# Parallele Programmierung FS25

Exercise Session 6

Jonas Wetzel

### Plan für heute

- Organisation
- Nachbesprechung Exercise 5
- Theory Recap
- Intro Exercise 6
- Exam Questions
- Kahoot

- Mein Name ist Jonas Wetzel
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- Feedback zur Session: <a href="https://forms.gle/qiDnqkfSP2NUQGvc9">https://forms.gle/qiDnqkfSP2NUQGvc9</a>

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- Falls ihr Feedback möchtet kommt bitte zu mir

## **Exam Preparation Session**

- Monday, March 31, 11:15 12:00
- Tuesday, April 1, 10:15 12:00

• HG F 5 / HG F 7

Hosted by Vera Schubert and Jackson Stanhope

### Wo sind wir jetzt?

Date	Title
Feb 17	Introduction & Course Overview
Feb 18	Java Recap and JVM Overview
Feb 24	Introduction to Threads and Synchronization (Part I)
Feb 25	Introduction to Threads and Synchronization (Part II)
Mar 3	Introduction to Threads and Synchronization (Part III)
Mar 4	Parallel Architectures: Parallelism on the Hardware Level
Mar 10	Basic Concepts in Parallelism
Mar 11	Divide & Conquer and Executor Service
Mar 17	DAG and ForkJoin Framework
Mar 18	Parallel Algorithms (Part I)
Mar 24	Parallel Algorithms (Part II)
Mar 25	Shared Memory Concurrency, Locks and Data Races
Mar 31	Virtual Threads
Apr 01	Exam Preparation (First Half)

### Plan für heute

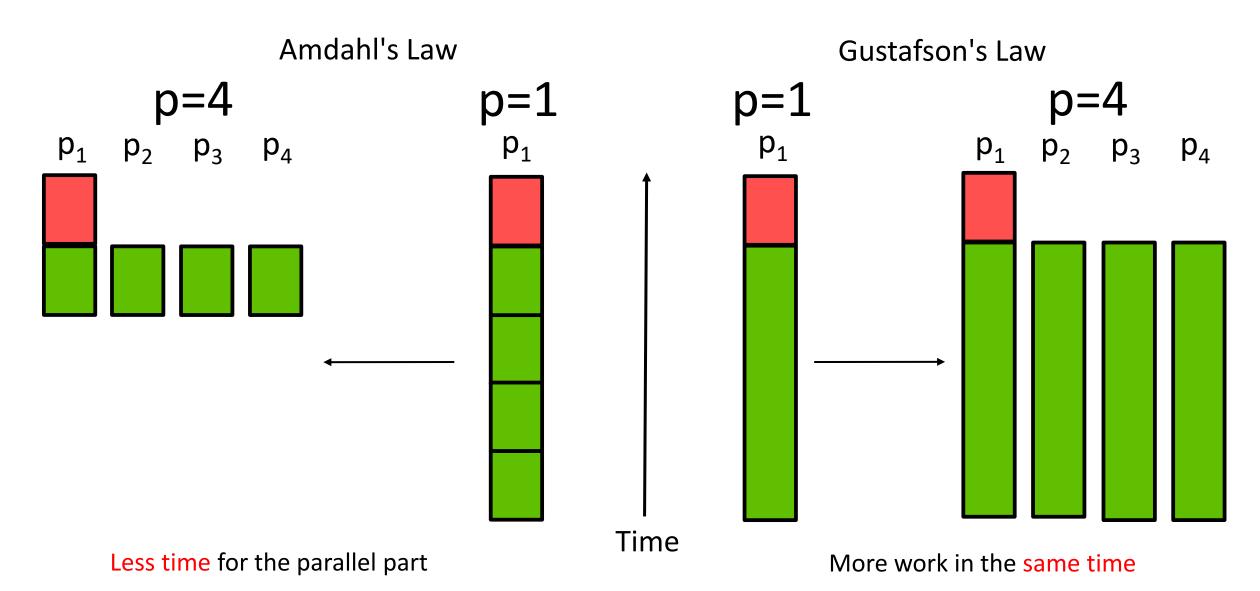
- Organisation
- Nachbesprechung Exercise 