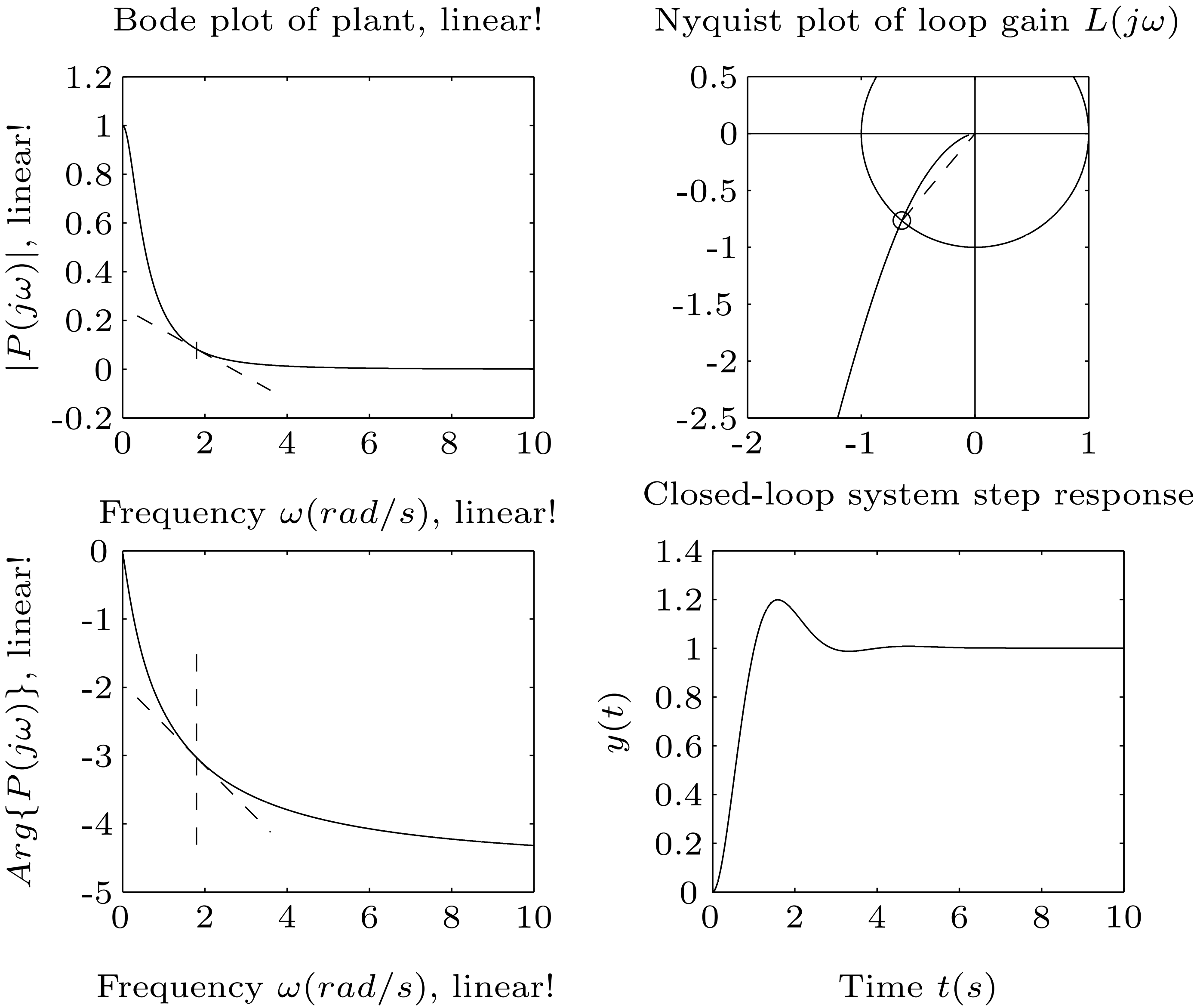
- Always start with a reasonable first design (say Ziegler-Nichols) to obtain a feeling for the realizable crossover frequencies and phase margins.

- Be careful not to mix up radians and degrees (formulas shown above are valid for radians).

Example

$$P(s) = \frac{1}{s^3 + 4 \cdot s^2 + 4 \cdot s + 1}$$

OK for $\{\omega_c, \varphi, \psi\} = \{1.9, 50^\circ, 66^\circ\}$ (see next page), but fails for, say, $\omega_c = 2$.


Bode plot of plant, linear!
$|P(j\omega)|$, linear!
Frequency $\omega(rad/s)$, linear!
$Arg\{P(j\omega)\}$, linear!
Frequency $\omega(rad/s)$, linear!
Nyquist plot of loop gain $L(j\omega)$
Closed-loop system step response
$y(t)$
Time $t(s)$

Predictive PI Control

Plants with delays are hard to control and the regular PID control structure is not well suited (in particular, the D part is not useful).

Instead use controllers that have a "prediction capability." Main point: no matter how complex that controller is, the delay must remain in the closed-loop system response, i.e., $S(s)$ *must* include the same delay.

Example: plant (or its approximation)

$$P(s) = \frac{k}{\tau \cdot s + 1} \cdot e^{-T \cdot s} \tag{11}$$

target sensitivity

$$S(s) = 1 - \frac{1}{s \cdot \sigma \cdot \tau + 1} \cdot e^{-s \cdot T} \tag{12}$$

Scalar σ is the "tuning" parameter.

The controller that produces this sensitivity must include an integrator and has the following form

$$
\begin{aligned}
C(s) &= \frac{1 - S(s)}{S(s) \cdot P(s)} \\
&= \frac{\tau \cdot s + 1}{k \cdot (\sigma \cdot \tau \cdot s + 1 - e^{-T \cdot s})}
\end{aligned}
\tag{13}
$$

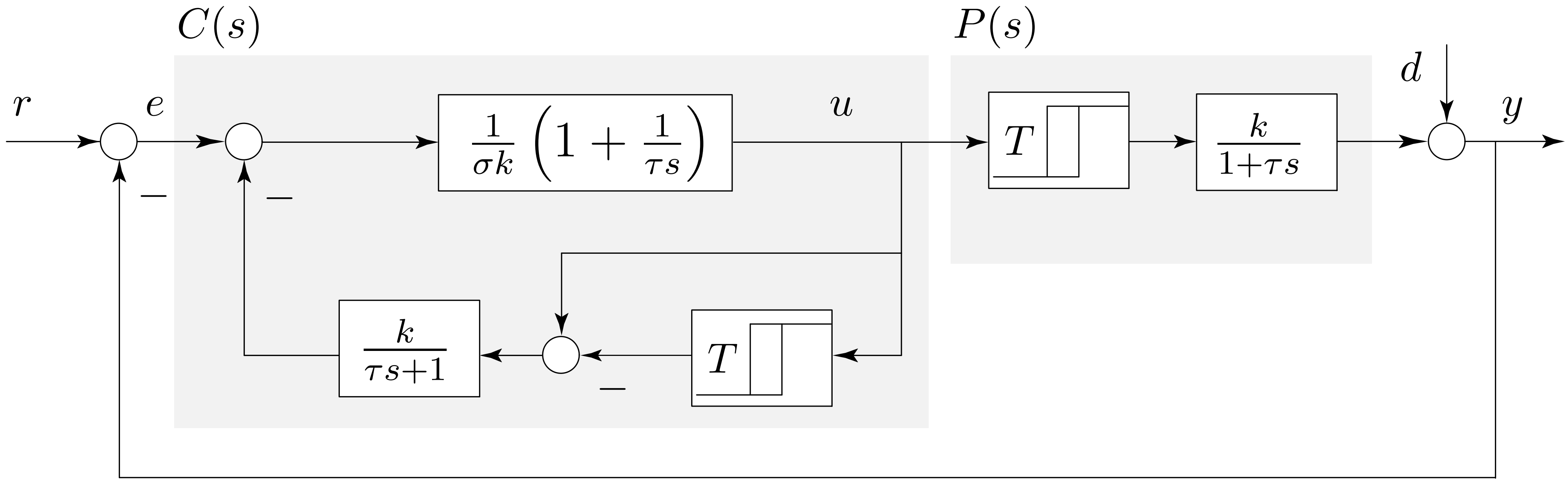
Plant inversion!

Alternative interpretation: the output $U(s)$ generated by this controller is found to be

$$U(s) = \frac{1}{\sigma \cdot k} \cdot (1 + \frac{1}{\tau \cdot s}) \cdot E(s) - \frac{1}{\sigma \cdot \tau \cdot s} \cdot (1 - e^{-T \cdot s}) \cdot U(s) \tag{14}$$

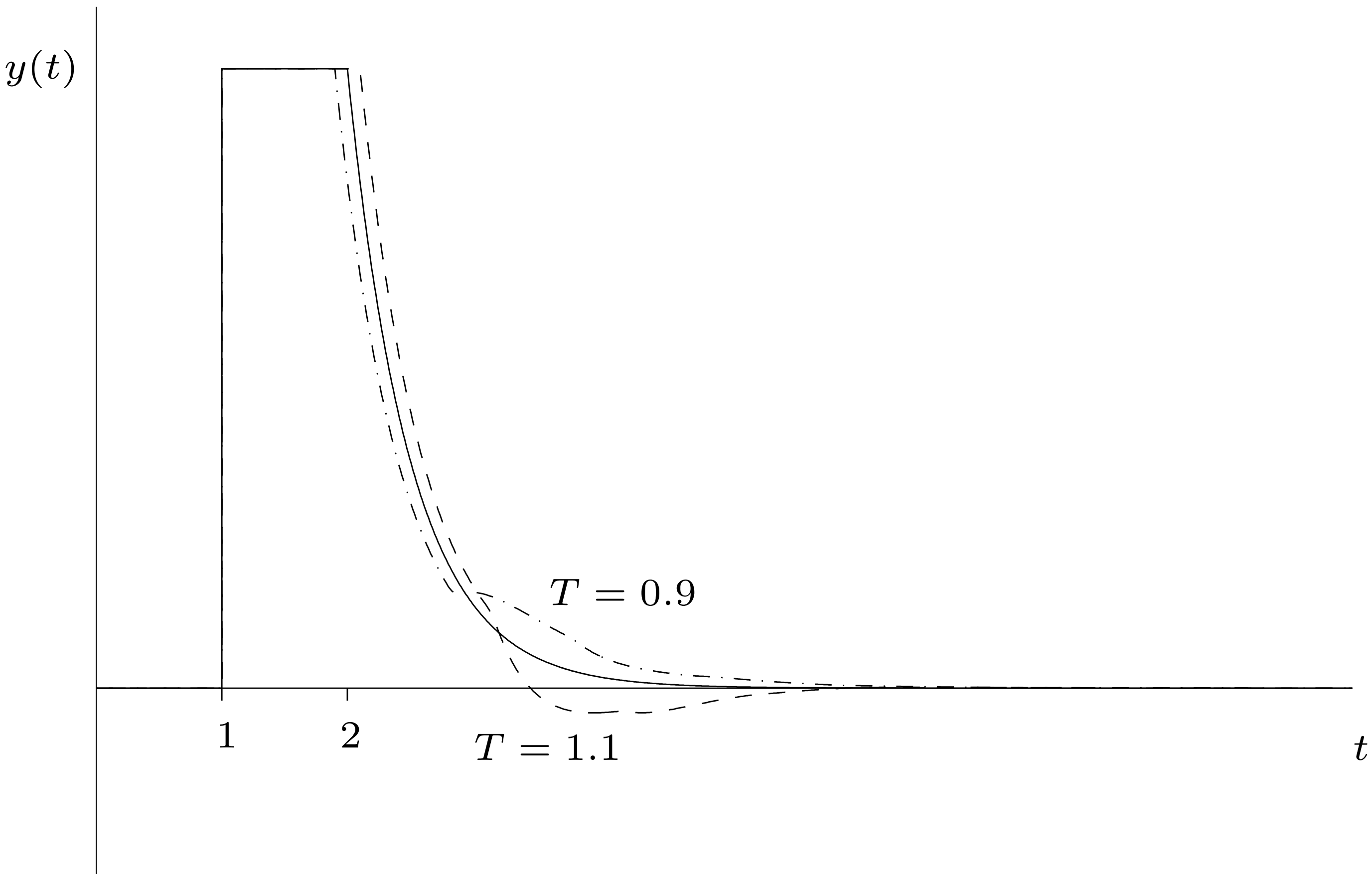
If no delay is present ($T = 0$), the resulting controller is a simple PI controller with σ the only tuning parameter that determines the dynamics of the disturbance rejection transients (recommended $0.5 < \sigma < 2$). The second part provides the predictive correction action.

Block diagram

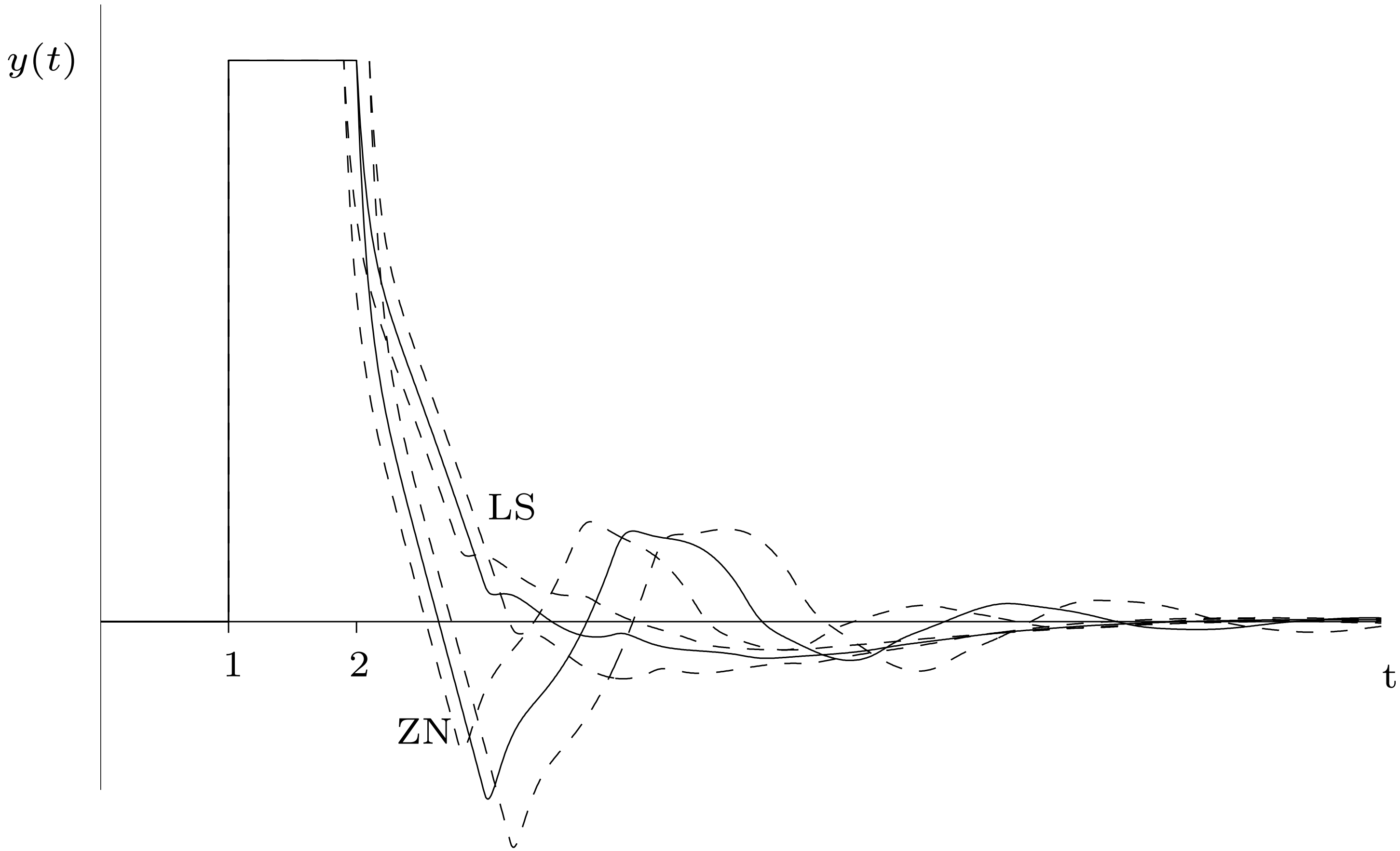


Interpretation: The D part of a PID controller does not make sense when the plant includes delays. Therefore, replace it by a term that penalizes differences in the control action (that – due to the delay – eventually will become differences in plant output).

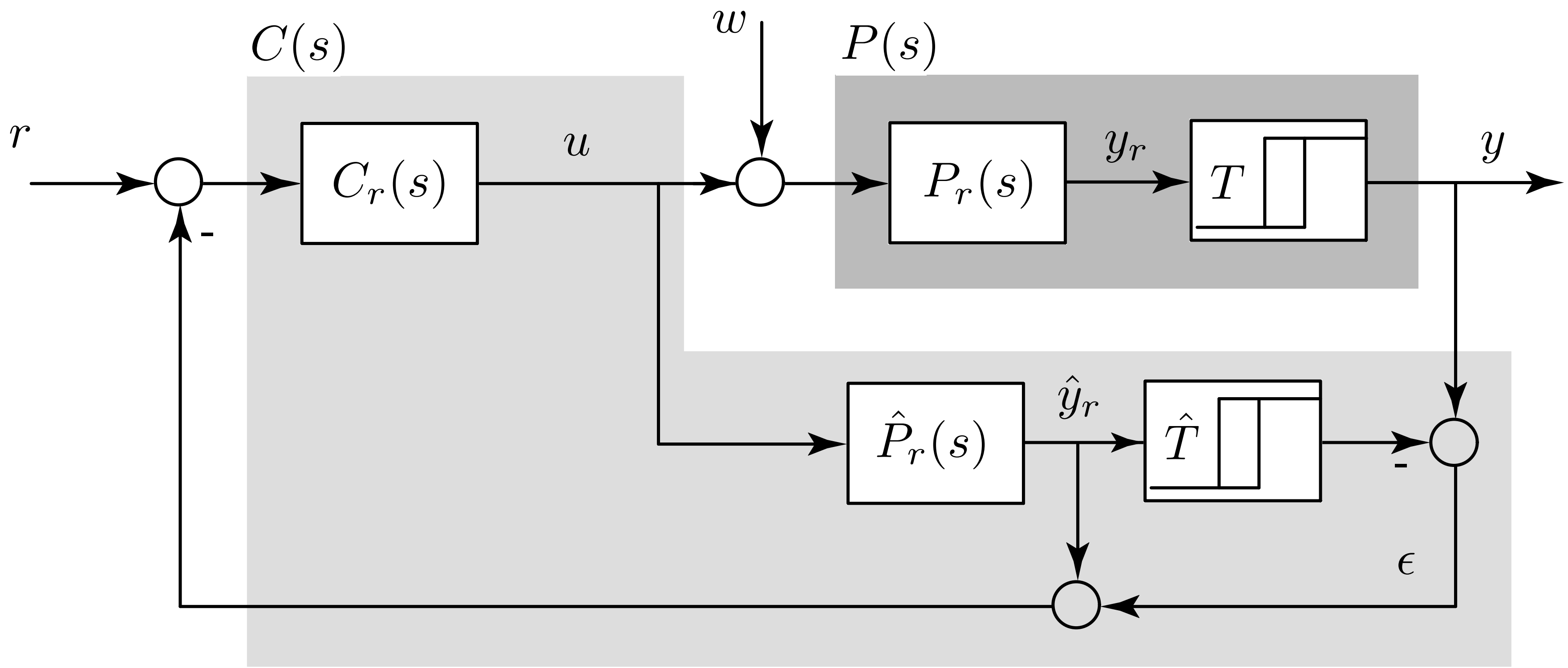
Closed-loop system response to step disturbance, $\sigma = 0.5$, dashed curves $\pm 10\%$ delay T error



As a reference, closed-loop system responses to step disturbance when using a standard PI controller (ZN = Ziegler-Nichols design, LS = loop-shaping design)



The generalization of the ideas presented in this section to arbitrary plants $P(s)$ is known as a *Smith Predictor*. More information on these controller structures can be found in [11].



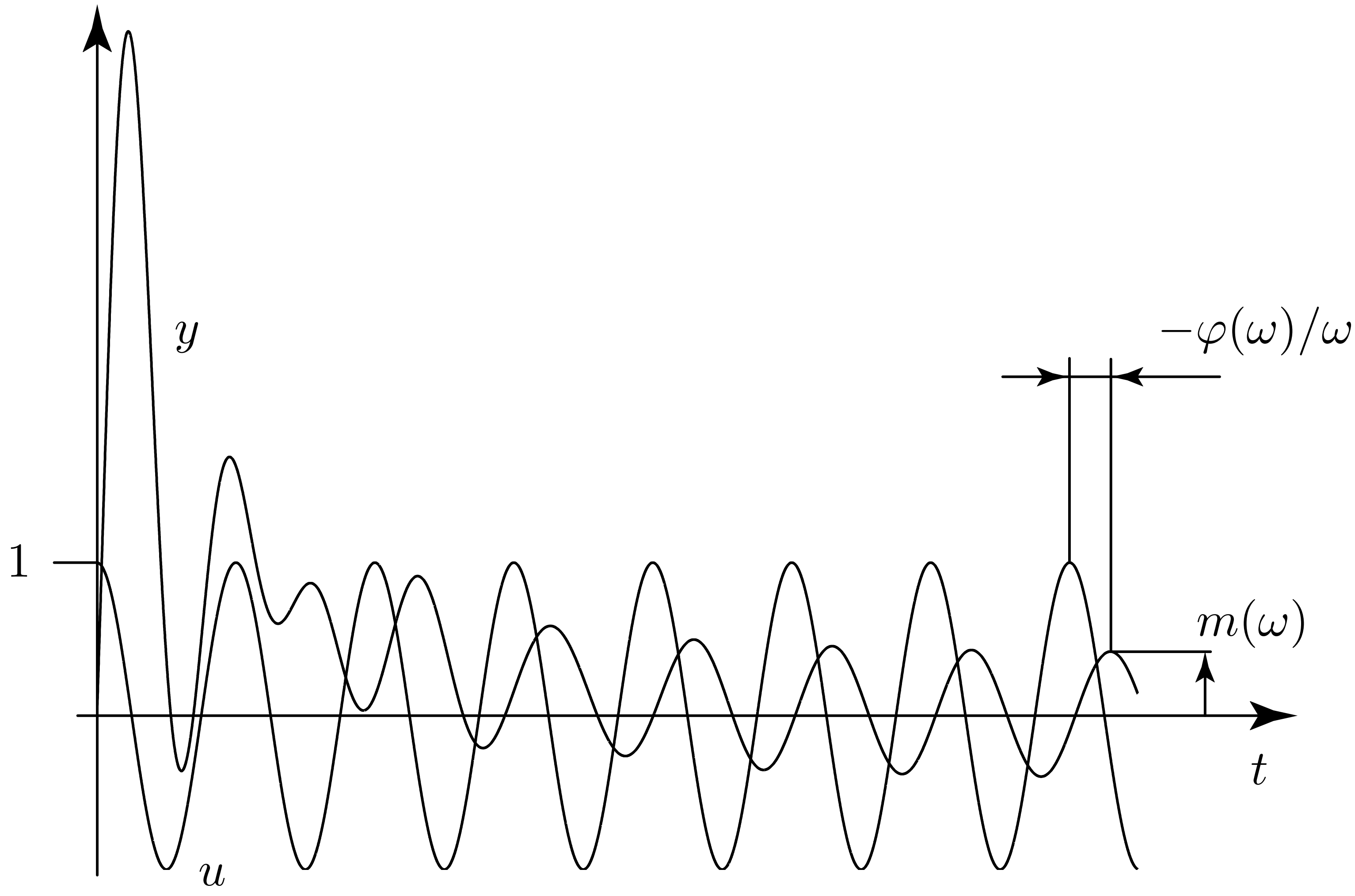
Key idea: design $C_r(s)$ as if $P(s) = P_r(s)$ (no delays present).

Lecture II

Loop Shaping

Case Study

Pro Memoria: Frequency Responses



System input

$$u(t) = \cos(\omega\, t) \tag{15}$$

System output in steady state

$$y_\infty(t) = |P(j\,\omega)| \cdot \cos(\omega\, t + \angle P(j\,\omega)) \tag{16}$$

Representation as Bode or Nyquist diagrams

Bode's law (here for asymptotically stable and miniphase systems)

$$\angle P(j\omega_0) - \angle P(0) \;\approx\; \frac{1}{\pi} \cdot k \cdot \frac{\pi^2}{2} = k \cdot \frac{\pi}{2} \tag{17}$$

where $k \cdot 20$ dB/dek is the slope of the magnitude plot.

Pro Memoria: Uncertainties

No model exactly describes a real system. All controller designs must acknowledge this fact and tolerate the expected errors ("robust" designs).

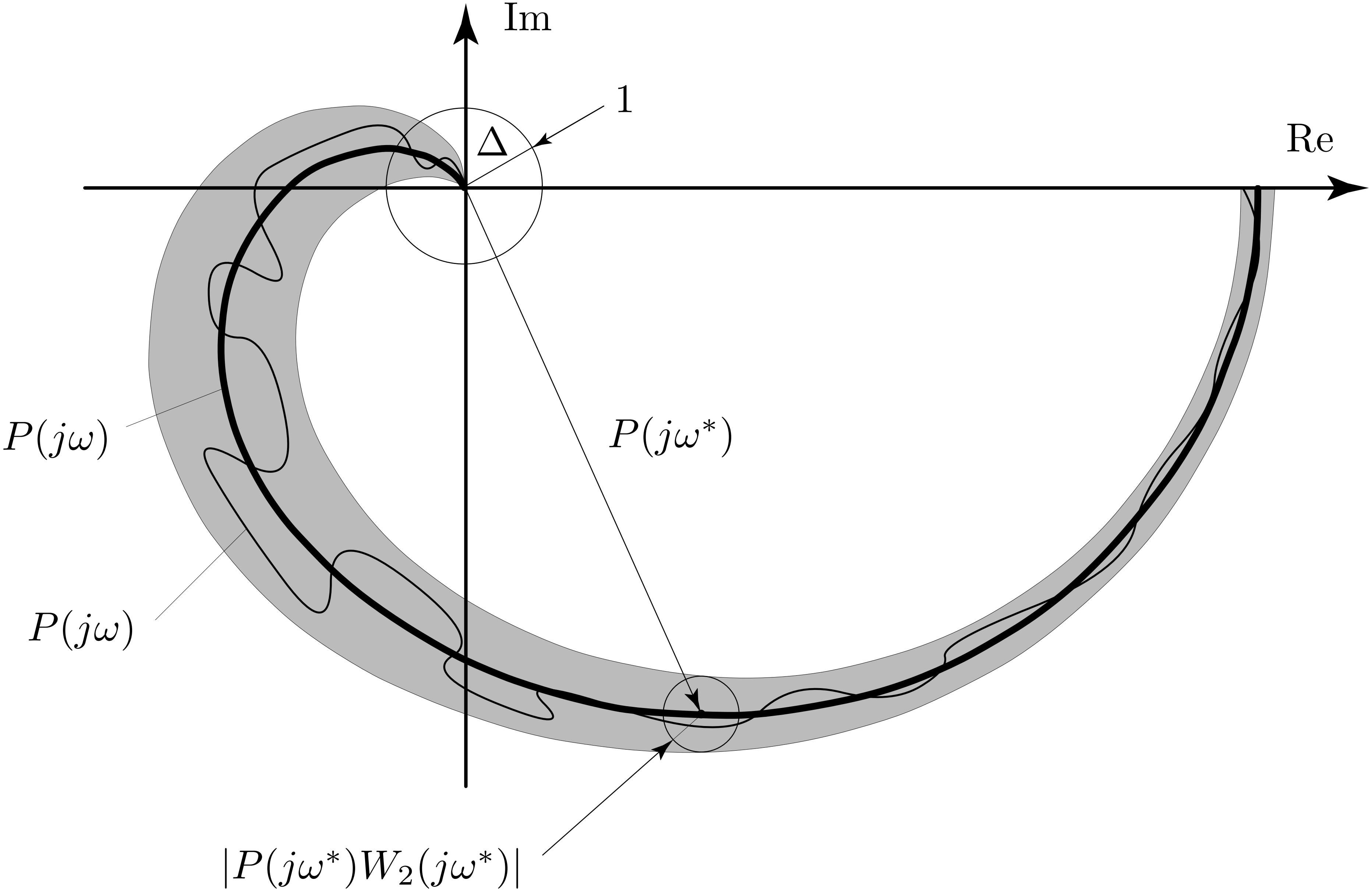
Uncertainties must be modelled; most simple approach

$$\mathcal{S} = \{P(s) \cdot (1 + \Delta(s) \cdot W_2(s)) \ \mid\ ||\Delta(s)|| \leq 1,\ \arg\{\Delta(s)\} \in [-\pi, \pi]\} \quad (18)$$

Underlying assumptions:

- There is a *linear* and *time-invariant* model $P_{\mathrm{t}}(s)$ which is not known but which exactly represents the system to be modeled.
- This "true" model is known to be included in a set $\mathcal{S}$ of models that is defined by the nominal model $P(s)$, an uncertainty bound $W_2(s)$ and an "uncertainty generator" $\Delta(s)$.

- Both $P(s)$ and $W_2(s)$ are problem-specific transfer functions. They must be estimated using one of the approaches described below.
- The variable $\Delta(s)$ represents an a-priori known mathematical object that generates a well-defined set of transfer functions. As such, $\Delta(s)$ has no physical interpretation, but it is a useful tool that permits an elegant mathematical formulation.
- The "true" system $P_{\mathrm{t}}(s)$ is only assumed to be in the set $\mathcal{S}$ and to have the same number of unstable poles as $P(s)$ has.



Im
1
Re
Δ
$P(j\omega)$
$P(j\omega^*)$
$P(j\omega)$
$|P(j\omega^*)W_2(j\omega^*)|$

Robust Stability

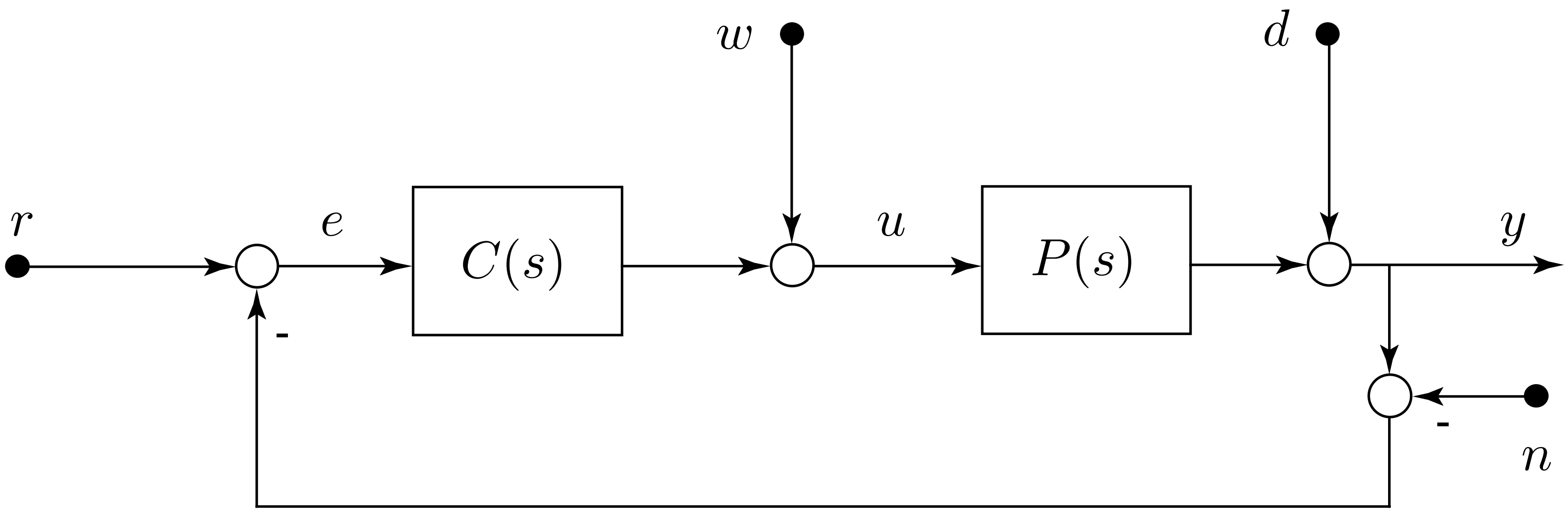
- Assume the nominal plant $P(s)$ and controller $C(s)$ yield an asymptotically stable closed-loop system $T(s)$.
- Assume the number of unstable poles of the "true" plant $P_{\mathrm{t}}(s)$ is the same as the number of unstable poles of $P(s)$.
- Then the "true" closed-loop system will be stable if the additional condition

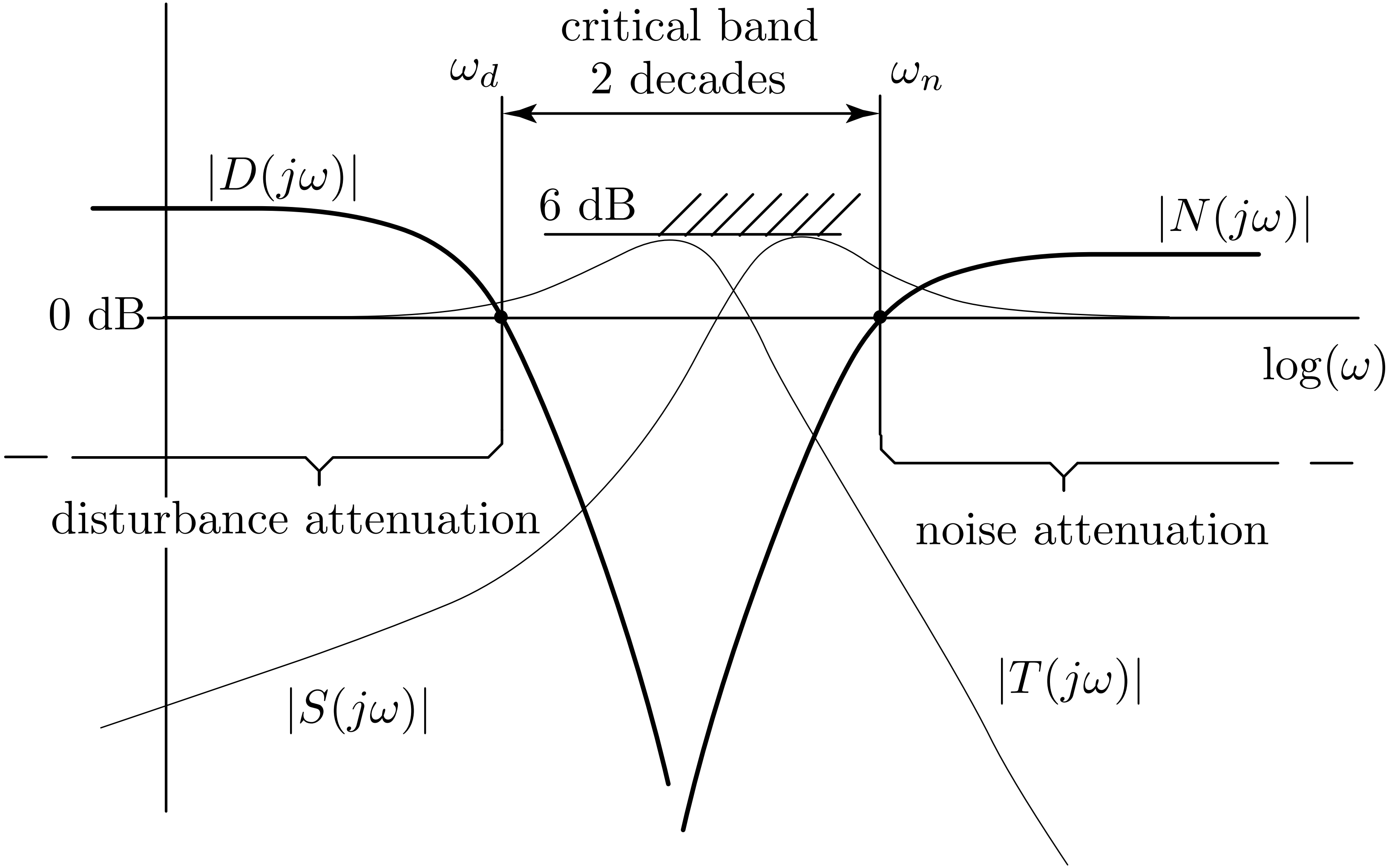
$$||T(s) \cdot W_2(s)||_\infty < 1 \tag{19}$$

is satisfied.

Advanced Loop Shaping

Standard Feedback Structure




critical band
2 decades
ω_d
ω_n
$|D(j\omega)|$
6 dB
$|N(j\omega)|$
0 dB
$\log(\omega)$
disturbance attenuation
noise attenuation
$|T(j\omega)|$
$|S(j\omega)|$

Pro Memoria: Constraints on Closed-Loop Systems

1 The disturbance d must be a low frequency signal whose spectrum is bounded by $\omega < \omega_d$. The noise n must be a high frequency signal whose spectrum is limited to frequencies $\omega > \omega_n$. A clear separation $\omega_d << \omega_n$ must be present.

2 The frequency ω_n imposes a first limit on the achievable cross-over frequency. Even if no other limitations are present, the condition $\omega_c < 0.1 \cdot \omega_n$ must be satisfied to avoid an unwanted amplification of the noise signal. Similarly, the inequality $\omega_d < 0.1 \cdot \omega_c$ must be satisfied to avoid an unwanted amplification of the disturbance signal.

3 The plant uncertainty limits the maximum bandwidth, i.e., the loop gain $|L(j\,\omega)|$ must be almost zero for all frequencies $\omega > \omega_2$ where ω_2 is the frequency where the uncertainty $|W_2(j\,\omega)|$ reaches the value 1.

4 Non-minimum phase zeros limit the achievable bandwidth. In the case of a single real non-minimum phase zero ζ^+, the cross-over frequency ω_c must be (substantially) smaller than ζ^+. Time delays $e^{-s\,T}$ have the same effect as non-minimum phase zeros ($\zeta^+ \approx 1/T$).

5 Unstable poles imply the opposite constraints, i.e., a system having one unstable real pole π^+ can only be stabilized if the open-loop transfer function has a cross-over frequency that is (substantially) larger than π^+.

6 Strong stabilization, which avoids the dangerous situation of having an unstable controller $C(s)$, cannot be achieved in all cases. Similarly, there are plants for which the achievable controller performance in terms of disturbance attenuation is limited. This is always the case when the plant features an unfavorable pole/zero pattern in the right-half complex plane.

Specifications for Feedback Systems

Specifications based on second-order system approximations discussed in RT I. Approximations:

$$\omega_c = \frac{1.7}{t_{90}} \tag{20}$$

$$\varphi = 71^\circ - 117^\circ \cdot \hat{\epsilon} \tag{21}$$

Only valid for "simple" loop gains $L(s)$! Caution when non-minimum phase zeros or unstable poles are present.

Advanced specs defined in the frequency domain. Ideas shown below applicable to MIMO systems as well.

Peaking Limitations

The output of the standard system can be expressed as

$$Y(j\omega) = S(j\omega) \cdot D(j\omega) + T(j\omega) \cdot N(j\omega) \tag{22}$$

Accordingly, as small sensitivity $S(j\omega)$ guarantees a good disturbance rejection behavior (and a good reference tracking behavior) and a small complementary sensitivity $T(j\omega)$ guarantees a good noise rejection behavior (and a good robustness against modeling errors).

However, it is not possible for $|S(j\omega)|$ and $|T(j\omega)|$ to be small at the same frequency ω. Moreover, at frequencies close to the cross-over frequency ω_c, *both* sensitivity and complementary sensitivity are likely to have a magnitude larger than 1. This peaking must be limited, otherwise disturbance *and* noise amplification occur.

A meaningful set of specifications, therefore, can have the form

$$||S||_\infty < S_{\max},\ ||T||_\infty < T_{\max}, \quad S_{\max}, T_{\max} > 1 \tag{23}$$

It is straightforward to derive the following two conditions for the loop gain $L(j\omega)$ that are equivalent to the two conditions (23). For the sensitivity, the following condition is obtained

$$||S||_\infty < S_{\max} \quad \Leftrightarrow \quad L(j\omega) \notin \{|1+z| \leq \frac{1}{S_{\max}} \mid z \in \mathbb{C}\} \tag{24}$$

For the complementary sensitivity, some intermediate steps are necessary to obtain the following condition for the loop gain $L(j\omega)$

$$||T||_\infty < T_{\max} \quad \Leftrightarrow \quad L(j\omega) \notin \{|\frac{T_{\max}^2}{T_{\max}^2 - 1} + z| \leq \frac{T_{\max}}{T_{\max}^2 - 1} \mid z \in \mathbb{C}\} \tag{25}$$

Geometrical interpretation of the condition on $S_{\max}$ in the Nyquist plane.

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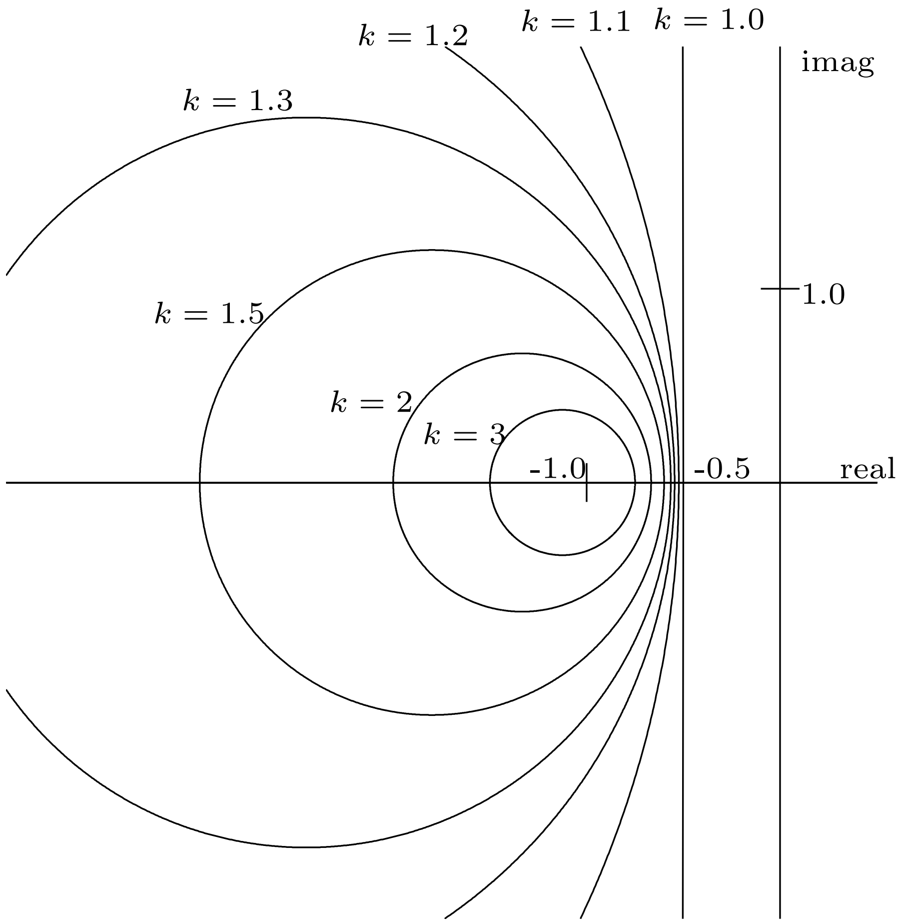

Special condition: $T_{\max} = 1$ requires $L(j\,\omega)$ to be only in the complex half plane defined by $Real\{z\} > -0.5$.

Multiplicative Specifications of the Sensitivity

Since sensitivity $S(s)$ has an important role in feedback control systems, a precise specification of that variable is used in advanced design methods. The main idea is to prescribe an upper bound for $|S(j\omega)|$ for *all* frequencies ω. Note that the phase of $S(j\omega)$ is not relevant.

The bound on $|S(j\omega)|$ can be expressed using a real rational transfer function $W_1(s)$ and the condition that must be met can be compactly stated as follows

$$||S(s) \cdot W_1(s)||_\infty < 1 \tag{26}$$

This condition can be interpreted in the Nyquist plane. In fact, the condition (26) is satisfied if for *each* frequency ω the condition

$$|S(j\omega)| \cdot |W_1(j\omega)| < 1 \tag{27}$$

is satisfied.

Since this condition is equivalent to the condition

$$|W_1(j\omega)| < |1 + L(j\omega)| \tag{28}$$

the interpretation in the Nyquist plane is that for each frequency ω the return difference should have a distance from the critical point that that is larger than $|W_1(j\omega)|$.

Geometric interpretation:

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A controller $C(s)$ that yields a closed loop system that satisfies the condition (26) for the nominal plant $P(s)$ satisfyies the *nominal performance condition*. If a controller $C(s)$ satisfies the combined condition

$$\xi(C(s)) = \| \, |W_1(s) \cdot S(s)| + |W_2(s) \cdot T(s)| \, \|_\infty < 1 \tag{29}$$

then this controller satisfies the *robust performance condition*. Such a controller will guarantee that the sensitivities of *all* plants in the set $\mathcal{S}$ defined by (18) satisfy the condition (26).

Again, the condition (29) can be best understood by analyzing it for each fixed frequency ω

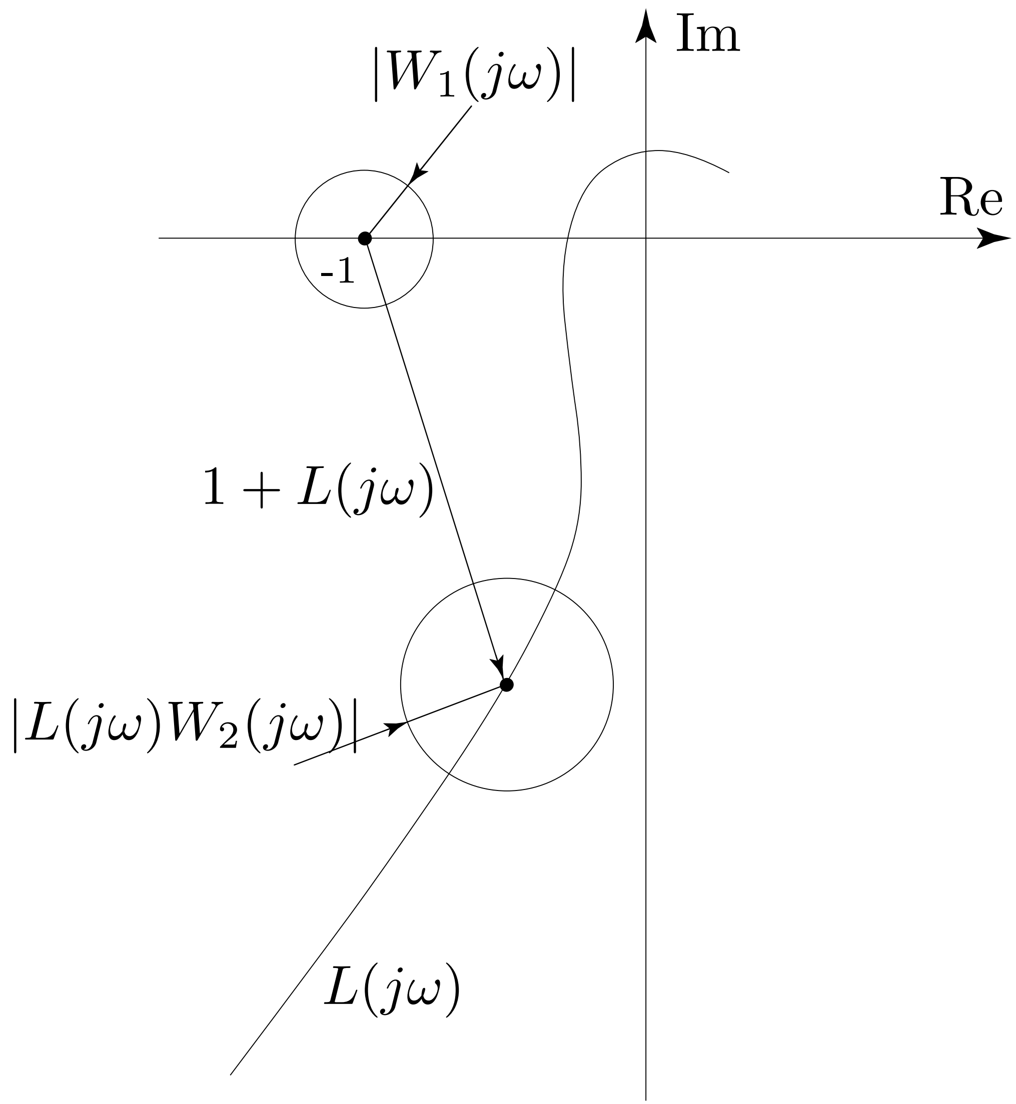
$$|W_1(j\omega) \cdot S(j\omega)| + |W_2(j\omega) \cdot T(j\omega)| < 1 \tag{30}$$

Inserting the definitions of $S(s)$ and $T(s)$, this inequality can be transformed to

$$|W_1(j\omega)| + |W_2(j\omega) \cdot L(j\omega)| < |1 + L(j\omega)| \tag{31}$$

Therefore, as illustrated in the following figure, the robust performance condition requires that for each frequency ω, the disk with radius $|W_1(j\omega)|$ centered at the critical point -1 and the disk with radius $|W_2(j\omega) \cdot L(j\omega)|$ centered at the nominal loop gain $L(j\omega)$ do not intersect at any frequency ω.

This can be interpreted as an extension of the robust stability theorem that simply required that the latter disk not include the point -1. This point is now a circle with frequency-dependent radius, such that the constraint (29) is more restricting than the robust stability constraint and, hence, guarantees sufficient performance in all possible cases.



Interpretation of the robust performance condition for one frequency ω in the Nyquist plane.

Unfortunately, even if a solution to the robust performance problem exists, there is no known procedure that permits one to compute the $C(s)$ that minimizes the scalar ξ for general $P(s)$, $W_1(s)$ and $W_2(s)$. The main problem is that the expression (29) is a nonlinear function of the controller $C(s)$. Systematic methods exist that permit to find suboptimal controllers $C(s)$. These approaches will be introduced in the second part of this course.

Since $L(s)$ depends linearly on $C(s)$, the problem oft finding a suitable controller $C(s)$ would be much easier to solve if the condition (29) could be re-formulated as a constraint on the open-loop gain $L(s)$. For very low and very high frequencies such approximation can be found as shown below. The key to that simplification is that at these frequencies the magnitude of the loop gain $|L(j\omega)|$ must be very large or very small, respectively.

In the first case ($\omega < 0.1 \cdot \omega_c \Rightarrow |L(j\omega)| >> 1$), the condition (29) is satisfied if

$$|L(j\omega)| > \frac{|W_1(j\omega)|}{1 - |W_2(j\omega)|} \tag{32}$$

This inequality can be interpreted in the following way:

- If the plant has no uncertainty, i.e., if $W_2(s) = 0$, the loop gain $|L(j\omega)|$ must be larger than $|W_1(j\omega)|$ to make sure that $1 > |W_1(j\omega) \cdot S(j\omega)| \approx |W_1(j\omega)|/|L(j\omega)|$.

- If the plant is uncertain, the loop gain must be larger in order to compensate for that uncertainty. Note that a solution exists only if $|W_2(j\omega)| < 1$ and, in practice, the inequality $|W_2(j\omega)| << 1$ must be satisfied. In other words, a good performance of the closed-loop system may only be expected if the plant model is precise.

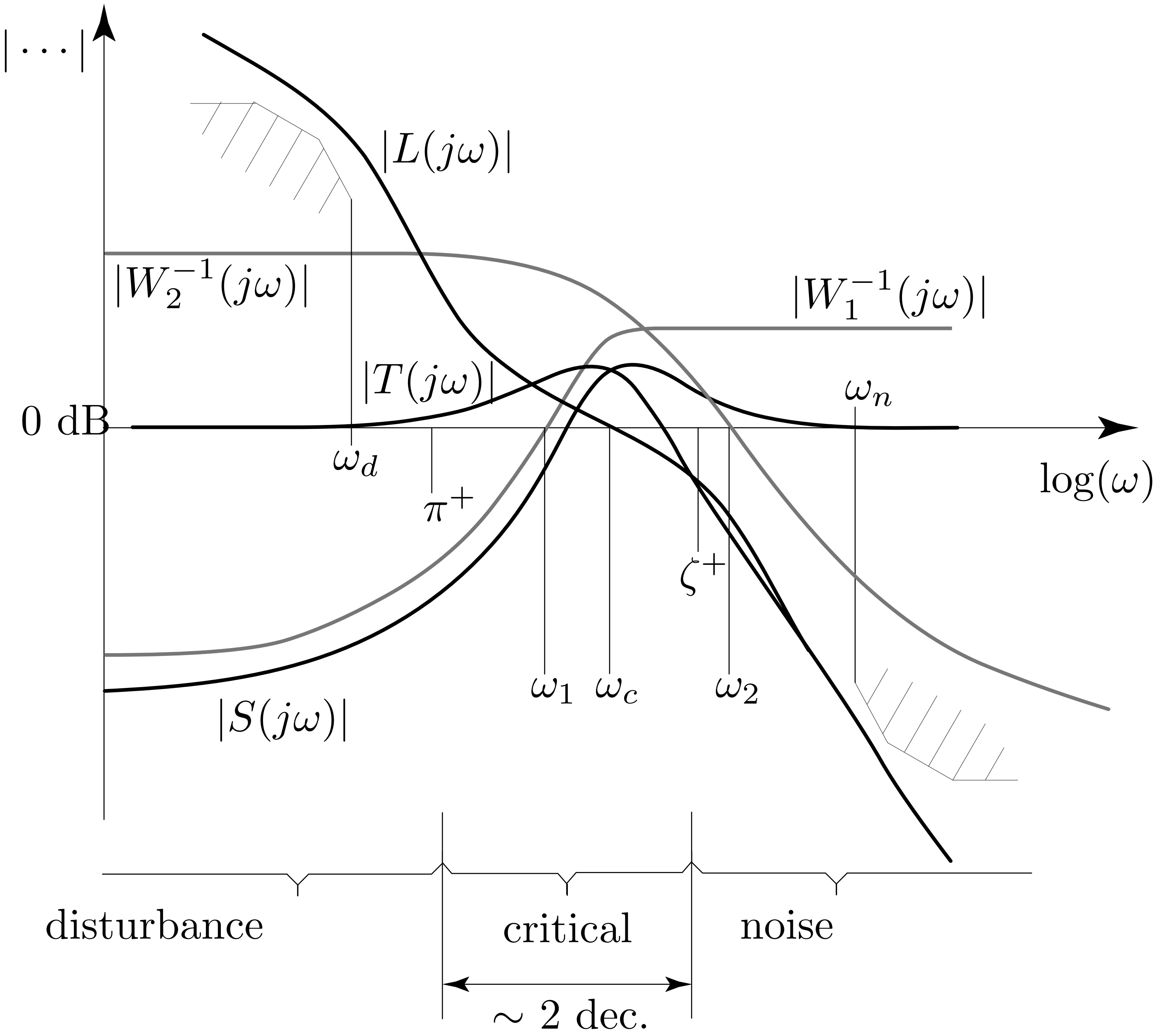
At very high frequencies ($\omega > 10 \cdot \omega_c \Rightarrow |L(j\omega)| << 1$), the same analysis yields the condition

$$|L(j\omega)| < \frac{1 - |W_1(j\omega)|}{|W_2(j\omega)|} \tag{33}$$

and the interpretation of this condition is that:

- If no performance conditions are imposed, i.e., if $W_1(s) = 0$, the loop gain $|L(j\omega)|$ must be smaller than $1/|W_2(j\omega)|$ to make sure that the robust stability condition is satisfied.

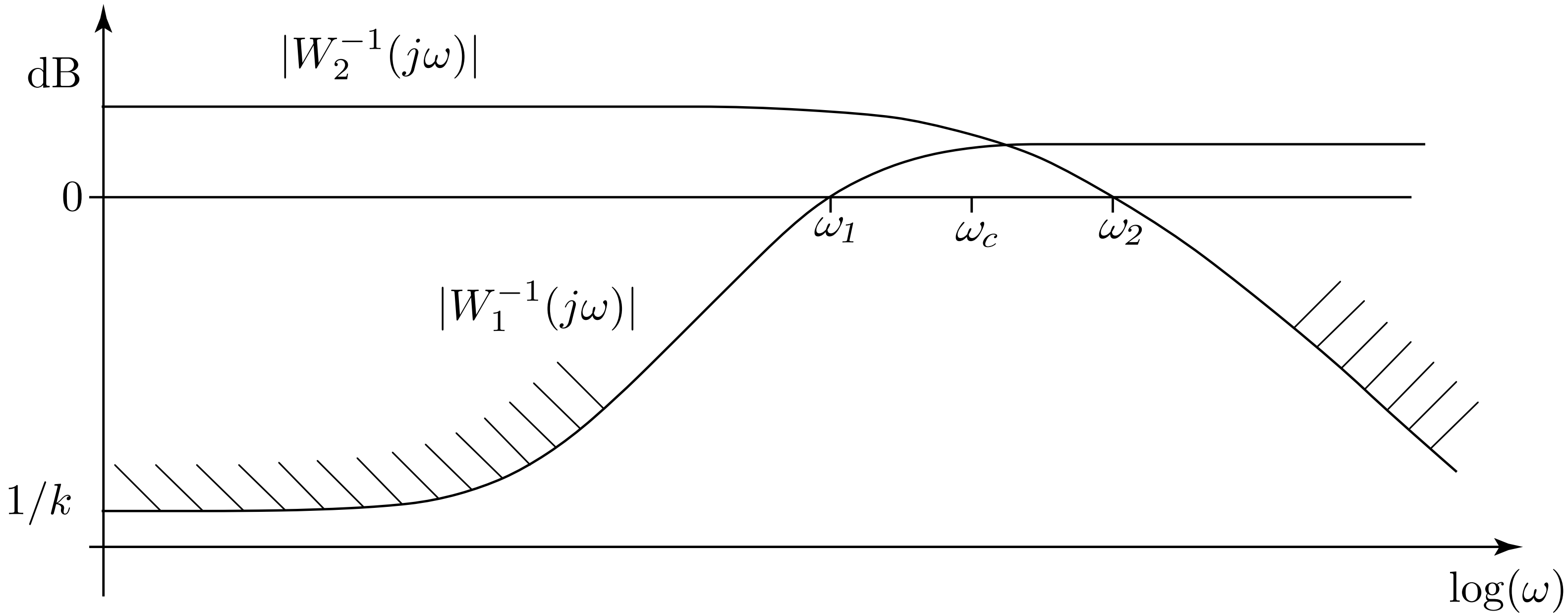
- If mere stability is not enough but some performance level is required as well, the loop gain must roll off even faster. Of course, $|W_1(j\omega)|$ may not be larger than 1 at these frequencies, i.e., some peaking of the sensitivity $S(s)$ is acceptable.



$|\cdots|$
$|L(j\omega)|$
$|W_2^{-1}(j\omega)|$
$|W_1^{-1}(j\omega)|$
$|T(j\omega)|$
ω_n
0 dB
ω_d
π^+
$\log(\omega)$
ζ^+
ω_1
ω_c
ω_2
$|S(j\omega)|$
disturbance
critical
noise
~ 2 dec.

Compatibility Conditions

Of course, the specification $W_1(s)$ must satisfy the constraints discussed above. The frequency ω_1 must be chosen to be compatible with the plant's non-minimum phase zeros and unstable poles and its uncertainty bounds. As mentioned, around ω_c *both* $|S(j\omega)|$ and $|T(j\omega)|$ will be larger than 1. Accordingly, this inequality may not be chosen too tightly.



Connection to MIMO Systems

One of the main reasons why the $W_1(s)$, $W_2(s)$ formalism is so widely used is that it can be easily generalized to MIMO systems. In this case, all assertions related to the maximum magnitude of a transfer function must be replaced by constraints on the maximum or minimum *singular value* of a transfer function matrix. These concepts are discussed in the second part of this class.

Time-Domain Formulations of Transfer Functions

Plant

$$\frac{d}{dt}x(t) = A \cdot x(t) + b \cdot u(t), \quad y(t) = c \cdot x(t) + d(t) \tag{34}$$

Controller

$$\frac{d}{dt}z(t) = F \cdot z(t) + g \cdot e(t), \quad u(t) = h \cdot z(t) \tag{35}$$

Remark: Equations shown for SISO systems (b, c, g, and h are vectors); the extension to MIMO systems (B, C, G, and H are vectors) is straightforward.

Open-loop gain $L(s)$:

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A & b \cdot h \\ 0 & F \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} 0 \\ g \end{bmatrix} \cdot e(t), \tag{36}$$

$$y(t) = \begin{bmatrix} c & 0 \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}$$

Return difference $1 + L(s)$:

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A & b \cdot h \\ 0 & F \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} 0 \\ g \end{bmatrix} \cdot e(t), \tag{37}$$

$$y(t) = \begin{bmatrix} c & 0 \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + [1] \cdot e(t)$$

Complementary sensitivity $T(s)$:

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A & b \cdot h \\ -g \cdot c & F \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} 0 \\ g \end{bmatrix} \cdot r(t), \tag{38}$$

$$y(t) = \begin{bmatrix} c & 0 \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}$$

Sensitivity $S(s)$:

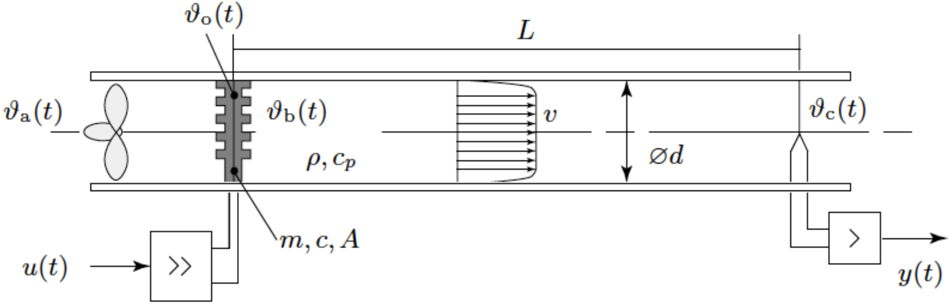
$$\frac{d}{dt}\begin{bmatrix} x(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A & b \cdot h \\ -g \cdot c & F \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -g \end{bmatrix} \cdot d(t), \tag{39}$$

$$y(t) = \begin{bmatrix} c & 0 \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + [1] \cdot d(t) \tag{40}$$

Fallstudie “Heissluftgebläse”

Modellierung

1. System/Umwelt abgrenzen (Ursache/Wirkung festlegen)



Schematische Darstellung des Systems

1. System/Umwelt abgrenzen (Ursache/Wirkung festlegen)

Die Ursache/Wirkungs-Kette ist im Bild 2 dargestellt.

Der Verstärker und das Thermoelement werden als statische und lineare Elemente modelliert.

Die Strömung im Rohr sei thermisch perfekt isoliert, ideal kolbenförmig und die Strömungsgeschwindigkeit ist gegenüber der Schallgeschwindigkeit sehr klein.

Die Drehzahl des Ventilators variiert zeitlich sehr langsam. Die relevanten Speicher sind demzufolge im Block 2 (Wärmespeicherung im Heizregister) und im Block 5 (Transportverzögerung).

2. Schritt: Bilanzen aufstellen

Block 2:

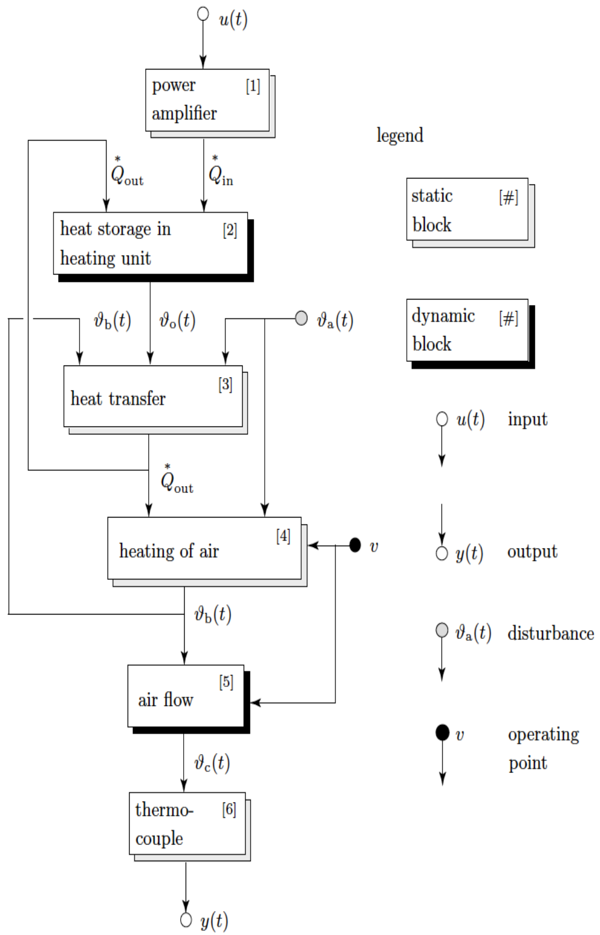
$$m \cdot c \cdot \frac{\mathrm{d}}{\mathrm{d}t} \vartheta_{\mathrm{o}}(t) = \overset{*}{Q}_{\mathrm{in}}(t) - \overset{*}{Q}_{\mathrm{out}}(t)$$

Das Heizregister hat überall die gleiche Temperatur (System mit konzentrierten Parametern). Gleichung (1) folgt aus dem 1. Hauptsatz für das Heizregister, wobei m und c die Masse und die spezifische Wärmekapazität des Heizregisters sind.

Block 5:

$$\vartheta_{\mathrm{c}}(t) = \vartheta_{\mathrm{b}}(t - T), \quad T = \frac{L}{v}$$

Eine perfekte Kolbenströmung ergibt eine reine Zeitverzögerung, wobei die Totzeit T konstant ist (da v konstant angenommen ist).



u(t)
power amplifier [1]
Q̊_in
Q̊_out
heat storage in heating unit [2]
ϑ_b(t)
ϑ_o(t)
ϑ_a(t)
heat transfer [3]
Q̊_out
heating of air [4]
v
ϑ_b(t)
air flow [5]
ϑ_c(t)
thermo-couple [6]
y(t)
legend
static block [#]
dynamic block [#]
u(t) input
y(t) output
ϑ_a(t) disturbance
v operating point

3. Schritt: Physikalische Phänomene

Block 1:

$$\overset{*}{Q}_{\mathrm{in}}(t) = k_u \cdot u(t), \quad 0 \leq u(t) \leq u_{\mathrm{max}}$$

Block 3:

$$\overset{*}{Q}_{\mathrm{out}}(t) = \alpha \cdot A \cdot (\vartheta_{\mathrm{o}}(t) - \frac{\vartheta_{\mathrm{a}}(t) + \vartheta_{\mathrm{b}}(t)}{2})$$

Block 4:

$$\vartheta_{\mathrm{b}}(t) = \left(v \cdot \rho \cdot \frac{\pi \cdot d^2}{4} \cdot c_p \right)^{-1} \cdot \overset{*}{Q}_{\mathrm{out}}(t) + \vartheta_{\mathrm{a}}(t)$$

Block 6:

$$y(t) = k_y \cdot \vartheta_{\mathrm{c}}(t)$$

4. Schritt: Vereinfachen

Zwischen den Blöcken 3 und 4 ergibt sich eine algebraische Schleife, d.h. die Temperatur ist definiert durch eine implizite Gleichung

$$\vartheta_b = f(\vartheta_a, \vartheta_0, v, \ldots, \vartheta_b).$$

Ein möglicher Ausweg aus dieser Situation besteht darin, solche Schleifen durch dynamische Zwischensysteme zu entkoppeln. Der Nachteil dieser Methode ist aber, dass dadurch die Systemordnung erhöht wird und man im Allgemeinen ein "steifes" System erhält. Besser ist es, wenn man diese Schleifen durch algebraische Manipulationen auflösen kann (was hier der Fall ist, siehe unten).

5. Schritt: Parametrieren
……

6. Schritt: Validieren
………

4. Schritt: Vereinfachen (Linearisieren und in Regelungsform bringen)

Die Dynamik des Heizrohrs wurde bereits linear formuliert. Will man aus den Zeitbereichs-Gleichungen eine Übertragungsfunktion herleiten, kann dies nicht automatisch erfolgen, da ein Totzeitelement und eine algebraische Schleife vorkommen. Nach diversen Schritten erhält man das Resultat

$$\begin{aligned} Y(s) = \mathrm{e}^{-T\cdot s} \cdot \frac{k_y \cdot k_2 \cdot k_u}{m \cdot c \cdot s + k_1} \cdot U(s) \\ + k_y \cdot \mathrm{e}^{-T\cdot s} \cdot \left(\frac{k_2 \cdot k_1}{m \cdot c \cdot s + k_1} + (1 - k_2) \right) \cdot \vartheta_{\mathrm{a}}(s) \end{aligned}$$

wobei die neuen Konstanten definiert sind durch

$$k_1 = \frac{\alpha \cdot A \cdot v \cdot \rho \cdot \pi \cdot d^2 \cdot c_p}{v \cdot \rho \cdot \pi \cdot d^2 \cdot c_p + 2 \cdot \alpha \cdot A}, \quad k_2 = \frac{4 \cdot \alpha \cdot A}{v \cdot \rho \cdot \pi \cdot d^2 \cdot c_p + 2 \cdot \alpha \cdot A}$$

4. Schritt: Vereinfachen (Linearisieren und in Regelungsform bringen)

Für den späteren Regelungsentwurf ist nur der Kanal u -> y interessant.

Diese Übertragungsfunktion kann noch kompakter geschrieben werden

$$Y(s) = P(s) \cdot U(s) = \frac{k}{\tau \cdot s + 1} \cdot \mathrm{e}^{-T \cdot s} \cdot U(s)$$

wobei die neuen Streckenparameter definiert sind durch

$$k = \frac{4 \cdot k_y \cdot k_u}{v \cdot \rho \cdot \pi \cdot d^2 \cdot c_p}, \quad \tau = \frac{m \cdot c}{k_1}$$

Parameter	Nominal Value	Interval	Unit
d	0.1	± 0.001	m
ρ	1.13	± 0.03	kg/m^3
v	4	± 1	m/s
c_p	1005	± 20	J/kg K
m	0.005	± 0.0001	kg
c	450	± 10	J/kg K
L	1	± 0.005	m
k_y	1	± 0.02	V/K
k_u	100	± 2	W/V
A	0.08	± 0.001	m^2
α	50	± 10	$W/m^2\,K$

Klassifizierung der vorkommenden Parameter

Abschätzung der Modellunsicherheiten

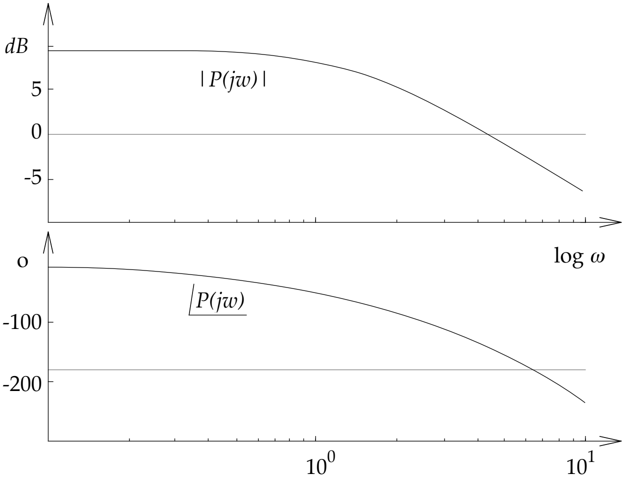
Es werden die folgenden Modellunsicherheiten betrachtet:

1 Parameterunsicherheiten, die wegen den effektiv nicht genau bekannten physikalischen Parametern entstehen.

2 Parameterunsicherheiten, die wegen variierenden Betriebspunkten entstehen.

3 Parameterunsicherheiten, die wegen den gewählten Approximationen und wegen den prinzipiellen Modellierungsfehlern entstehen.

$$Y(s) = P(s) \cdot U(s) = \frac{k}{\tau \cdot s + 1} \cdot \mathrm{e}^{-T \cdot s} \cdot U(s)$$



$k_{\text{nom}} = 3.00$, $T_{\text{nom}} = 0.27$ s, $\tau_{\text{nom}} = 0.63$ s.

Die Nyquistkurven aller Strecken der Familie

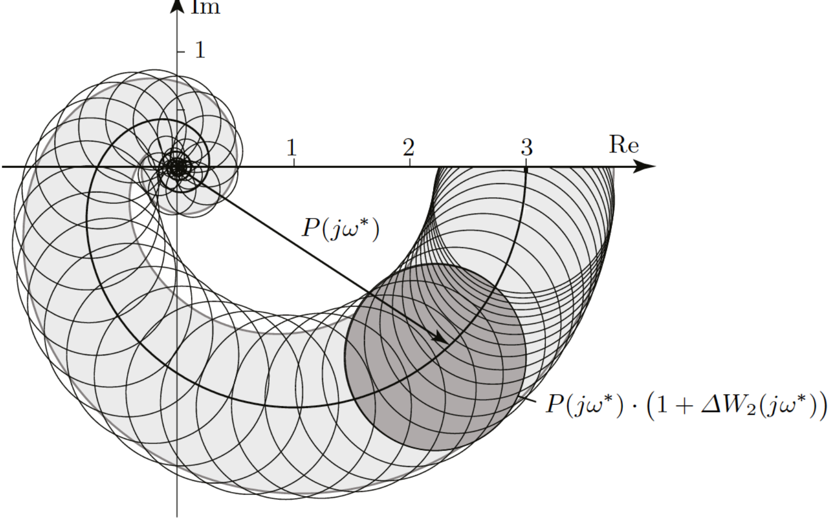
$$\Theta = \left\{ \frac{k \cdot e^{-\delta s}}{\tau \cdot s + 1} \quad \middle| \quad k \in [k_{min}, k_{\max}], \tau \in [\tau_{min}, \tau_{\max}], \delta \in [\delta_{min}, \delta_{\max}] \right\}$$

sind im Bild 5 dargestellt (hellgraues Gebiet). Die dick ausgezogene Nyquistkurve stellt den Nominalfall dar um welchen jeweils die durch die Gewichtung

$$W_2(s) = 0.26 \cdot \frac{0.65 \cdot s + 1}{0.24 \cdot 0.65 \cdot s + 1}$$

definierten Kreise angegeben sind, wobei der übliche multiplikative Ansatz zur Beschreibung der Unsicherheitsschranken benutzt wird

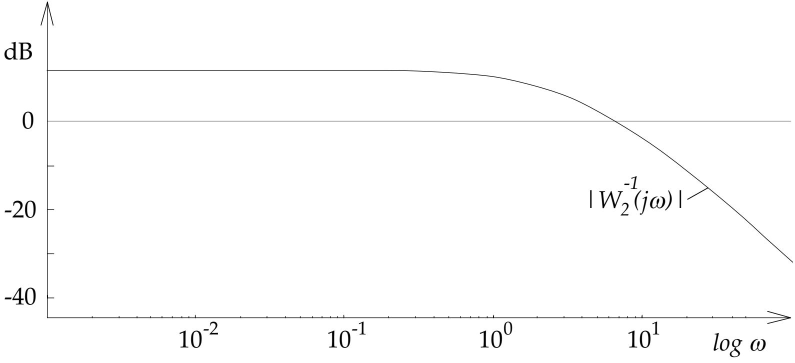
$$P_t(s) \in \left\{ P(s)\left(1 + W_2(s)\Delta(s)\right) \mid \ |\Delta(j\omega)| \leq 1 \right\}$$



Nyquistkurven der nominalen und aller möglichen Frequenzgänge

Damit das geschlossene System gegen Sensorrauschen unempfindlicher ist, wird die Unsicherheit der Strecke bei hohen Frequenzen wie folgt erweitert

$$W_2(s) = 0.26 \cdot \frac{0.65 \cdot s + 1}{0.24 \cdot 0.65 \cdot s + 1} \cdot (0.06 \cdot s + 1)^2$$



Erweiterte Unsicherheitsschranke

Spezifikationen

Geschlossener Kreis, Zeitbereich:

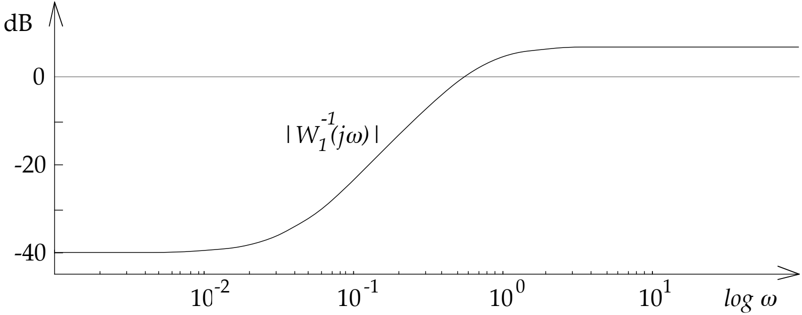
1. keine bleibende Regelabweichung
2. möglichst geringes Überschwingen
3. Anstiegszeit etwa 1.5 Sekunden

Offener Kreis, Frequenzbereich

1. Regler mit I-Anteil
2. Mind. 70 Grad Phasenreserve
3. Durchtrittsfrequenz etwa grösser als 1 rad/s

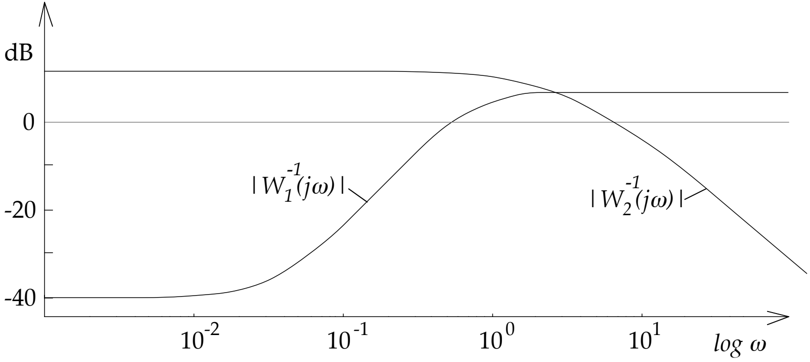
Die Gewichtung $W_1(s)$ wird dementsprechend wie folgt gewählt

$$W_1(s) = 100 \cdot \frac{(1.6 \cdot s + 1)^2}{(15 \cdot 1.6 \cdot s + 1)^2}$$



Obere Grenze der Sensitivität

Da der Regler ein I-Teil haben wird, kann $W_1(0) < \infty$ gewählt werden. Um ω_c herum muss die Sensitivität >1 werden. Zudem muss $W_1(s)$ mit der Unsicherheitsschranke $W_2(s)$ kompatibel sein, was hier der Fall ist.



Obere Schranken für die $S(j\omega)$ und $T(j\omega)$

Gemäss RT-II lautet die Bedingung für die gewünschte „robuste Regelgüte“:

$$\max_{\omega \in \mathbb{R}} \left(|S(j\,\omega) \cdot W_1(j\,\omega)| + |T(j\,\omega) \cdot W_2(j\,\omega)| \right) \leq 1$$

Leider liefert diese Bedingung nicht direkt die Anforderungen an den offenen Regelkreis. Approximativ muss bei sehr tiefen Frequenzen gelten

$$|L(j\,\omega)| > \frac{|W_1(j\,\omega)|}{1 - |W_2(j\,\omega)|}$$

und bei sehr hohen Frequenzen muss die folgende Bedingung erfüllt sein

$$|L(j\,\omega)| < \frac{1 - |W_1(j\,\omega)|}{|W_2(j\,\omega)|}$$

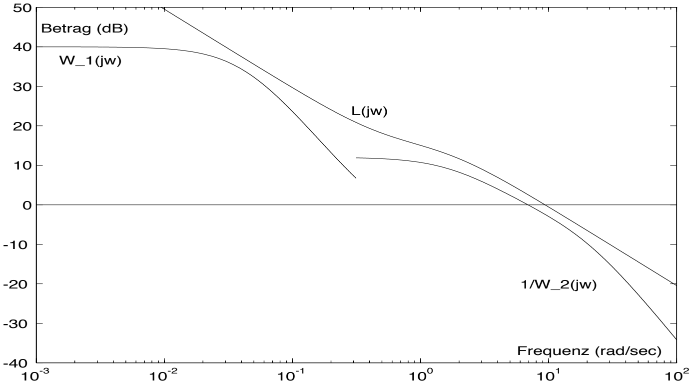
d.h. bei sehr tiefen Frequenzen muss $|L(j\omega)|$ grösser ca. 130 sein muss. Da die Unsicherheitsschranke ein Hochpass ist muss die Kreisverstärkung gegen null gehen (was unbedingt erforderlich ist).

Reglerauslegung

1. Iteration: PID Regler

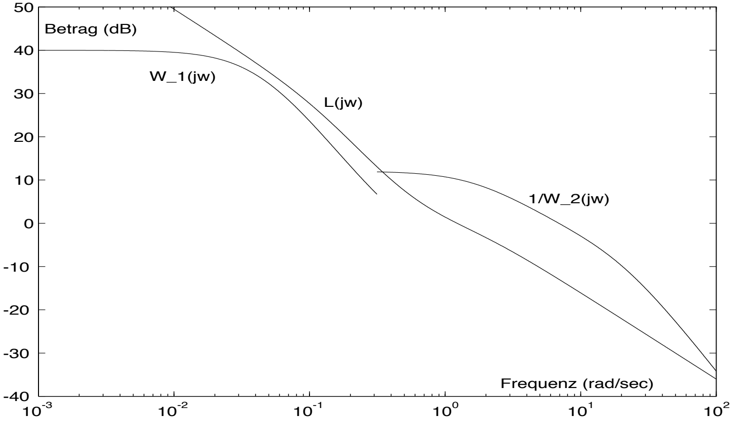
$$C_1(s) = k_\mathrm{p} \cdot \left(1 + \frac{1}{T_\mathrm{i} \cdot s}\right), \quad k_\mathrm{p} = 2,\ T_\mathrm{i} = 2\,\mathrm{s}$$

Integrationswirkung bei etwa 0.5 rad / sec beendet.



2. Iteration: Lag-Element (Absenkung des Betrags bei mittleren Frequenzen)

$$C_2(s) = k_{\mathrm{p}} \cdot \left(1 + \frac{1}{T_{\mathrm{i}} \cdot s}\right) \cdot \frac{T_{\mathrm{L}} \cdot s + 1}{\alpha_{\mathrm{L}} \cdot T_{\mathrm{L}} \cdot s + 1}, \quad T_{\mathrm{L}} = 1.3\,\mathrm{s},\ \alpha_{\mathrm{L}} = 6$$



3. Iteration: Tiefpass:

$$C_3(s) = k_\mathrm{p} \cdot \left(1 + \frac{1}{T_\mathrm{i} \cdot s}\right) \cdot \frac{T_\mathrm{L} \cdot s + 1}{\alpha_\mathrm{L} \cdot T_\mathrm{L} \cdot s + 1} \cdot \frac{1}{\tau_\mathrm{LP} \cdot s + 1}, \quad \tau_\mathrm{LP} = 0.03\,\mathrm{s}$$

Um die Phasenreserve aber auf den gewünschten Wert anzuheben, wird zum Schluss noch ein Lead 2. Ordnung in Serie geschaltet

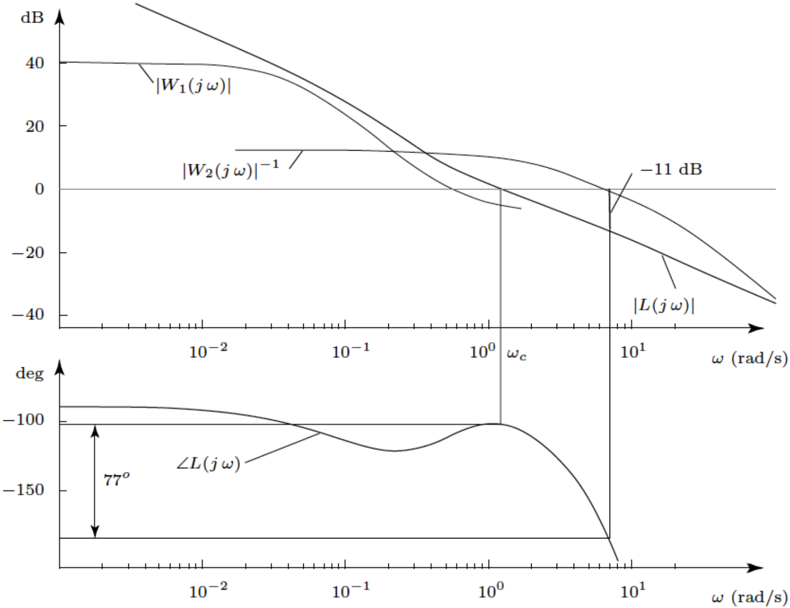
$$C(s) = k_\mathrm{p} \cdot \left(1 + \frac{1}{T_\mathrm{i} \cdot s}\right) \cdot \frac{T_\mathrm{L} \cdot s + 1}{\alpha_\mathrm{L} \cdot T_\mathrm{L} \cdot s + 1} \cdot \frac{1}{\tau_\mathrm{TP} \cdot s + 1} \cdot C_\mathrm{L2}(s)$$

wobei

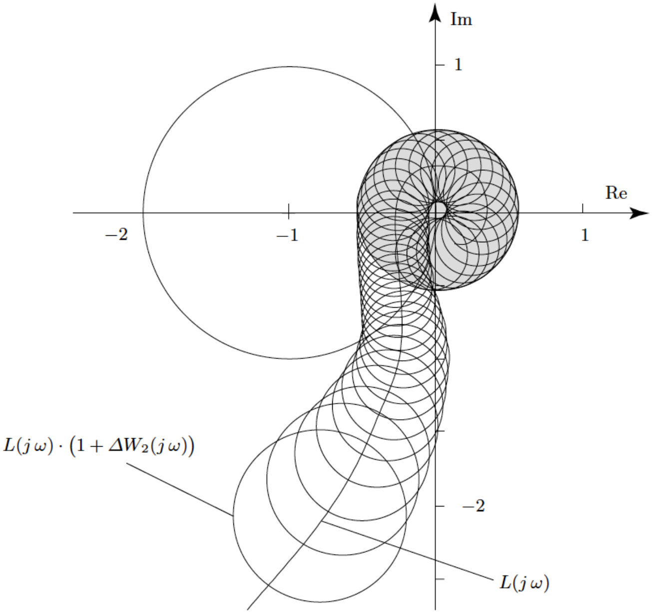
$$C_\mathrm{L2}(s) = k_\mathrm{L2} \cdot \frac{s^2 + 2 \cdot \zeta \cdot (1-\epsilon) \cdot \omega_0 \cdot s + (1-\epsilon)^2 \cdot \omega_0^2}{s^2 + 2 \cdot \zeta \cdot (1+\epsilon) \cdot \omega_0 \cdot s + (1+\epsilon)^2 \cdot \omega_0^2}$$

mit den Parametern

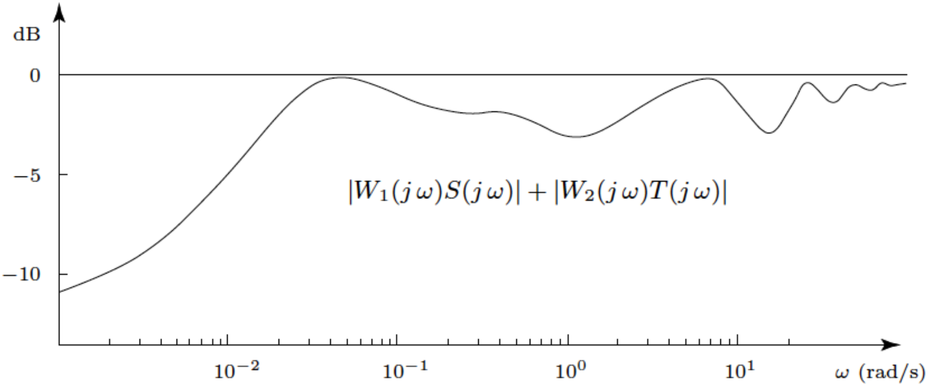
$$\zeta = 0.7,\ \omega_0 = 9\,\mathrm{rad/s},\ \epsilon = 0.17,\ k_\mathrm{L2} = \frac{(1+\epsilon)^2}{(1-\epsilon)^2} = 1.987$$



dB
40
20
0
−20
−40
$|W_1(j\,\omega)|$
$|W_2(j\,\omega)|^{-1}$
−11 dB
$|L(j\,\omega)|$
10^{-2}
10^{-1}
10^{0}
ω_c
10^{1}
ω (rad/s)
deg
−100
−150
$\angle L(j\,\omega)$
77^o
10^{-2}
10^{-1}
10^{0}
10^{1}
ω (rad/s)



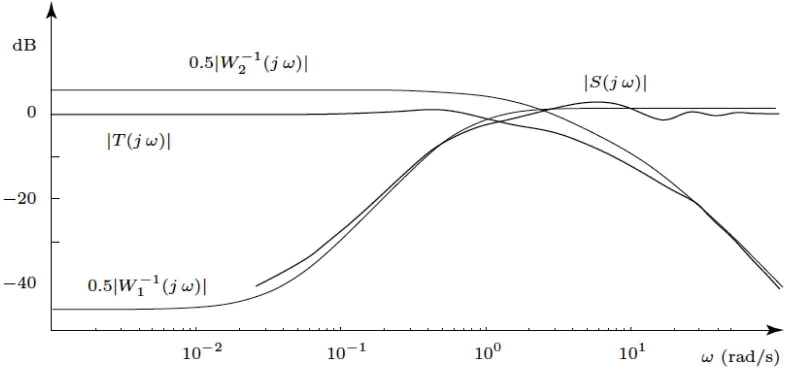
Im
1
Re
−2
−1
1
−2
$L(j\,\omega) \cdot \big(1 + \Delta W_2(j\,\omega)\big)$
$L(j\,\omega)$



"Robuste Regelgüte" des geregelten Systems

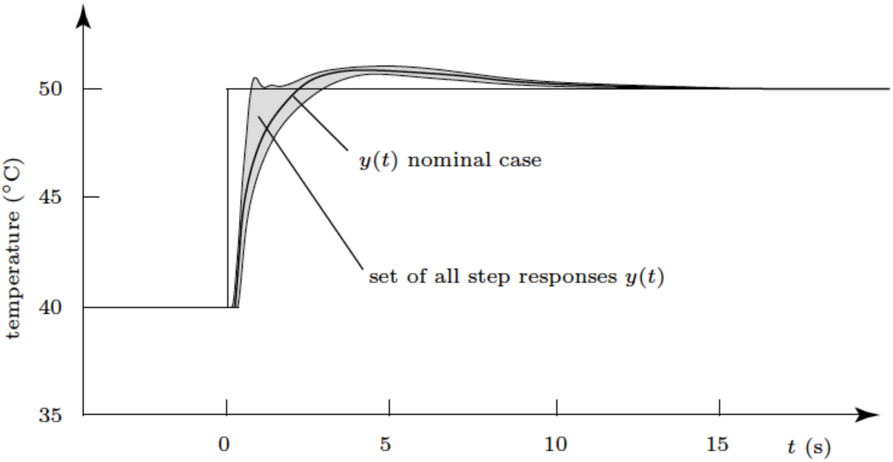
Alternatives konservatives, aber einfacher zu erfüllendes Kriterium:

$$\max_{\omega \in \mathbb{R}} \left(|S(j\,\omega) \cdot W_1(j\,\omega)| \right) \leq \frac{1}{2} \quad \text{and} \quad \max_{\omega \in \mathbb{R}} \left(|T(j\,\omega) \cdot W_2(j\,\omega)| \right) \leq \frac{1}{2}$$



Beachte: Dieses Kriterium ist *nicht* erfüllt.

Über die Qualität einer Regelung entscheidet natürlich das zeitliche Verhalten im geschlossenen Kreis.



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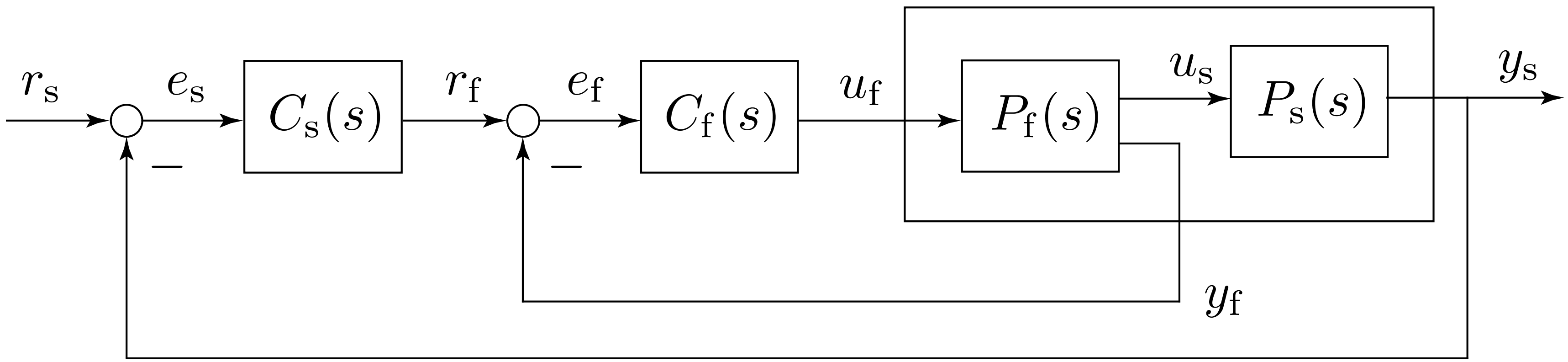
Lecture III

Cascaded Control Systems

Root-Locus Design Methods

Numerical Optimization

Cascaded Control Systems



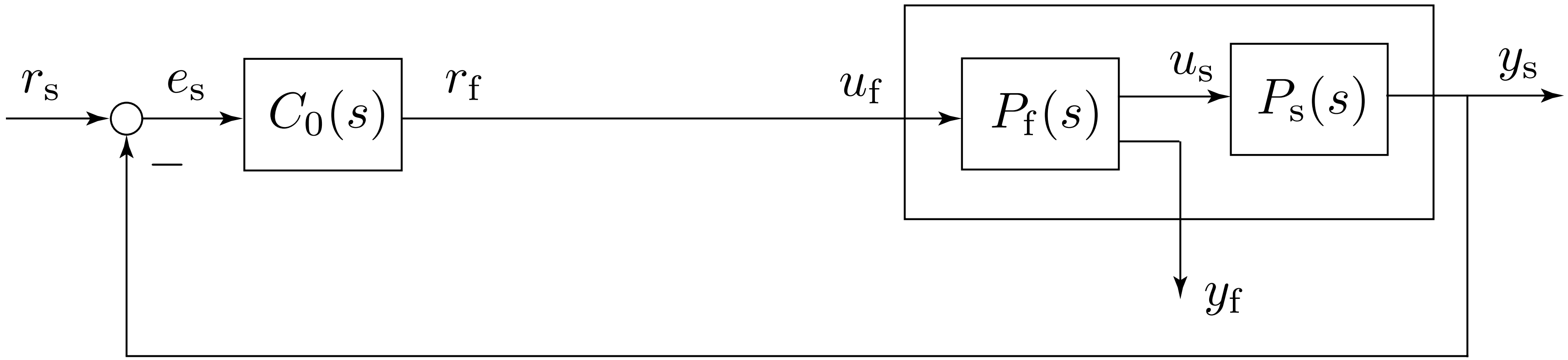
Design inner ("fast") loop without considering outer loop: main objective "speed."

Design outer ("slow") loop with inner loop closed: main objective "accuracy."

Example (delay in both measurements)

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{dt}}x(t) &= v(t) \\ \frac{\mathrm{d}}{\mathrm{dt}}v(t) &= -x(t) - v(t) + u_{\mathrm{f}}(t) \\ y_{\mathrm{s}}(t) &= x(t-T) \\ y_{\mathrm{f}}(t) &= v(t-T) \end{aligned}$$

.



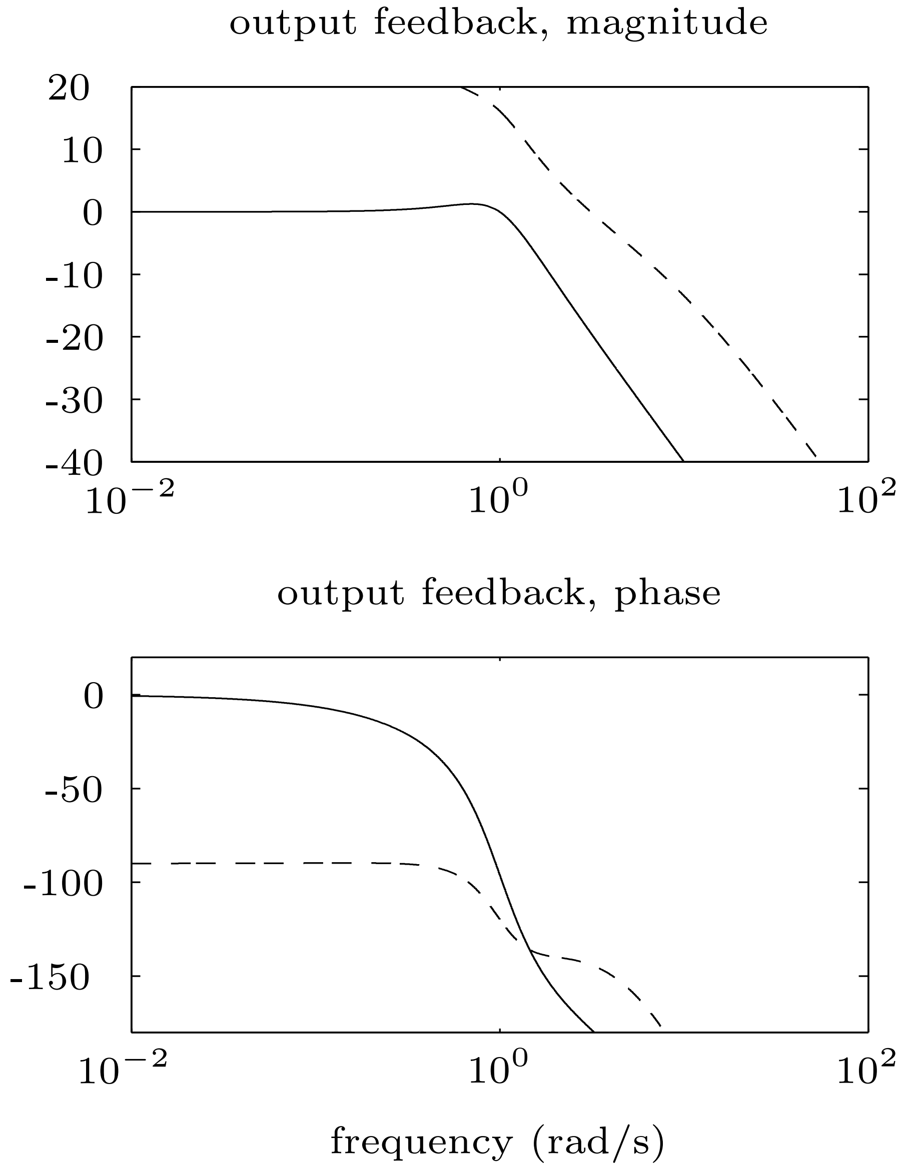
Standard (SISO) output-feedback plant transfer function $u_f \to y_s$

$$P_0(s) = \frac{1}{s^2 + s + 1} \cdot e^{-s \cdot 0.1}$$

Stabilizing PID controller (rather "aggressive")

$$C_0(s) = k_{p,0} \cdot \frac{T_{i,0} \cdot T_{d,0} \cdot s^2 + T_{i,0} \cdot s + 1}{T_{i,0} \cdot \tau_0 \cdot s^2 + T_{i,0} \cdot s}$$

with $k_{p,0} = 6.12$, $T_{i,0} = 1.24$ s, $T_{d,0} = 0.476$ s, and $\tau_0 = 0.0952$ s.


output feedback, magnitude
20
10
0
-10
-20
-30
-40
10^{-2}
10^{0}
10^{2}
output feedback, phase
0
-50
-100
-150
10^{-2}
10^{0}
10^{2}
frequency (rad/s)

Fast transfer function $u_\mathrm{f} \to y_\mathrm{f}$

$$P_\mathrm{f}(s) = \frac{s}{s^2 + s + 1} \cdot e^{-s \cdot 0.1}$$

Slow transfer function $u_\mathrm{s} \to y_\mathrm{s}$

$$P_\mathrm{s}(s) = \frac{1}{s} \cdot e^{-s \cdot 0.1}$$

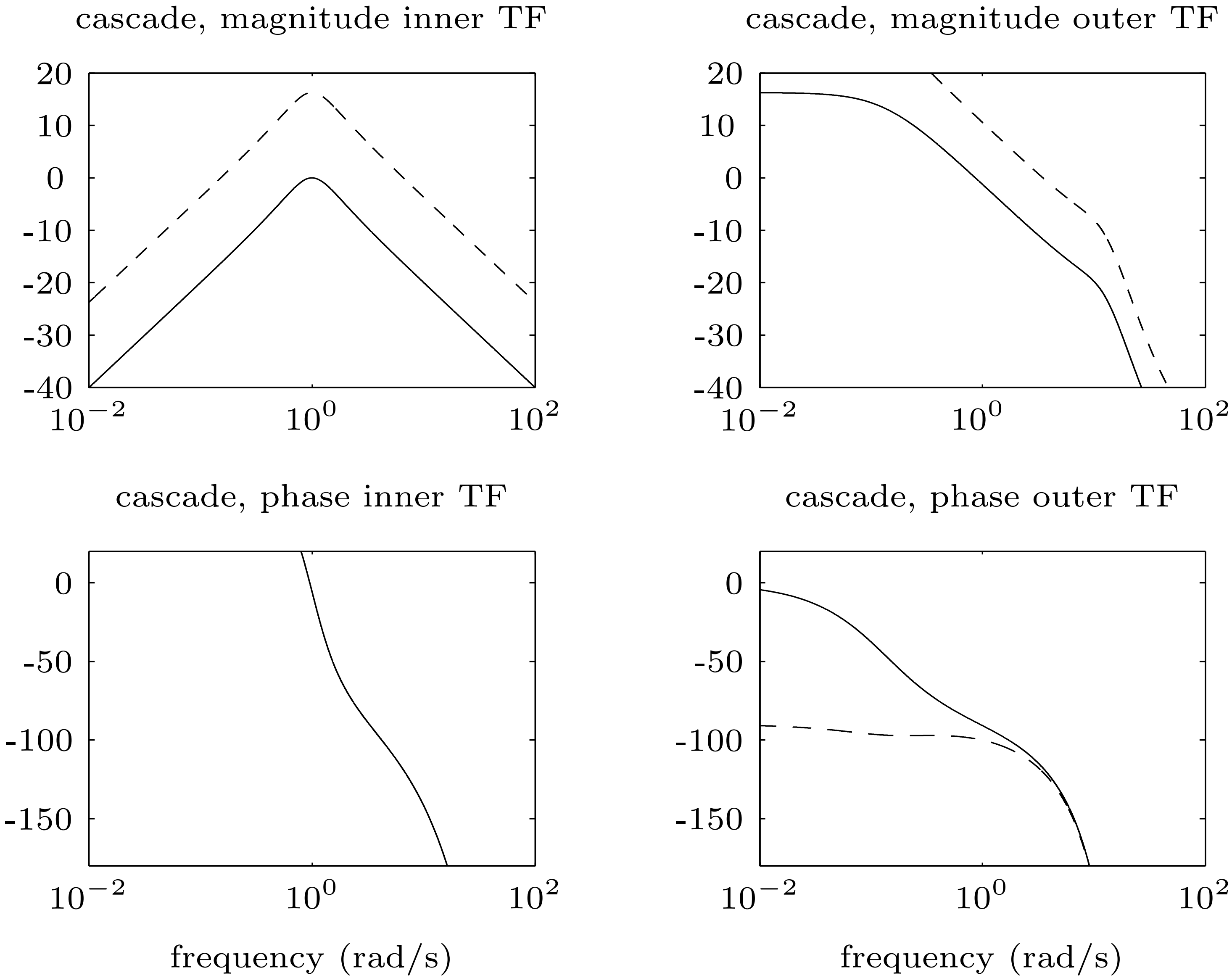
Fast (inner) controller is a simple P controller

$$C_\mathrm{f}(s) = k_\mathrm{p,f}$$

with gain $k_\mathrm{p,f} = 6.5$ (40% of the critical gain k^* of the inner loop).

System transfer function from $r_\mathrm{f} \to y_\mathrm{s}$, with the inner controller $C_\mathrm{f}(s)$ active, is given by

$$P_\mathrm{i}(s) = \frac{k_\mathrm{p,f}}{s^2 + (1 + k_\mathrm{p,f} \cdot e^{-s \cdot 0.1}) \cdot s + 1} \cdot e^{-s \cdot 0.1}$$



cascade, magnitude inner TF
cascade, magnitude outer TF
cascade, phase inner TF
cascade, phase outer TF
20
10
0
-10
-20
-30
-40
-50
-100
-150
10^{-2}
10^{0}
10^{2}
frequency (rad/s)
frequency (rad/s)

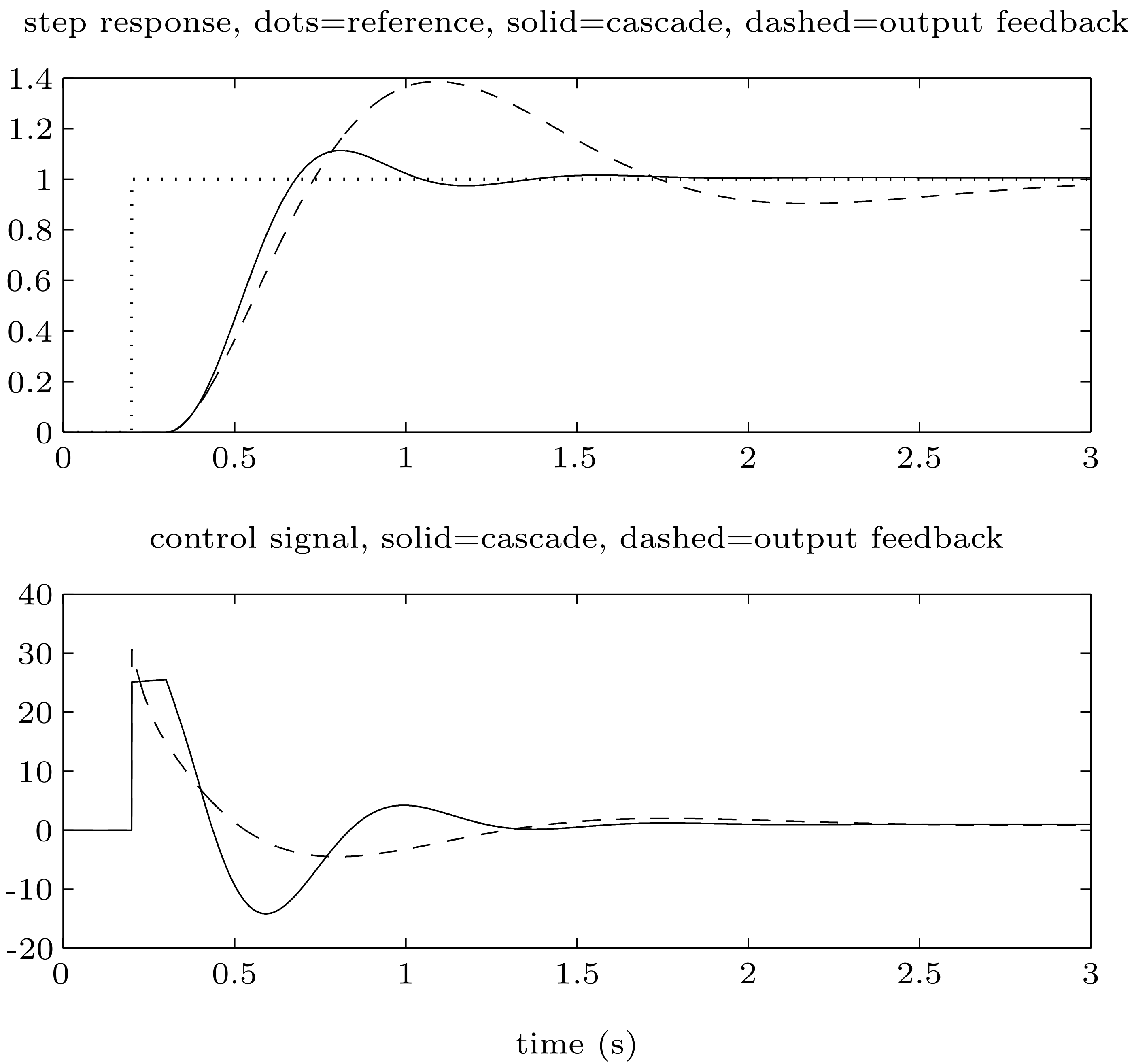
The critical frequency of the transfer function $P_\mathrm{i}(s)$ ($r_\mathrm{f} \rightarrow y_\mathrm{s}$ with the inner loop closed) is 9.0 rad/s, i.e., substantially larger than the critical frequency of the transfer function $u_\mathrm{f} \rightarrow y_\mathrm{s}$, which is only 3.3 rad/s.

Therefore, $C_\mathrm{s}(s)$ can be chosen to be more "aggressive" than $C_0(s)$

$$C_\mathrm{s}(s) = k_{\mathrm{p},s} \cdot \frac{T_\mathrm{i,s} \cdot s + 1}{T_\mathrm{i,s} \cdot s}$$

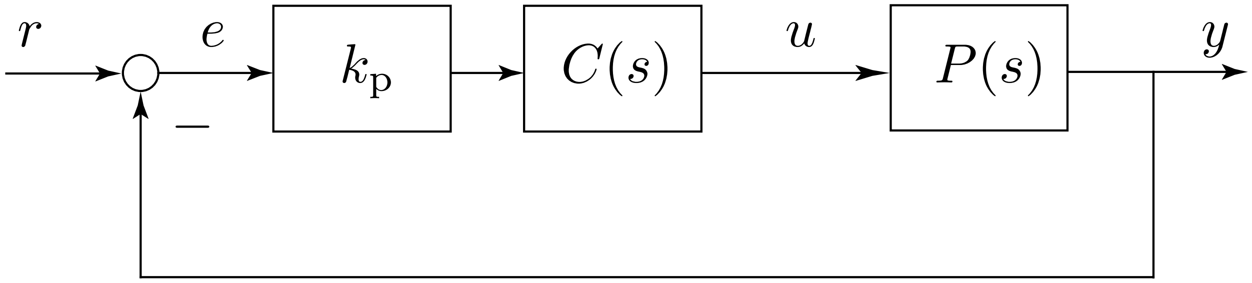
with $k_{\mathrm{p},s} = 3.85$ and $T_\mathrm{i,s} = 6.1$ s (see $L_\mathrm{i}(j\,\omega) = P_\mathrm{i}(j\,\omega) \cdot C_\mathrm{s}(j\,\omega)$ (dashed curves).

Accordingly, a better closed-loop behavior results. Notice that this comes *not* at the price of a higher control input energy.


step response, dots=reference, solid=cascade, dashed=output feedback
control signal, solid=cascade, dashed=output feedback
time (s)

Root Locus Methods

Basic idea: plot poles of closed-loop system as a function of k_p



.

Open loop gain $L(s) \cdot k_{\mathrm{p}} = P(s) \cdot C(s) \cdot k_{\mathrm{p}}$

Transfer function assumed to be

$$L(s) = \frac{b_m\, s^m + \ldots + b_1\, s + b_0}{s^n + \ldots + a_1\, s + a_0} = \frac{b(s)}{a(s)}$$

Closed-loop poles defined by

$$1 + k_{\mathrm{p}} \cdot L(s) = 0 \quad \Rightarrow \quad p(s, k_{\mathrm{p}}) = a(s) + k_{\mathrm{p}} \cdot b(s) = 0$$

$p(s, k_{\mathrm{p}}) = a(s) + k_{\mathrm{p}} \cdot b(s)$ is the characteristic polynomial of the closed-loop system.

Main Rules

- The root locus is a set of curves in the complex plane
- The curves start ($k_\mathrm{p} = 0$) at the poles π_i of the open loop system, $p(\pi_i, 0) = a(\pi_i) = 0$
- For k_p approaching ∞, m of the curves approach the m finite zeros ζ_i of the open loop system, $p(\zeta_i, \infty) = b(\zeta_i) = 0$
- The $n - m = r$ other curves go to ∞ approaching straight asymptotes
- The center of these asymptotes is at $\sigma_a + j\,0$ (see next slide)
- The angle of the asymptotes is δ_i (see next slide)
- A point in the complex plane is part of the root locus iff it satisfies the *phase condition* (see below)

The asymptotes all start at the point $\sigma_a + j \cdot 0$

$$\sigma_a = \frac{1}{n-m} \cdot \left(\sum_{i=1}^{n} \mathrm{Re}(\pi_i) - \sum_{i=1}^{m} \mathrm{Re}(\zeta_i) \right)$$

The angle δ_i of the i-th asymptote to the real axis is given by

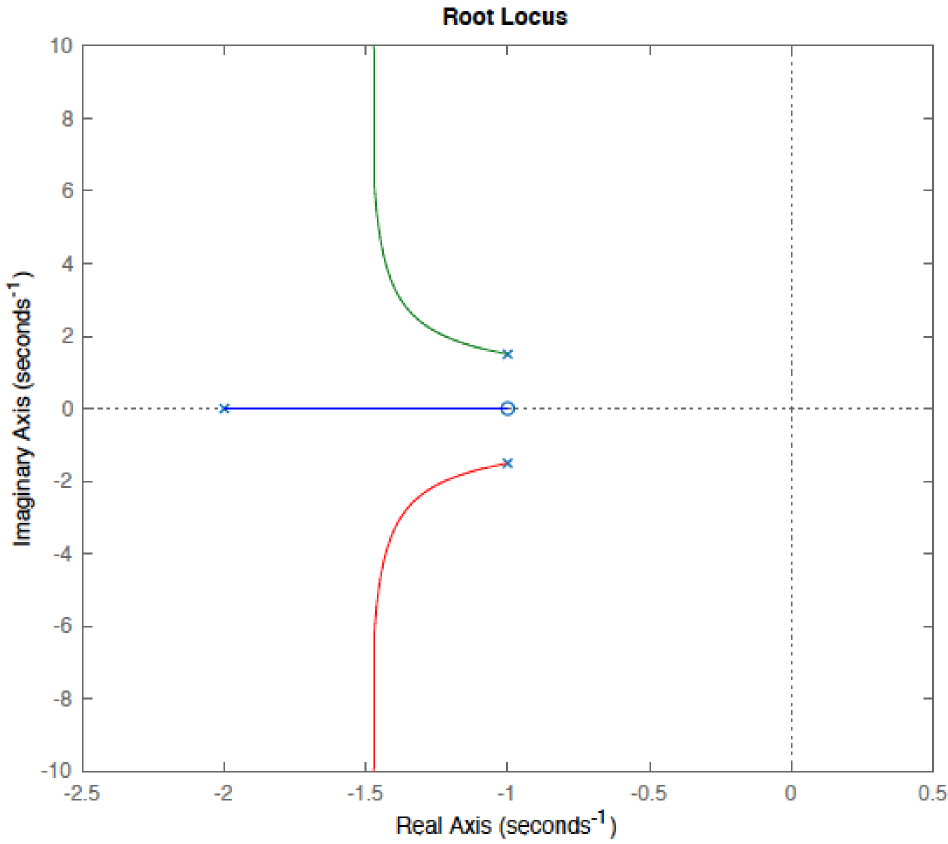
$$\delta_i = \frac{\pi}{n-m} \cdot (2 \cdot (i-1) + 1), \quad i = 1, \ldots, n-m$$

Example:

$$L(s) = \frac{s+1}{(s+2) \cdot (s^2 + 2\,s + 13/4)}, \quad \pi_1 = -2, \;\; \pi_{2,3} = -1 \pm j\; 1.5, \quad \zeta_1 = -1$$

with

$$\sigma_a = \frac{1}{3-1} \cdot (-2-1-1+1) = -1.5, \quad \delta_1 = \frac{\pi}{3-1} \cdot 1, \;\; \delta_2 = \frac{\pi}{3-1} \cdot 3$$


Root Locus
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
10
8
6
4
2
0
-2
-4
-6
-8
-10
-2.5
-2
-1.5
-1
-0.5
0
0.5

Example:

$$P(s) = \frac{1}{s + 1.5}$$

Specifications: (1) zero steady-state error, (2) t_{90} of 1.3 s, and (3) $\hat{\epsilon}$ of 2%.

Specification (1) requires

$$C_1(s) = \frac{1}{s}$$

Second-order system approximation

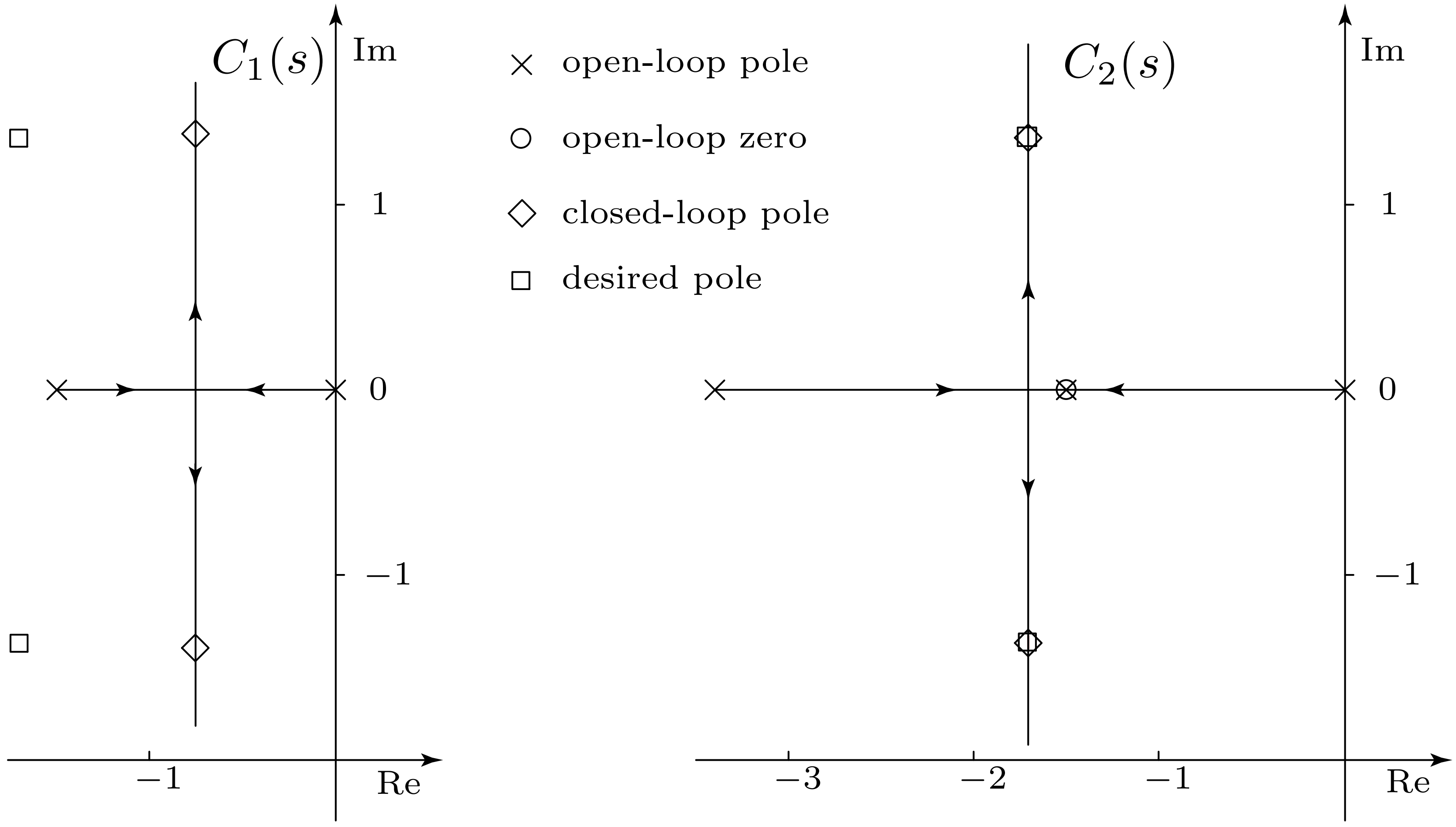
$$\delta = \frac{-\ln(0.02)}{\sqrt{\pi^2 + \ln^2(0.02)}} \approx 0.78$$

and

$$\omega_0 = (0.14 + 0.4 \cdot \delta) \cdot \frac{2 \cdot \pi}{1.3} \approx 2.2 \text{ rad/s}$$

Therefore, the desired pole location is given by

$$\pi_{\text{des}} = -\delta \cdot \omega_0 \pm j\,\sqrt{1 - \delta^2} \cdot \omega_0 \approx -1.70 \pm j\,1.37 \text{ rad/s}$$



$C_1(s)$
Im
1
0
−1
−1
Re
open-loop pole
open-loop zero
closed-loop pole
desired pole
$C_2(s)$
Im
1
0
−1
−3
−2
−1
Re

Result with $C_1(s)$ (left plot): intersection of the asymptotes with the real axis at -0.75 rad/s. Even for the best $k_\mathrm{p} \approx 2.5$ the closed-loop system is too slow and not sufficiently well damped.

Accordingly, the controller $C_1(s)$ must be augmented

$$C_2(s) = \frac{1}{s} \cdot \frac{s + 1.5}{s + a}$$

The plant's pole at -1.5 rad/s is cancelled by the zero of $C_2(s)$, and a new open-loop pole $s = -a$ is chosen such that the root-locus will include the desired pole locations. Using the rule for σ_a the result is

$$a = -2 \cdot \mathrm{Re}(\pi_\mathrm{des}) = 3.4 \text{ rad/s}$$

The gain $k_\mathrm{p} = 4.75$ yields a perfect match.

Argument condition: A point $z \in \mathbb{C}$ is part of the root-locus iff it satisfies the phase condition

$$\sum_{i=1}^{m} \angle(z - \zeta_i) - \sum_{i=1}^{n} \angle(z - \pi_i) = -\pi \pm k \cdot 2\,\pi, \;\; k = 0, 1, 2, \cdots$$

“Proof”:

$$1 + k_{\mathrm{p}} \cdot L(z) = 0 \;\;\Rightarrow\;\; L(z) = -1/k_{\mathrm{p}} \;\;\Rightarrow\;\; \angle L(z) = -\pi \pm k \cdot 2\,\pi, \;\; k = 0, 1, 2, \cdots$$

and

$$L(s) = b_m \cdot \frac{(s - \zeta_1) \cdot (s - \zeta_2) \cdot \ldots \cdot (s - \zeta_m)}{(s - \pi_1) \cdot (s - \pi_2) \cdot \ldots \cdot (s - \pi_n)}$$

Example Argument Condition:

The plant to be controlled is

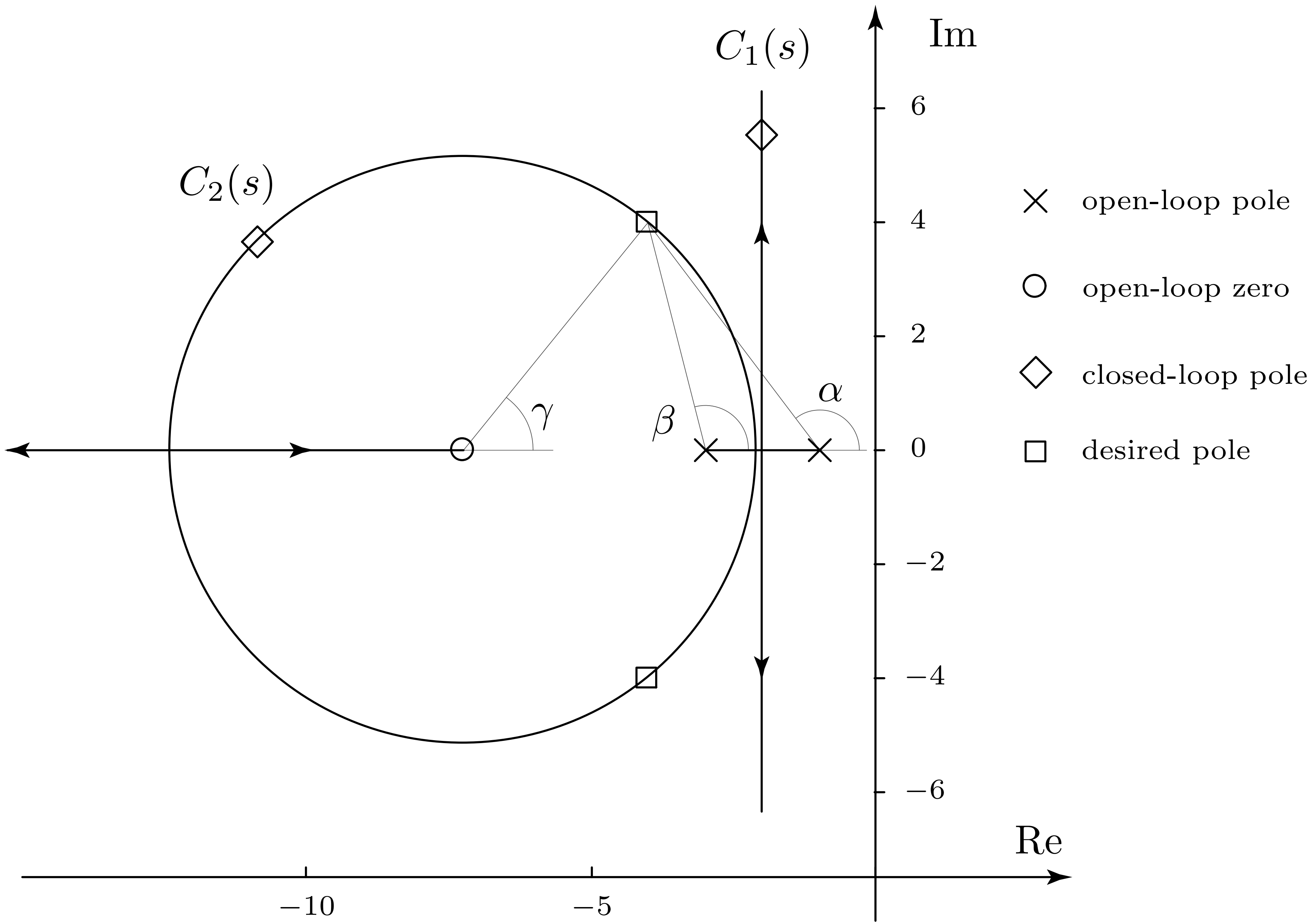
$$P(s) = \frac{1}{(s+1) \cdot (s+3)}$$

This plant has no finite zeros and two real poles at $\pi_1 = -1$ rad/s and $\pi_2 = -3$ rad/s.

The desired pole location is assumed to be $-4 \pm j \cdot 4$ rad/s.

Using a simple compensator $C_1(s) = 1$, the root-locus indicated with $C_1(s)$ is obtained.

Since the desired pole location does not satisfy the argument condition, the root-locus does not pass through that point.


Im
Re
$C_1(s)$
$C_2(s)$
α
β
γ
6
4
2
0
−2
−4
−6
−10
−5
open-loop pole
open-loop zero
closed-loop pole
desired pole

To correct for that fact, a real zero at $-\zeta$ can be added to the controller

$$C_2(s) = s + \zeta$$

The value of ζ can be computed using the argument condition

$$\alpha = \frac{\pi}{2} + \arctan\frac{3}{4} \approx 2.21\ (127^\circ) \quad \beta = \pi/2 + \arctan\frac{1}{4} \approx 1.82\ (104^\circ)$$

The argument condition is satisfied if

$$\gamma = \alpha + \beta - \pi$$

Inserting the above values for α and β yields

$$\gamma \approx 0.889\ (51^\circ)$$

Therefore,

$$\zeta = 4 + 4/\tan(\gamma) \approx 7.25\,\mathrm{rad/s}$$

and with $k_\mathrm{p} \approx 4$ the desired closed-loop poles are attained.

Numerical Multicriteria Optimization

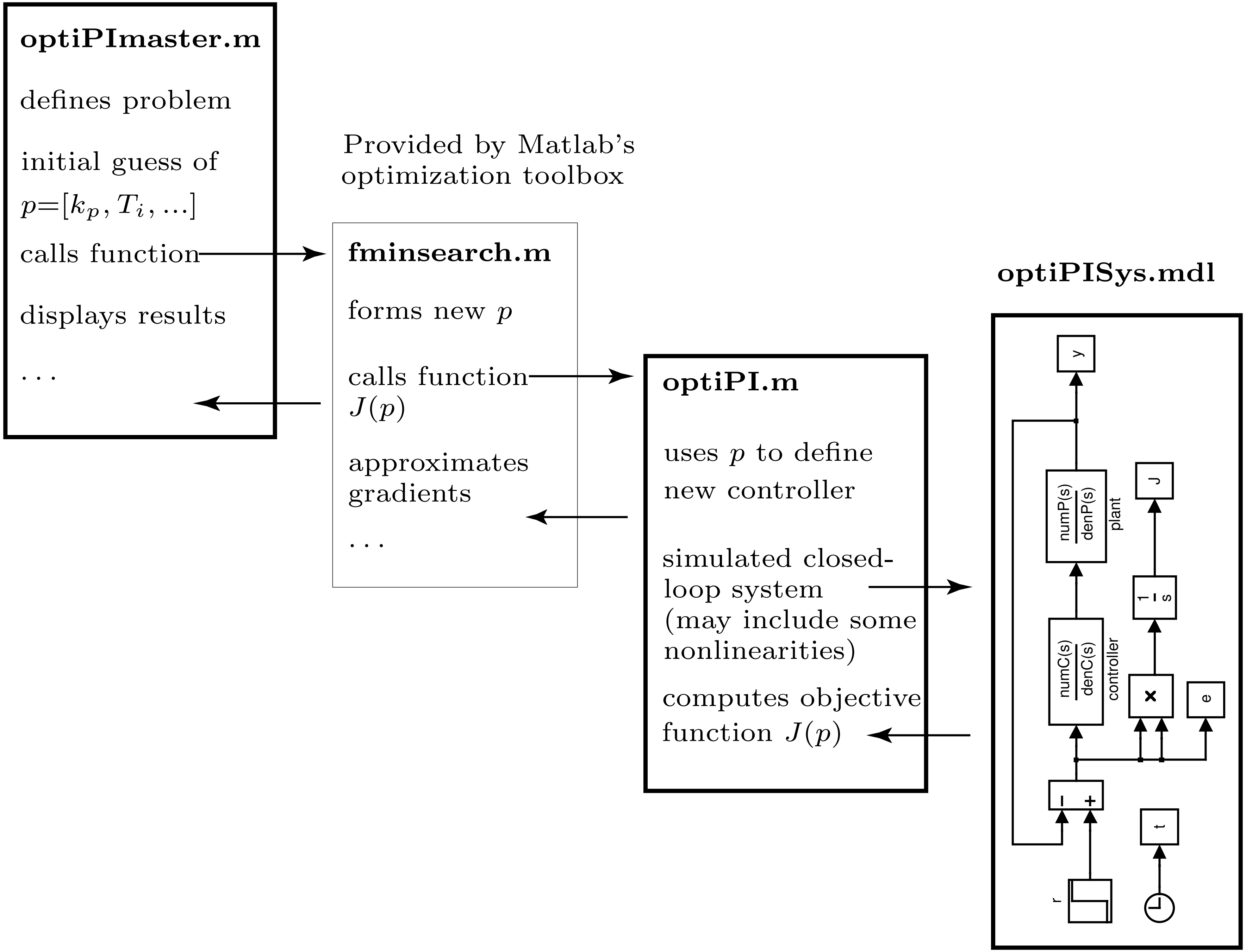
Basic idea: fix controller structure and find "optimal" parameters using numerical optimization routines. Idea explained using an example. Can be extended to more complex formulations (including nonlinear effects, etc.).

Main difficulties: numerical approaches can never guarantee that a solution is optimal or even "good." In the best case, only local optima can be found, but the solvers can altogether fail. Good initial guesses for the parameters to be optimized important.

Example plant

$$P(s) = \frac{1}{(s^2+s+1)\cdot(s^4+4s^3+6s^2+4s+1)}$$

Plant of high-order but "benign." Critical gain $k^* = 1.75$ and period $T^* = 9.7$. Start values for PI controller chosen according to ZN rules.



optiPImaster.m
defines problem
initial guess of
p=[kp, Ti, ...]
calls function
displays results
...
Provided by Matlab's
optimization toolbox
fminsearch.m
forms new p
calls function
J(p)
approximates
gradients
...
optiPI.m
uses p to define
new controller
simulated closed-
loop system
(may include some
nonlinearities)
computes objective
function J(p)
optiPISys.mdl
y
numP(s)
denP(s)
plant
J
1
s
numC(s)
denC(s)
controller
e
t
r

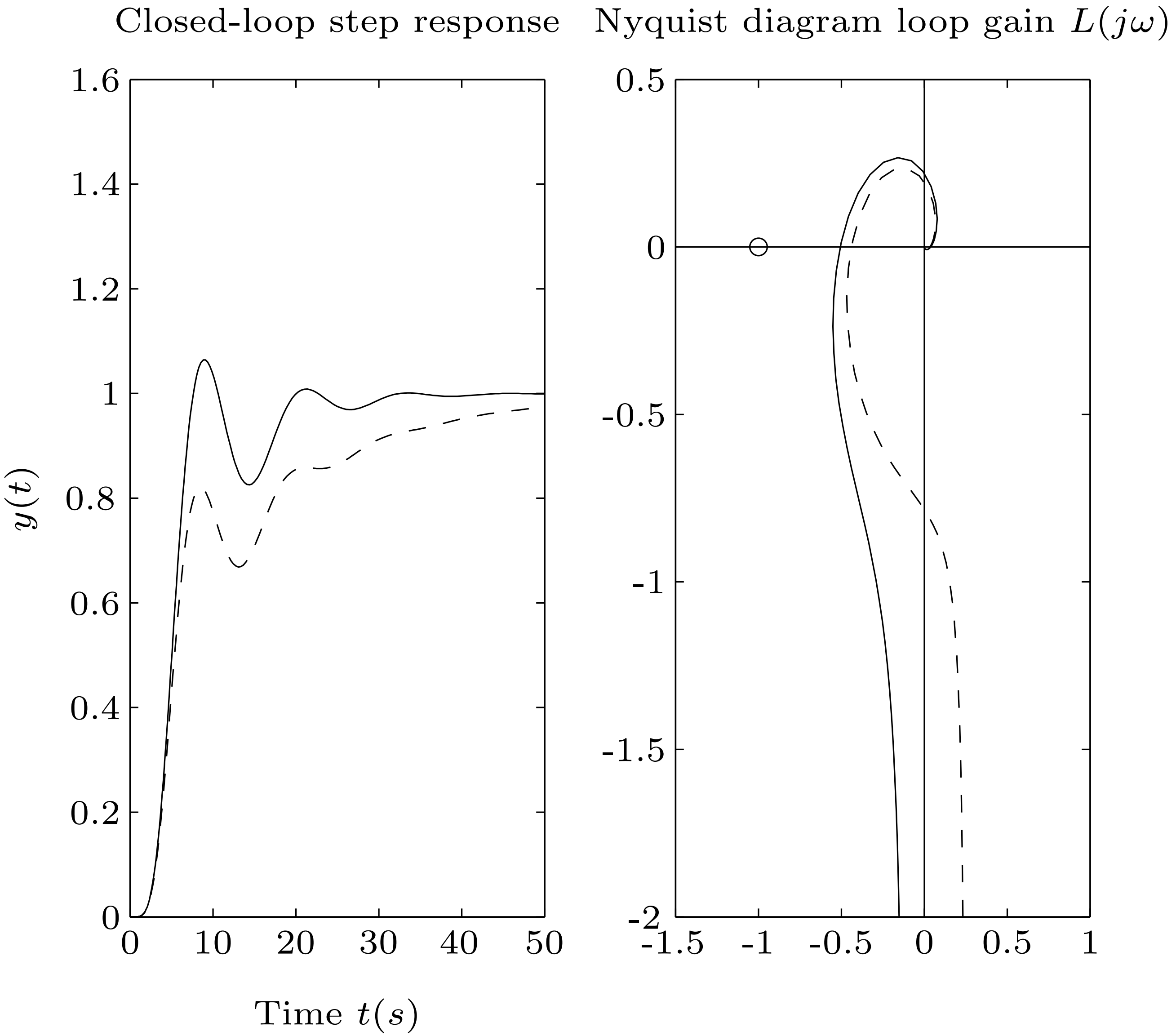
The .m file "optiPI.m" computes the actual objective function

$$J(k_p, T_i) = \mu_1 \cdot \int_0^\infty e^2(t)\,dt + \mu_2 \cdot \max_t(y(t) - 1) + \mu_3 \cdot (1 - \min_\omega(|1 + L(j\omega)|))$$

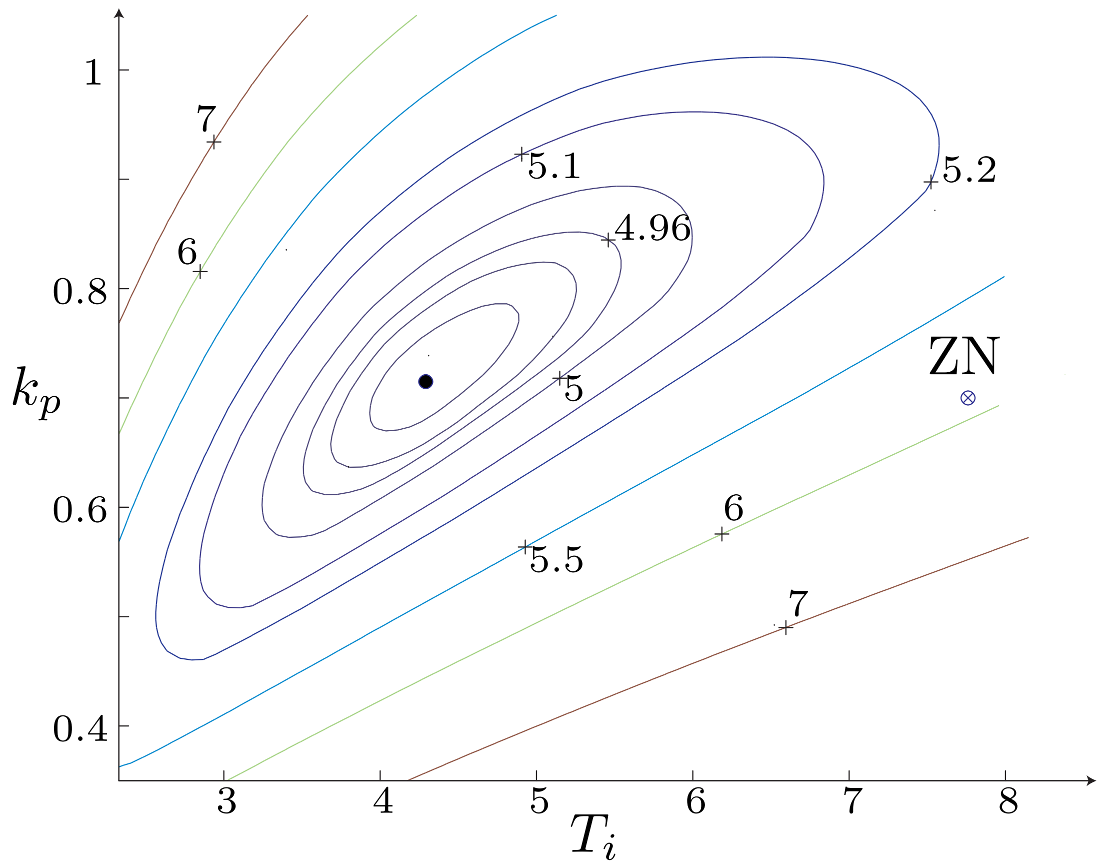
Remarks:

- For a step change in the reference signal $r(t)$, the objective function penalizes the squared error integral and the maximum overshoot of the closed-loop system (time-domain behavior). In the frequency domain, the minimum return difference of the open-loop system (robustness) is maximized.
- The three weights $\mu_{1,2,3}$ are fixed a priori and permit to decide what impact each of the penalized effect has on the objective function. They represent the design iteration parameters.
- Of course the integration cannot be made for $t_{stop} = \infty$; t_{stop} is chosen finite but must be so large that the error $e(t_{stop})$ is negligibly small.
- Matlab uses variable encapsulation on each subroutine level. Use the "global" command to exchange parameters across levels.

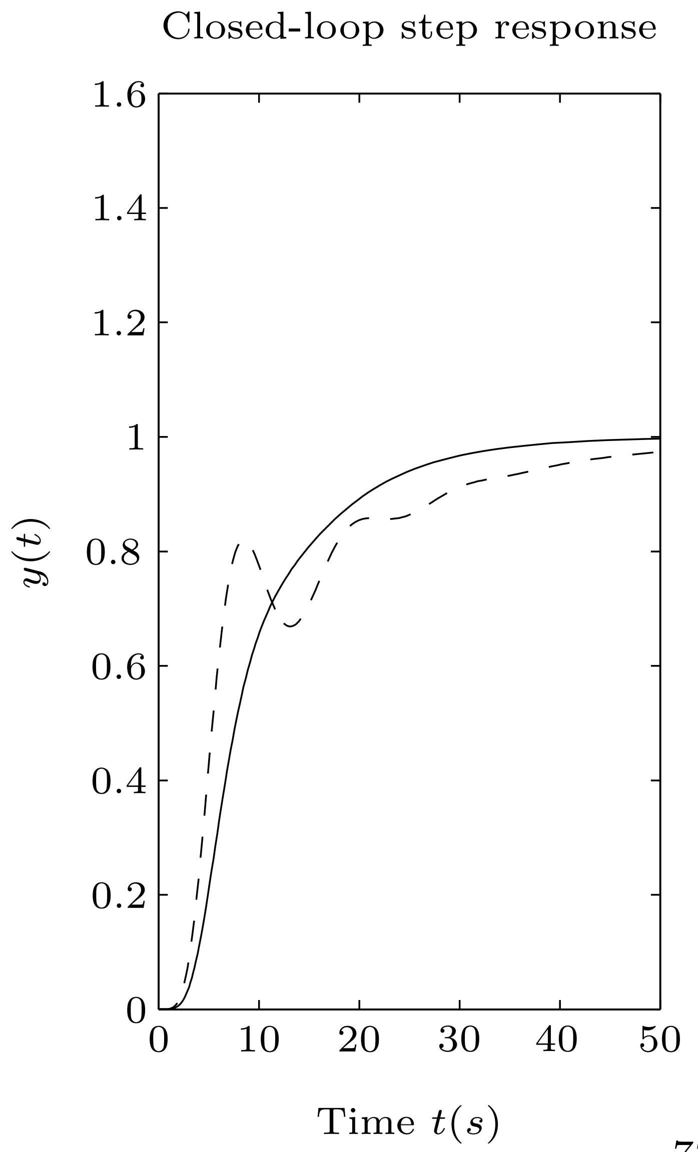
$$\mu_1 = 1,\ \mu_2 = 1,\ \mu_3 = 1\ \Rightarrow\ k_p = 0.724,\ T_i = 4.34\ \ (\mathrm{ZN} : k_p = 0.7,\ T_i = 7.76)$$

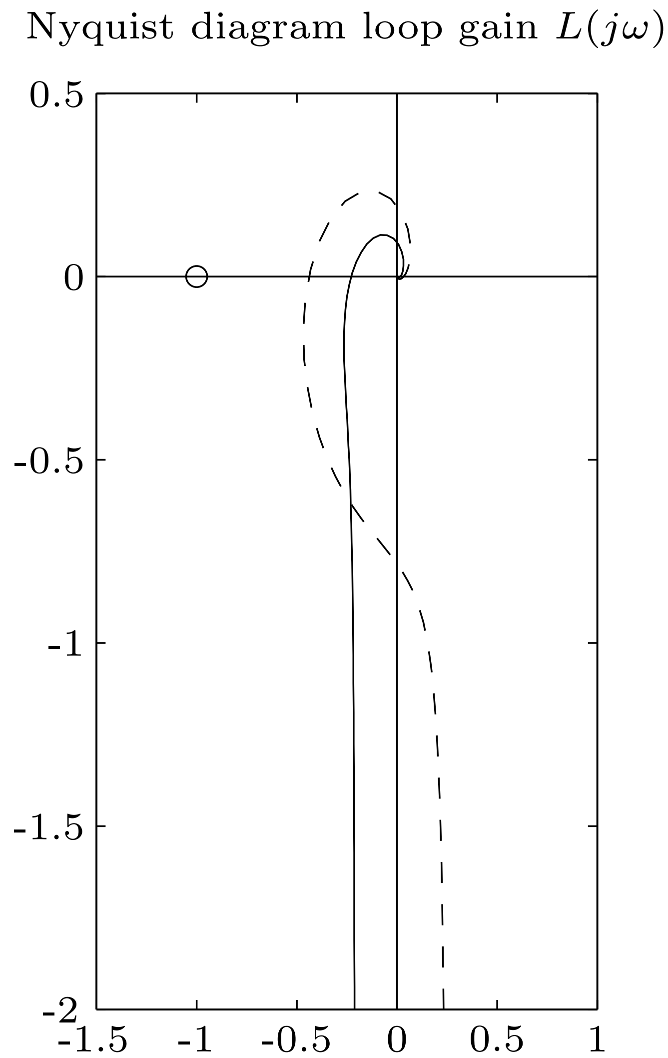


$$\mu_1 = 1,\ \mu_2 = 1,\ \mu_3 = 1$$



$$\mu_1 = 1, \mu_2 = 1,\ \mu_3 = 20\ \Rightarrow\ k_p = 0.273,\ T_i = 2.83\ \ (\mathrm{ZN}: k_p = 0.7,\ T_i = 7.76)$$





Lecture IV

Algebraic Stability Criteria

Control Systems Realization

Algebraic Stability Criteria – Hurwitz Criterion

The polynomial

$$p(s) = a_n \cdot s^n + a_{n-1} \cdot s^{n-1} + \ldots + a_1 \cdot s + a_0$$

has n roots ρ_i, $i = 1, \ldots, n$ which satisfy the equation $p(\rho_i) = 0$.

If the coefficients a_i are real, all roots will bei either real or form complex conjugated pairs.

The explicit computation of the roots ρ_i is possible for $n \leq 4$, although for $n = \{3, 4\}$ the corresponding "formulas" (Cardano's equations) are rather cumbersome. For $n > 4$ only numerical procedures exist.

If $p(s)$ is the characteristic polynomial of a LTI system one is particularly interested to know if all of its roots have negative real part. This question can be answered for any finite n using the approach of Hurwitz.

Definition Hurwitz matrix $H_n \in I\!R^{n \times n}$

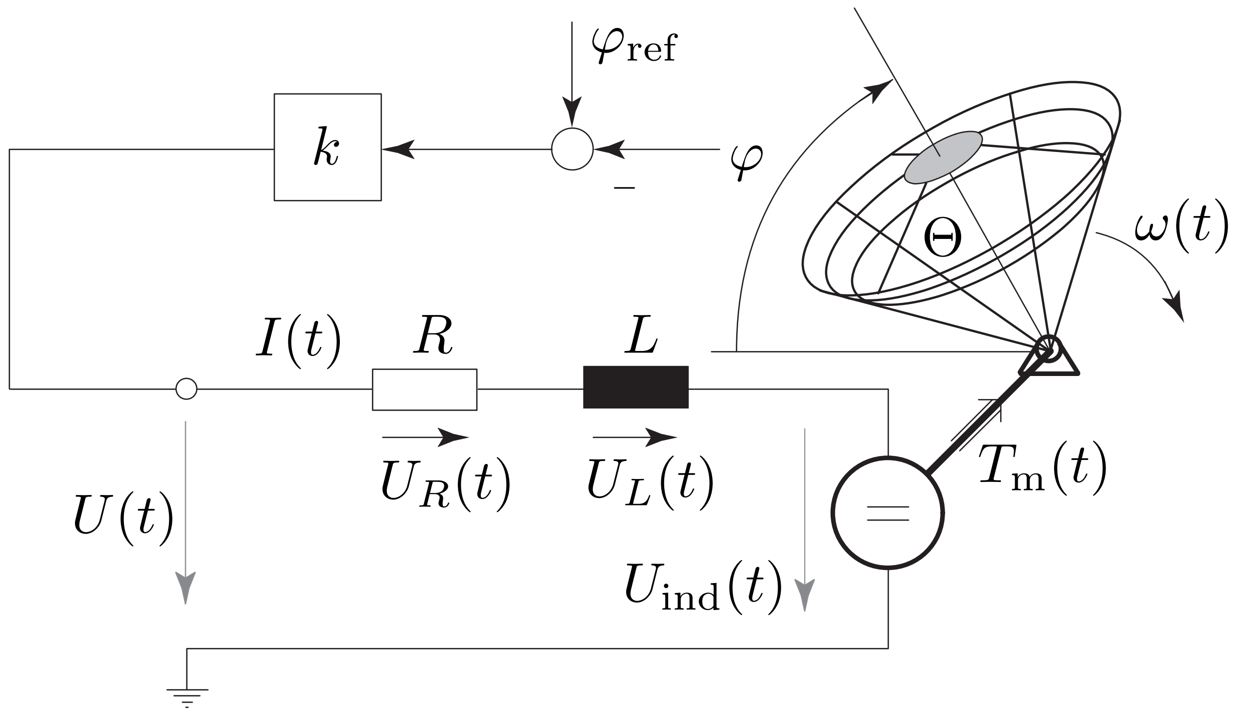
$$H_n = \begin{bmatrix} a_{n-1} & a_n & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ a_{n-3} & a_{n-2} & a_{n-1} & a_n & 0 & \cdots & \cdots & \cdots & 0 \\ a_{n-5} & a_{n-4} & a_{n-3} & a_{n-2} & a_{n-1} & a_n & 0 & \cdots & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & \cdots & 0 & a_0 & a_1 & a_2 & a_3 & a_4 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & a_0 & a_1 & a_2 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & a_0 \end{bmatrix}$$

Submatrices H_i and determinants $d_i = \det(H_i)$ with $i = 1, \ldots, n$

$$
\begin{aligned}
H_1 &= [a_{n-1}] \Rightarrow \; d_1 = a_{n-1} \\
H_2 &= \begin{bmatrix} a_{n-1} & a_n \\ a_{n-3} & a_{n-2} \end{bmatrix} \Rightarrow \; d_2 = a_{n-1} \cdot a_{n-2} - a_n \cdot a_{n-3} \\
H_3 &= \begin{bmatrix} a_{n-1} & a_n & 0 \\ a_{n-3} & a_{n-2} & a_{n-1} \\ a_{n-5} & a_{n-4} & a_{n-3} \end{bmatrix} \Rightarrow \; d_3 = \begin{array}{l} d_2 \cdot a_{n-3} \\ \quad -a_{n-1} \cdot (a_{n-1} \cdot a_{n-4} - a_n \cdot a_{n-5}) \end{array} \\
H_n &= \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \end{bmatrix} \Rightarrow \; d_n = d_{n-1} \cdot a_0
\end{aligned}
$$

Roots ρ_i are all in the complex left half plane iff all Hurwitz determinants d_i, $i = 1, \ldots, n$ are strictly positive.

Example: Antenna Control System



Time-domain equations open-loop system

$$
\begin{aligned}
L \cdot \tfrac{\mathrm{d}}{\mathrm{dt}} I(t) &= U(t) - R \cdot I(t) - \kappa \cdot \omega(t) \\
\Theta \cdot \tfrac{\mathrm{d}}{\mathrm{dt}} \omega(t) &= \kappa \cdot I(t) \\
\tfrac{\mathrm{d}}{\mathrm{dt}} \varphi(t) &= \omega(t) \\
U(t) &= k \cdot \big(\varphi_{\mathrm{ref}}(t) - \varphi(t)\big)
\end{aligned}
$$

Laplace transformation of closed-loop system

$$
\Sigma(s) = \frac{k \cdot \kappa}{\Theta \cdot L \cdot s^3 + \Theta \cdot R \cdot s^2 + \kappa^2 \cdot s + k \cdot \kappa} = \frac{k \cdot \kappa}{a(s)}
$$

According to the Hurwitz criterion the following conditions must be met for the closed-loop system to remain asymptotically stable

$$\begin{aligned}
H_1 &= \Theta \cdot R && \Rightarrow d_1 = \Theta \cdot R > 0 \\
H_2 &= \begin{bmatrix} \Theta \cdot R & \Theta \cdot L \\ k \cdot \kappa & \kappa^2 \end{bmatrix} && \Rightarrow d_2 = \Theta \cdot R \cdot \kappa^2 - \Theta \cdot L \cdot k \cdot \kappa > 0 \\
H_3 &= [\ldots] && \Rightarrow d_3 = k \cdot \kappa \cdot d_2 > 0
\end{aligned}$$

Since all system parameters are positive, the only nontrivial condition for the controller gain k is

$$0 < k < k_{\max} = \frac{R \cdot \kappa}{L}$$

Interval Coefficient Polynomials

Definition *interval coefficient* polynomials

$$p(s,a) = [\underline{a}_n, \overline{a}_n] \cdot s^n + [\underline{a}_{n-1}, \overline{a}_{n-1}] \cdot s^{n-1} + \ldots + [\underline{a}_0, \overline{a}_0]$$

where the coefficient $\underline{a}_i \leq a_i \leq \overline{a}_i$ are constant, but unknown. Only their bounds, which satisfy the conditions

$$0 < \underline{a}_i \leq \overline{a}_i, \quad i = 0, \ldots, n$$

are known. Notice: the stability of the two extreme polynomials

$$p(s,\underline{a}) = \underline{a}_n \cdot s^n + \underline{a}_{n-1} \cdot s^{n-1} + \ldots \underline{a}_1 \cdot s + \underline{a}_0$$

$$p(s,\overline{a}) = \overline{a}_n \cdot s^n + \overline{a}_{n-1} \cdot s^{n-1} + \ldots + \overline{a}_1 \cdot s + \overline{a}_0$$

is necessary, but not sufficient for the stability of the complete interval family.

Example

$$p(s,a) = [1,1] \cdot s^3 + [0.5,1] \cdot s^2 + [0.5,1] \cdot s + [0.2,0.5]$$

has two stable extreme polynomials

$$p_2(s) = 1 \cdot s^3 + 1 \cdot s^2 + 1 \cdot s + 0.5 \qquad \Rightarrow \lambda_{1/2} = -0.176 \pm j\,0.861,\ \ \lambda_3 = -0.648$$

$$p_4(s) = 1 \cdot s^3 + 0.5 \cdot s^2 + 0.5 \cdot s + 0.2 \quad \Rightarrow \lambda_{1/2} = -0.037 \pm j\,0.684,\ \ \lambda_3 = -0.427$$

Nevertheless, the polynomial

$$p_1(s) = 1 \cdot s^3 + 0.5 \cdot s^2 + 0.5 \cdot s + 0.5$$

which is also part of the family has two unstable roots

$$\lambda_{1/2} = +0.1195 \pm j\,0.8138$$

Kharitonov Theorem

Define four *vertex polynomials*

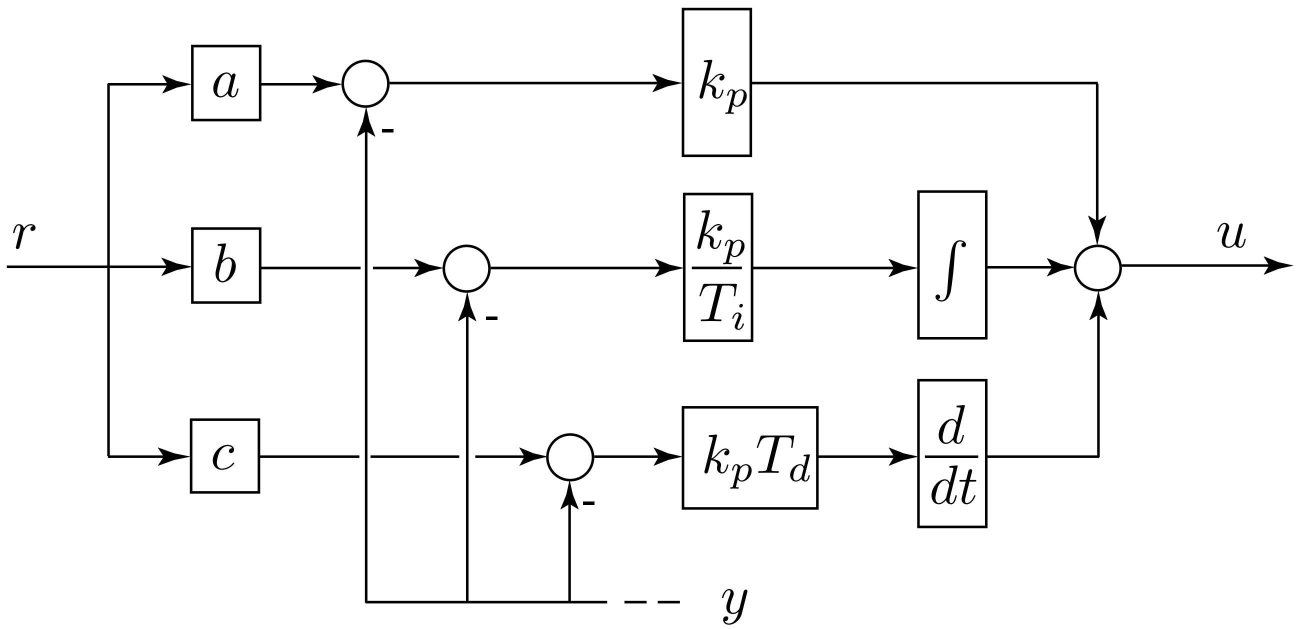
$$\begin{aligned} p_1(s) \quad &= \overline{a}_0 + \underline{a}_1 \cdot s + \underline{a}_2 s^2 + \overline{a}_3 \cdot s^3 + \ldots \\ p_2(s) \quad &= \overline{a}_0 + \overline{a}_1 \cdot s + \underline{a}_2 s^2 + \underline{a}_3 \cdot s^3 + \ldots \\ p_3(s) \quad &= \underline{a}_0 + \overline{a}_1 \cdot s + \overline{a}_2 s^2 + \underline{a}_3 \cdot s^3 + \ldots \\ p_4(s) \quad &= \underline{a}_0 + \underline{a}_1 \cdot s + \overline{a}_2 s^2 + \overline{a}_3 \cdot s^3 + \ldots \end{aligned}$$

Kharitonov Theorem: *all* polynomials in the interval polynomial family

$$p(s, a) = [\underline{a}_n, \overline{a}_n] \cdot s^n + [\underline{a}_{n-1}, \overline{a}_{n-1}] \cdot s^{n-1} + \ldots + [\underline{a}_0, \overline{a}_0]$$

have all of their roots in the complex left half plane iff the four vertex polynomials have all of their roots in the complex left half plane.

Setpoint weighting for PID Controllers

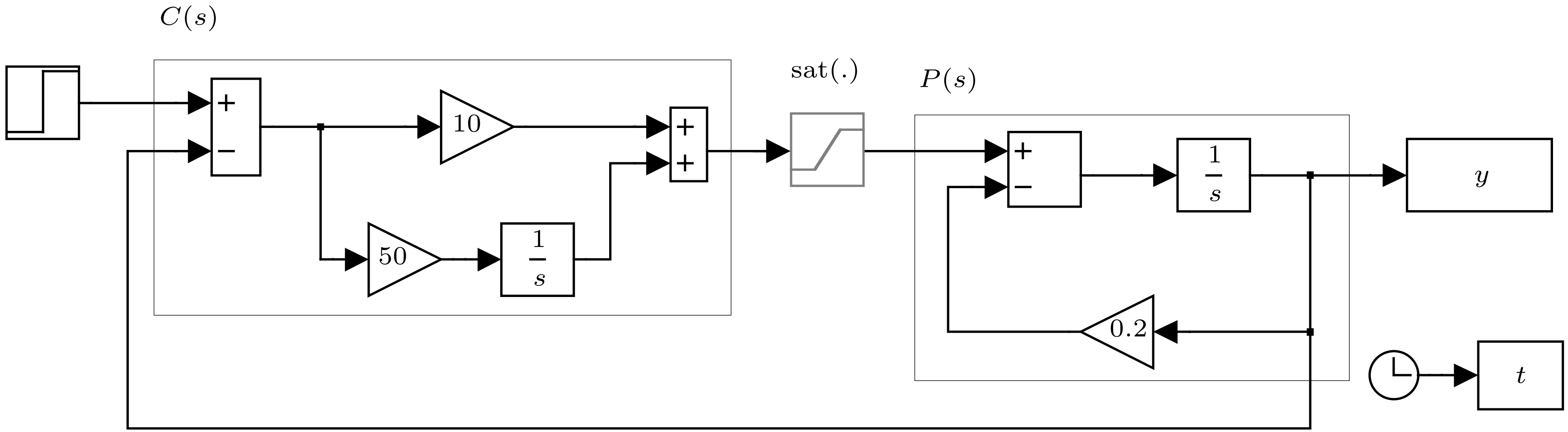


Usually $b = 1$ and $c = 0$, and a is an additional design parameter (see for instance Aström-Hägglund rules). Also an additional "roll-off" term

$$C(s) = \frac{1}{\tau \cdot s + 1}$$

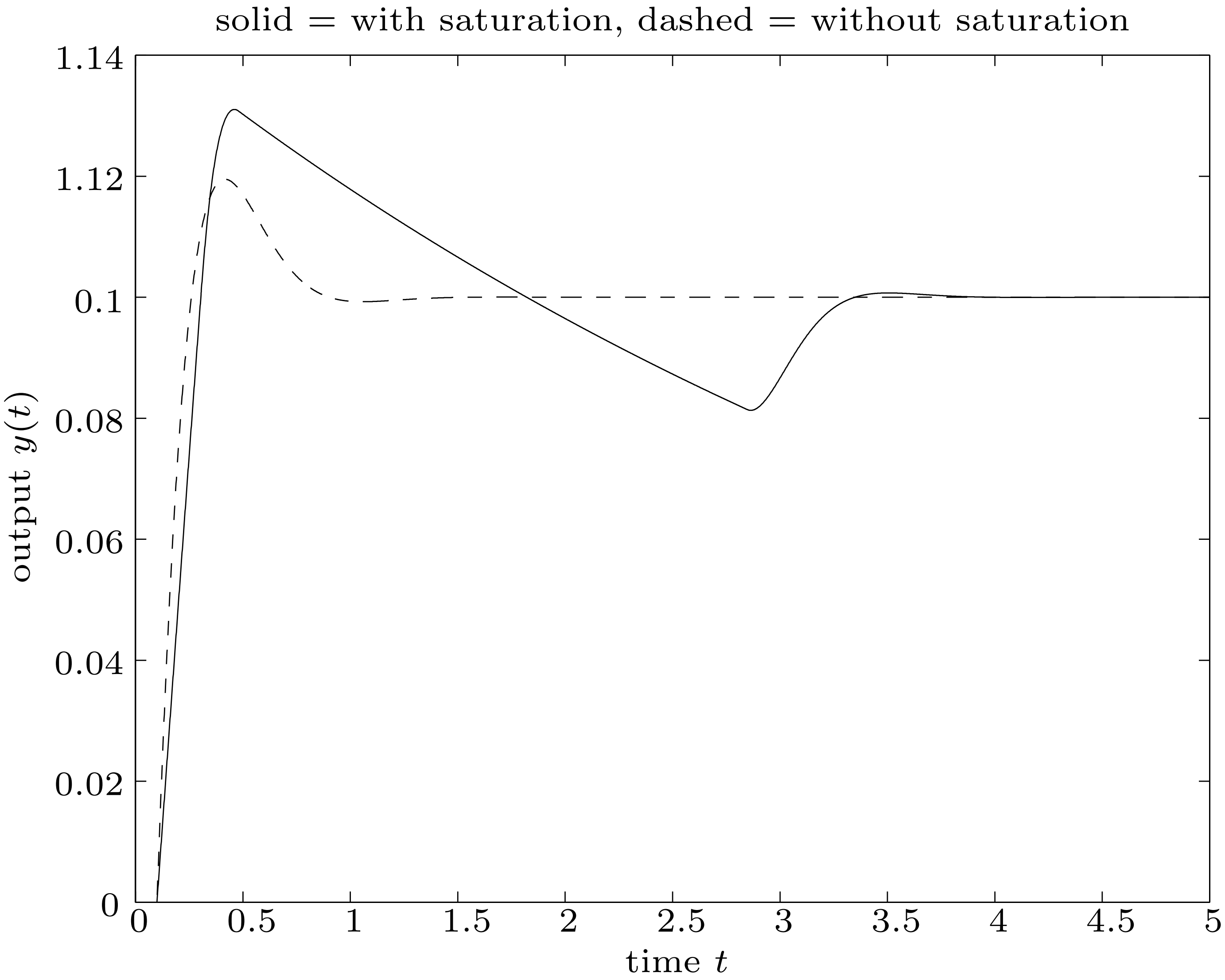
is added usually, either in series to the PID element or to the D element. Roll-off time constant τ substantially smaller than $\min\{T_d, T_i\}$.

Reset Windup

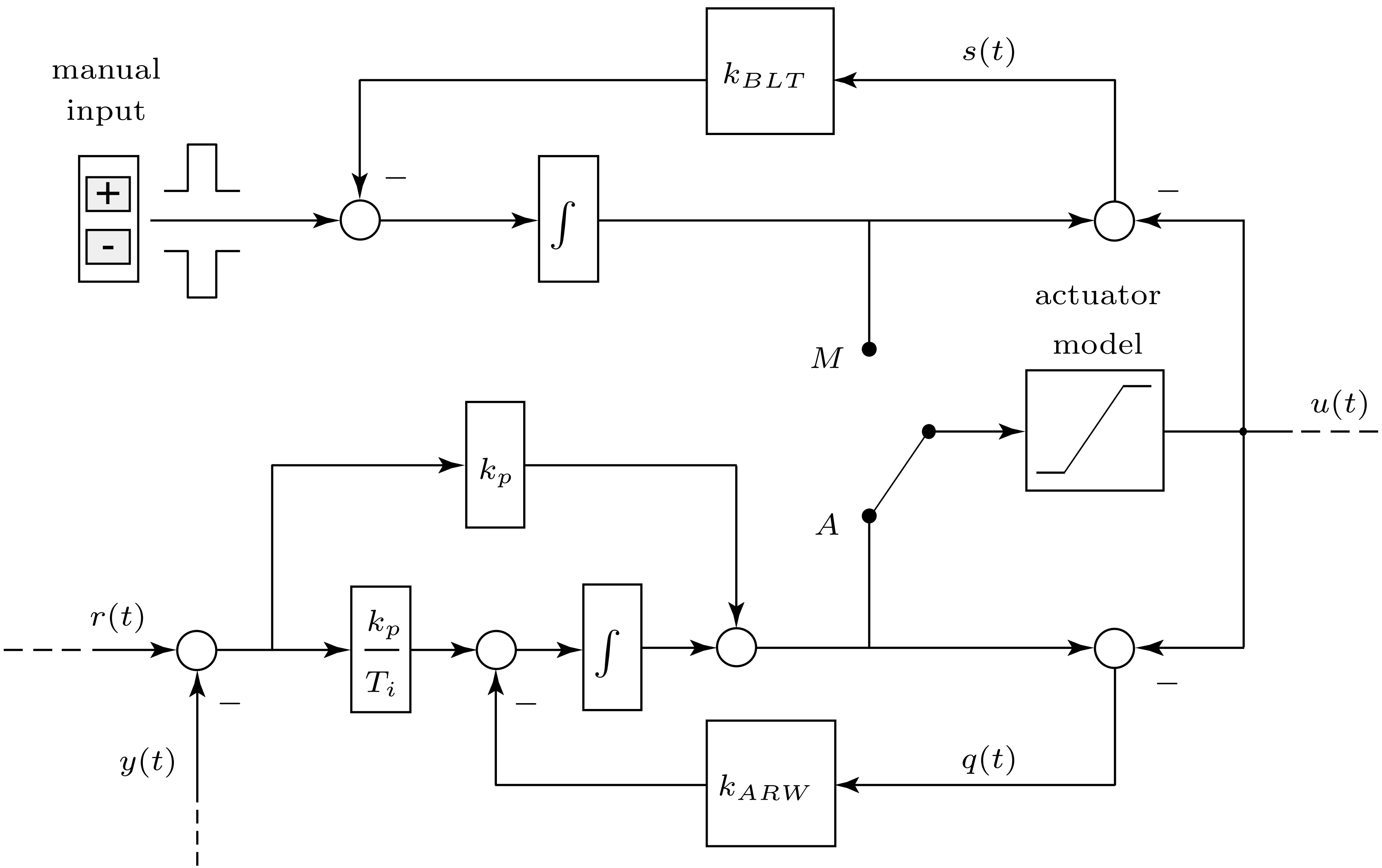


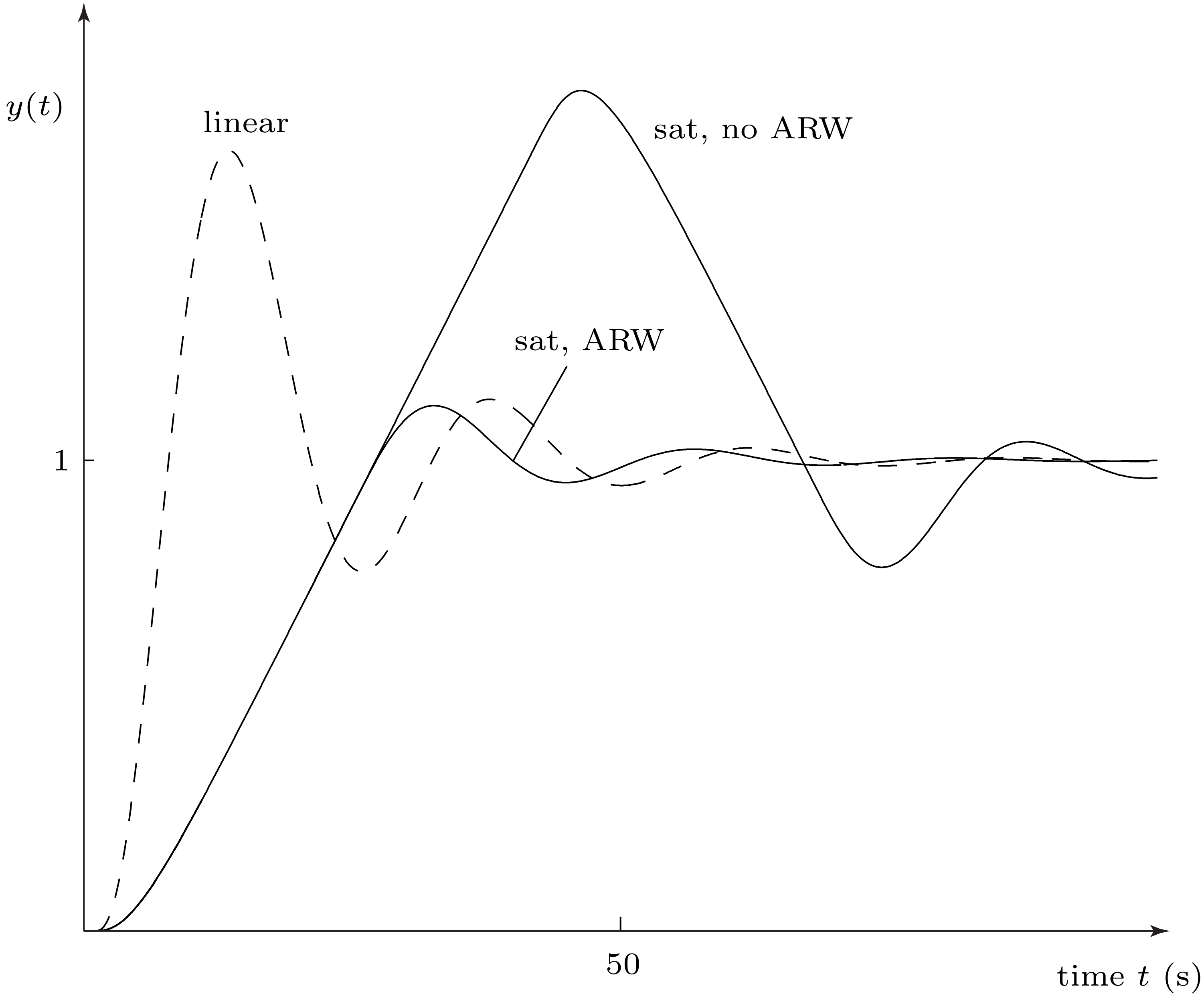
Linear design fine.

What happens if the actuators saturate in range (power, force, . . .)?


solid = with saturation, dashed = without saturation
output $y(t)$
time t
1.14
1.12
0.1
0.08
0.06
0.04
0.02
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

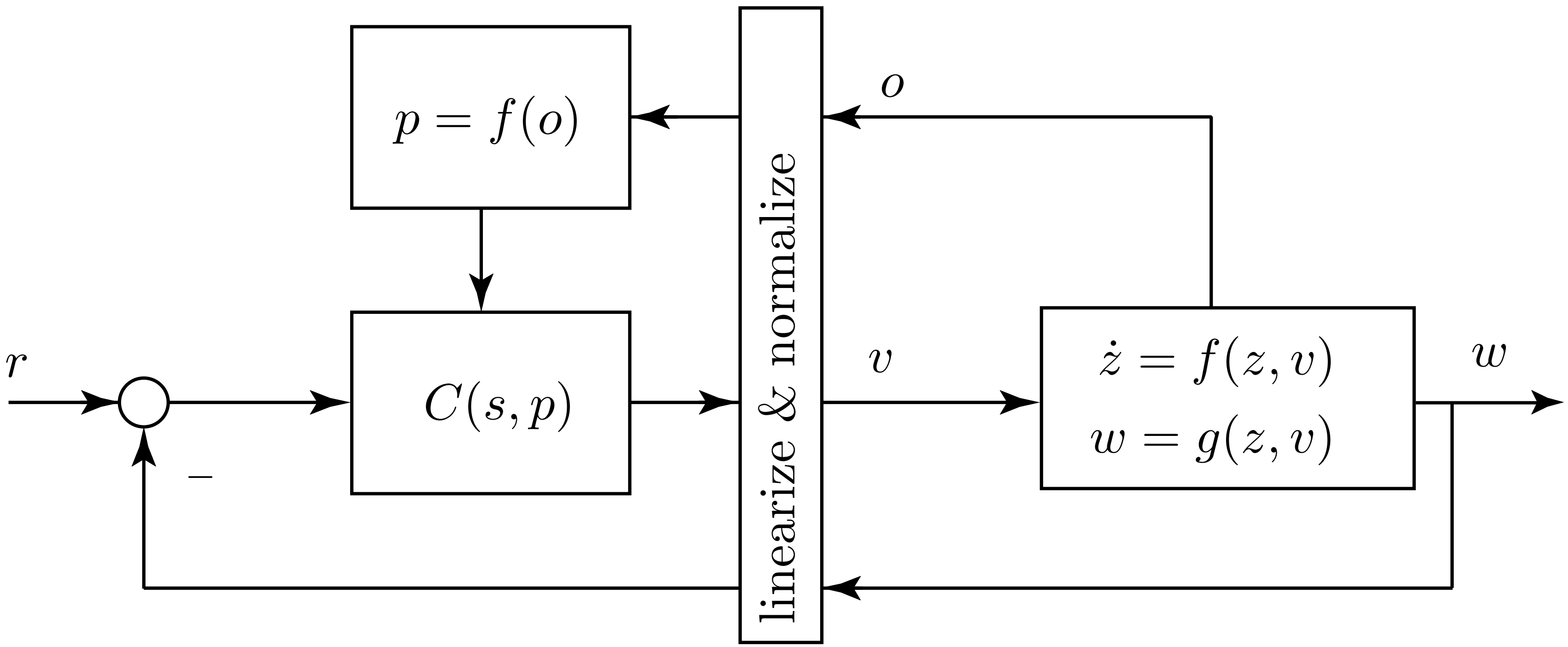
Anti reset windup and bumpless transfer





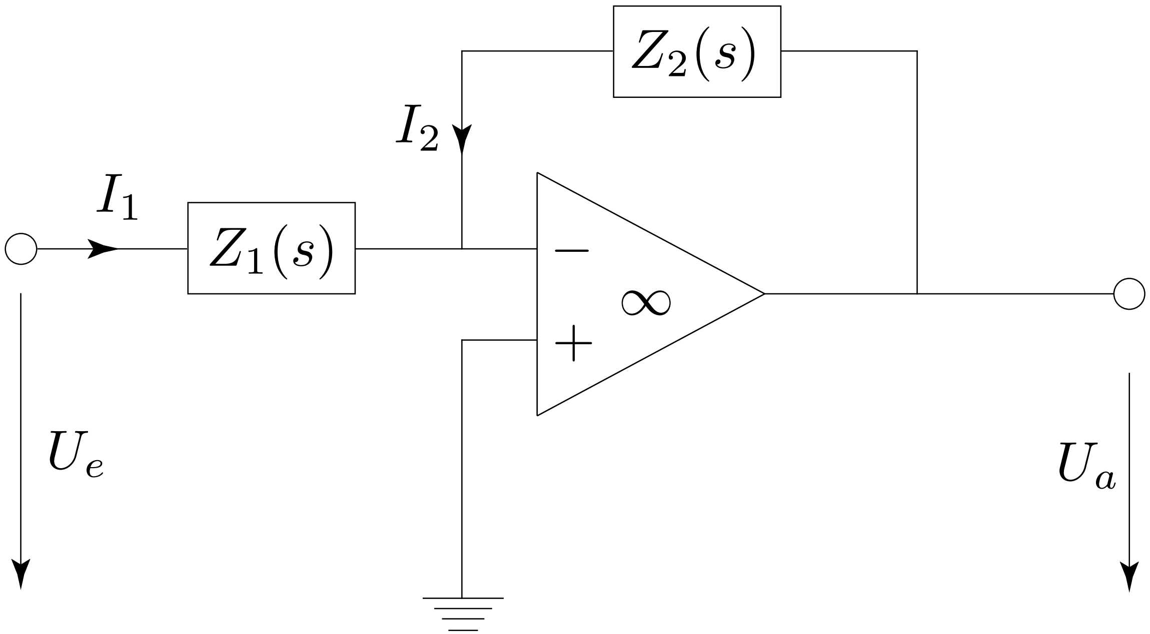
y(t)
linear
sat, no ARW
sat, ARW
1
50
time t (s)

Gain Scheduling



System variable o is available that defines the operating point. This o changes much slower than w and v. Linearizing the plant in each o and designing a controller for each o yields, e.g. PI controllers with parameters $k_{\mathrm{p}}(o)$ and $T_{\mathrm{i}}(o)$. During operation, these parameters are changed according to the actual operating point.

Realization with Analog Components and Op-Amps



$$I_1(s) = \frac{U_e(s)}{Z_1(s)}, \quad I_2(s) = \frac{U_a(s)}{Z_2(s)}$$

Gain$= \infty \Rightarrow I_1 + I_2 = 0$, therefore

$$C(s) = \frac{U_a(s)}{U_e(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Elements (not systems!):

$$\begin{array}{lcl} \text{resistance} & : & Z_R(s) = R \\ \text{capacitance} & : & Z_C(s) = \frac{1}{s \cdot C} \\ \text{inductance} & : & Z_L(s) = s \cdot L \end{array}$$

Connections:

$$\begin{array}{lcl} \text{series} & : & Z_{tot}(s) = Z_1(s) + Z_2(s) \\ \text{parallel} & : & Z_{tot}(s) = Z_1(s) \cdot Z_2(s) / [Z_1(s) + Z_2(s)] \end{array}$$

Example: realize

$$C(s) = -\frac{T \cdot s + 1}{\alpha \cdot T \cdot s + 1}$$

with $T = 1\ s$ and $\alpha = 0.5$.

$$C(s) = -\frac{1/(s \cdot C_2) + R_2}{1/(s \cdot C_1) + R_1} = -\frac{s + 1}{0.5 \cdot s + 1}$$

$$\Rightarrow \quad \left(\frac{1}{s \cdot C_2} + R_2\right)(1 + 0.5 \cdot s) = (1 + s)\left(\frac{1}{s \cdot C_1} + R_1\right)$$

$$\Rightarrow \quad s^2 \cdot 0.5 \cdot R_2 + s \cdot \left(R_2 + \frac{0.5}{C_2}\right) + \frac{1}{C_2} = s^2 \cdot R_1 + s \cdot \left(R_1 + \frac{1}{C_1}\right) + \frac{1}{C_1}$$

Therefore

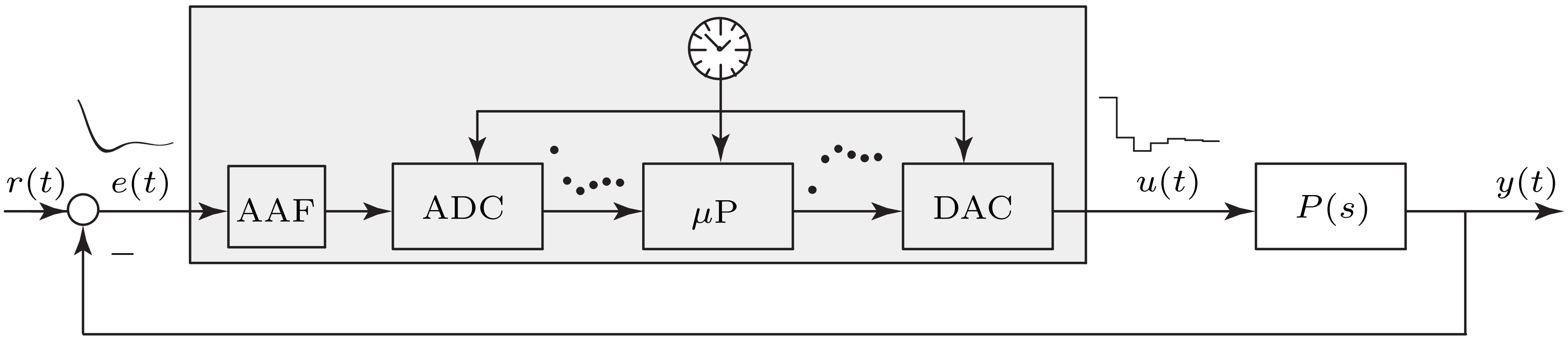
$$s^2\colon\ 0.5 \cdot R_2 = R_1 = R \qquad s^0\colon\ C_2 = C_1 = C \qquad s^1\colon\ R = \tfrac{0.5}{C}$$

and a possible choice is $C = 5\ \mu F$ and $R = 100\ k\Omega$.

Realization with Digital Computers

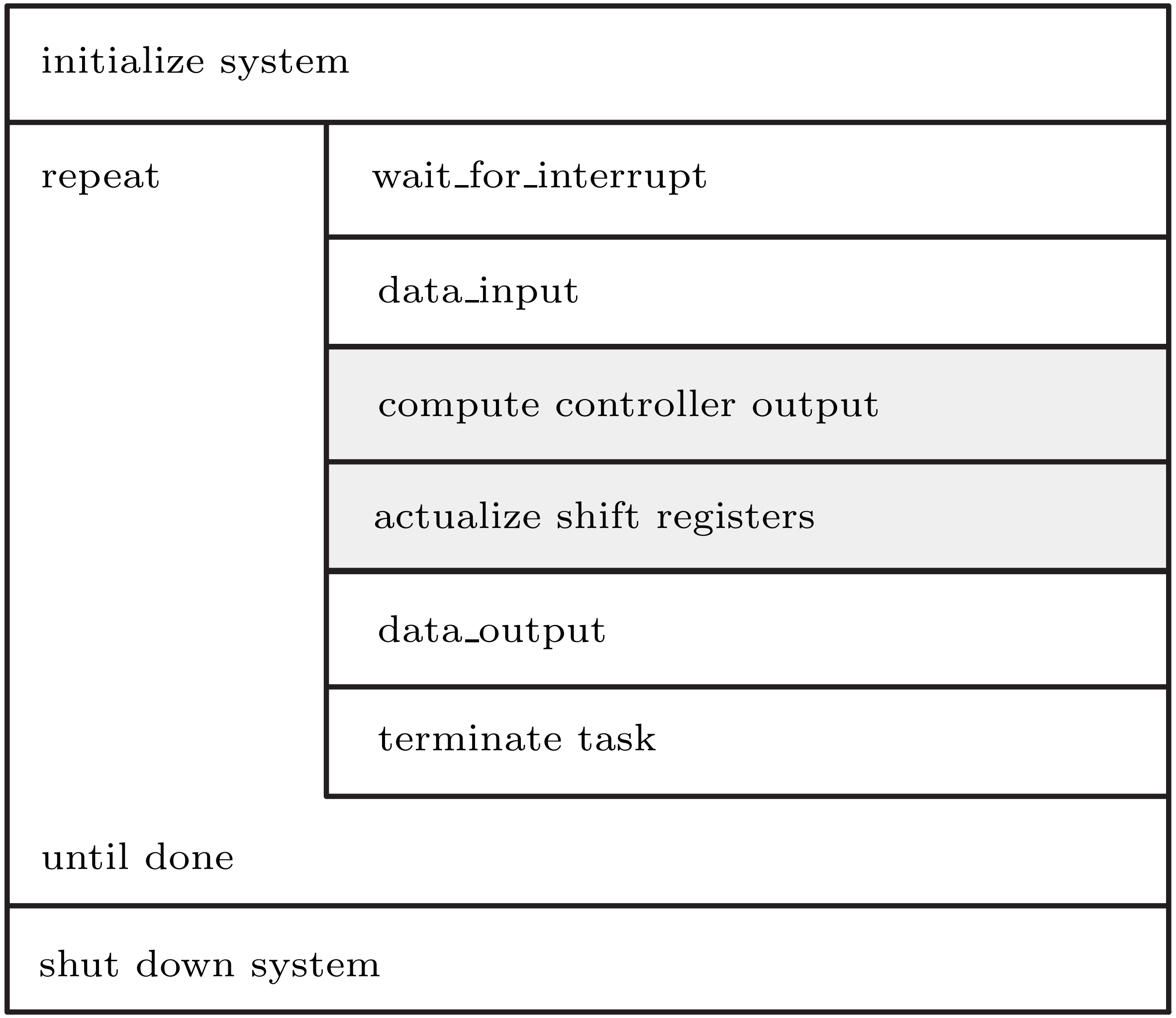
Zero-order holding and sampling

$$u(t) = u(k) \quad \forall\, t \in [k \cdot T_s, (k+1) \cdot T_s)$$



Emulation = Find $C(z)$ that produces approximately the same closed-loop behavior as with $C(s)$.

Approach OK as long as $T_s < T_c/10$ with $\omega_c = 2\pi/T_c$.


initialize system
repeat
wait_for_interrupt
data_input
compute controller output
actualize shift registers
data_output
terminate task
until done
shut down system

Commutation Diagram of Design Approaches

$$
\begin{tikzcd}[column sep=huge, row sep=large]
P(s) \arrow[rr, "\text{continuous-time synthesis}"] \arrow[dd, "\text{ZOH}"', "T_s \text{ large}"' near end] \arrow[ddr, bend right, "a"'] \arrow[drr, bend left, "b"] & & C(s) \arrow[d, "\text{emulation}", "T_s \text{ small}"'] \\
 & & \tilde{C}(z) \\
P(z) \arrow[r, "\text{discrete-time synthesis}"'] & C(z) &
\end{tikzcd}
$$

Discrete-time signal $x(k)$, $k = 0, 1, \ldots$.

Definition $\mathcal{Z}$ transformation

$$x(k) \;\leftrightarrow\; X(z) = \sum_{k=0}^{\infty} x(k) \cdot z^{-k}$$

Properties:

- linearity $\mathcal{Z}(x + y) = \mathcal{Z}(x) + \mathcal{Z}(y)$
- shift vorwards $x(k + 1) \leftrightarrow z \cdot X(z) - z \cdot x_0$
- shift backwards $x(k - 1) \leftrightarrow z^{-1} \cdot X(z)$

Pro Memoria: In the frequency domain, multiplication with s^{-1} is equivalent to integration and multiplication with s to differentiation.

Approximate integration in the time domain using Heun's rule (trapezoidal approximation, T_s is the sampling time)

$$\int_0^{N \cdot T_s} f(t)\, dt \approx q_N$$

where q_N is defined by the iteration $(k = 0, \ldots, N)$

$$q_{k+1} = q_k + \frac{f(k \cdot T_s + T_s) + f(k \cdot T_s)}{2} \cdot T_s, \quad q_0 = 0$$

Apply shift property of $\mathcal{Z}$ transformation

$$z \cdot Q(z) = Q(z) + \frac{z \cdot F(z) + F(z)}{2} \cdot T_s$$

and therefore

$$Q(z) = \frac{z+1}{z-1} \cdot \frac{T_s}{2} \cdot F(z)$$

Accordingly, the integration operator s^{-1} can be approximated by

$$s^{-1} \approx \frac{z+1}{z-1} \cdot \frac{T_s}{2}$$

or the differentiaton operator s by

$$s \approx \frac{z-1}{z+1} \cdot \frac{2}{T_s}$$

This result is known as the Tustin transformation or emulation.

The emulated controller in the discrete-time domain is found using

$$\widetilde{C}(z) = C(s)|_{s=\frac{2}{T_s} \cdot \frac{z-1}{z+1}}$$

Example

$$C(s) = \frac{k_p}{\tau \cdot s + 1} \quad \rightarrow \quad \widetilde{C}(z) \approx \frac{k_p \cdot (z+1)}{(2\frac{\tau}{T_s}+1)\cdot z + (1 - 2\frac{\tau}{T_s})} = \frac{U(z)}{E(z)}$$

This can be written as

$$U(z) \cdot \left[(2\frac{\tau}{T_s} + 1) \cdot z + (1 - 2\frac{\tau}{T_s}) \right] = E(z) \cdot k_p \cdot (z+1)$$

or

$$U(z) \cdot \left[(2\frac{\tau}{T_s} + 1) + (1 - 2\frac{\tau}{T_s}) \cdot z^{-1} \right] = E(z) \cdot k_p \cdot (1 + z^{-1})$$

Back to time-domain

$$u_k \cdot (2\frac{\tau}{T_s} + 1) + u_{k-1} \cdot (1 - 2\frac{\tau}{T_s}) = k_p \cdot (e_k + e_{k-1})$$

or

$$u_k = (2\frac{\tau}{T_s} + 1)^{-1} \left[k_p \cdot (e_k + e_{k-1}) - u_{k-1} \cdot (1 - 2\frac{\tau}{T_s}) \right]$$

Rewrite

$$u_k = (2\frac{\tau}{T_s} + 1)^{-1}\left[k_p \cdot (e_k + e_{k-1}) - u_{k-1} \cdot (1 - 2\frac{\tau}{T_s})\right]$$

as computer program with $a = k_p/(2\frac{\tau}{T_s} + 1)$, $b = (1 - 2\frac{\tau}{T_0})/(2\frac{\tau}{T_s} + 1)$

```
e_new = analog_input();

u_new = a*(e_new + e_old) - b*u_old;

analog_output(u_new);

u_old = u_new;

e_old = e_new;
```

Some Remarks:

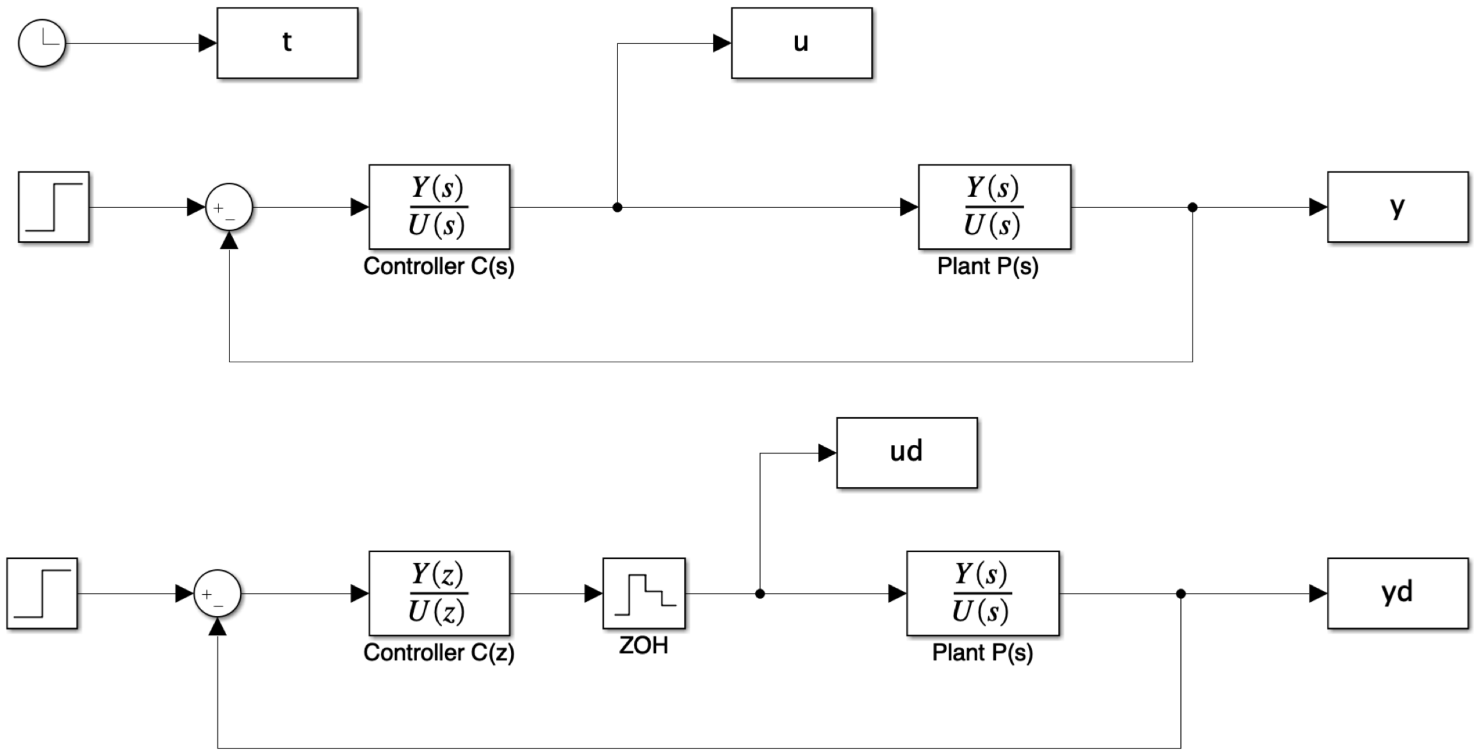
The emulation approach is very popular and is often used, particularly in rapid prototyping systems.

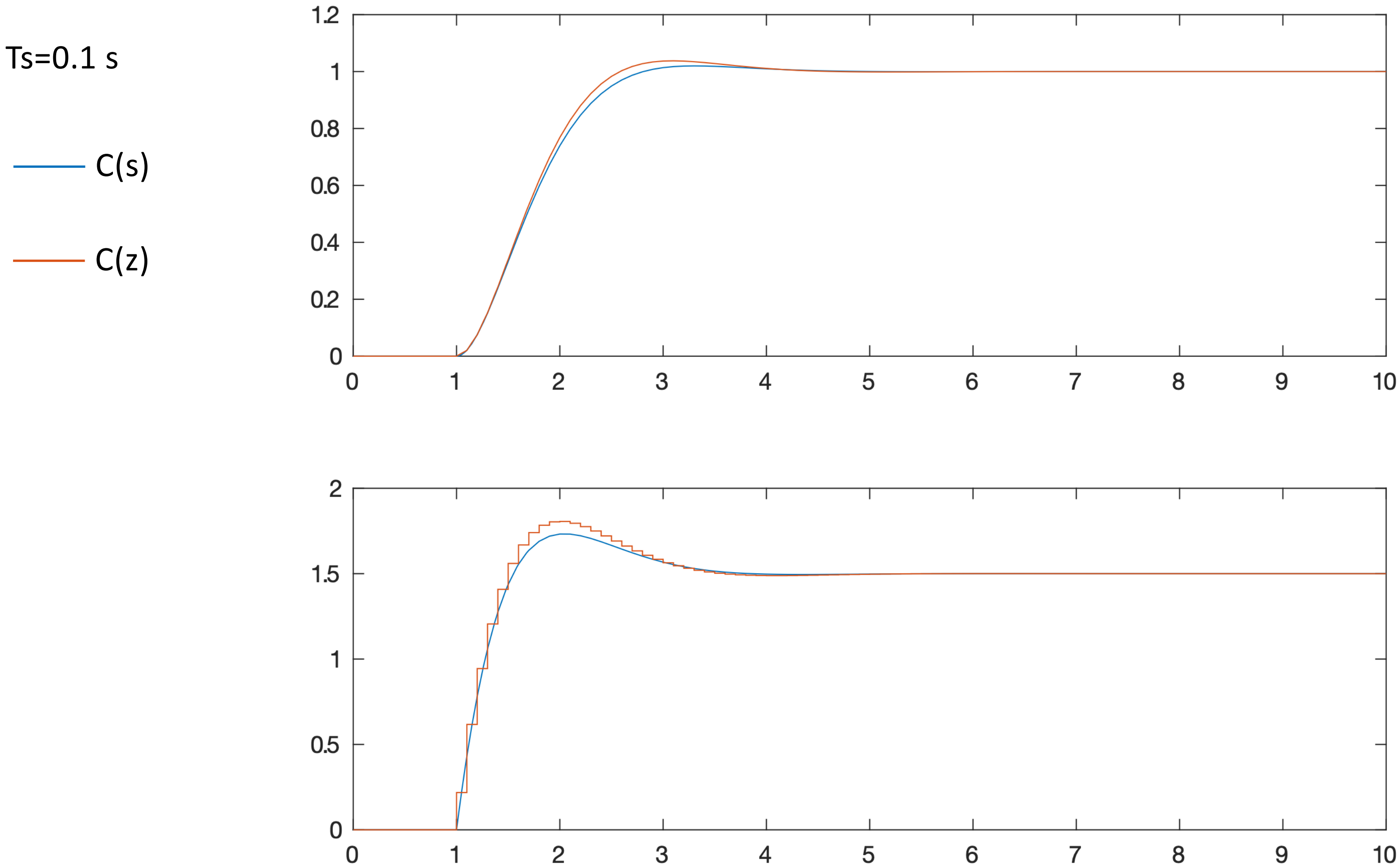
However, it must be emphasized that the emulation approach is *not* guaranteed to yield satisfactory results.

Specifically, if the sampling time T_s is relatively large compared to the system's time constants, a badly damped or even unstable closed-loop system can result.

In these cases, a correct discrete-time design process can substantially improve the closed-loop behavior of the control systems. Matlab offers tools to compute the emulated controller coefficients (type `help c2dm` for more information).

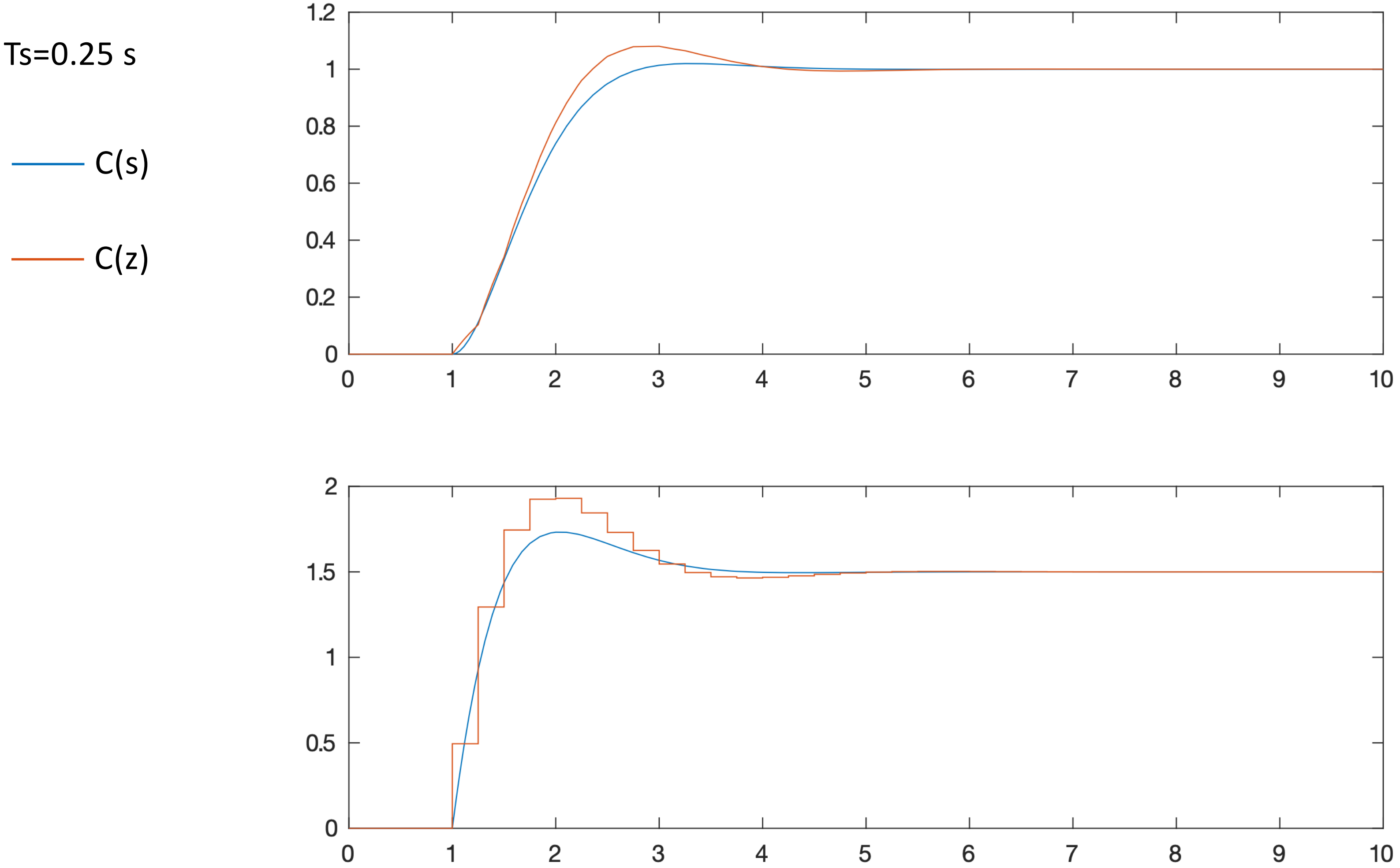
```
np=[0,0,1]
dp=[0,1,1.5]
kp=4.75
a=3.4
nc=kp*[0,1,1.5]
dc=[1,a,0]
roots(conv(np,nc))
roots(conv(np,nc)+conv(dp,dc))

Ts=0.25
Tstop=10
td=[0:Ts:Tstop]';
[ncd,dcd]=c2dm(nc,dc,Ts,'tustin')

sim('ExTustin')

subplot(211)
plot(t,y,t,yd)
subplot(212)
plot(t,u),hold on
stairs(td,ud), hold off
subplot(111)
```

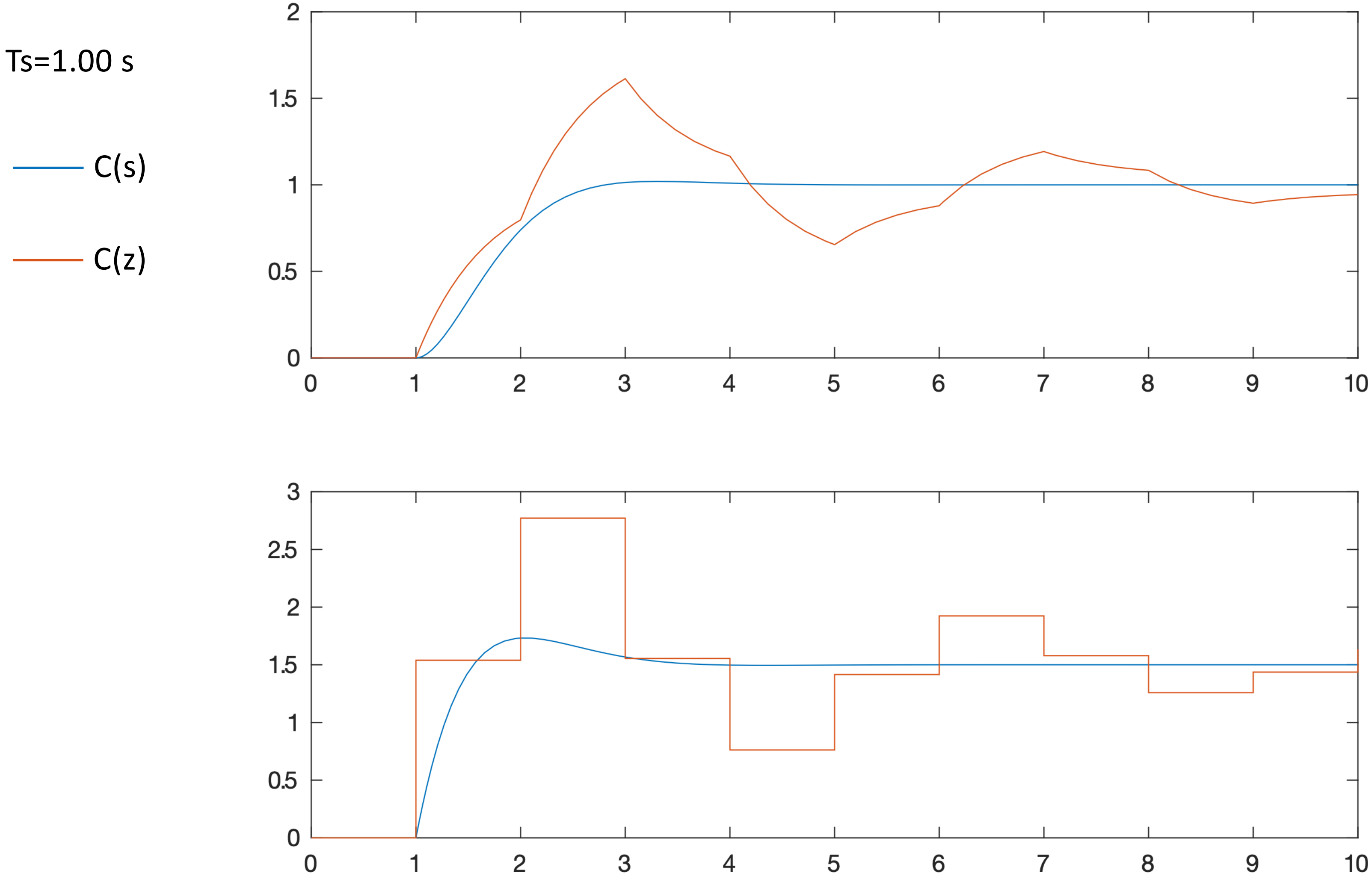




Ts=0.1 s
C(s)
C(z)
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8
9
10

Ts=0.25 s




Ts=1.00 s
C(s)
C(z)
2
1.5
1
0.5
0
0 1 2 3 4 5 6 7 8 9 10
3
2.5
2
1.5
1
0.5
0
0 1 2 3 4 5 6 7 8 9 10

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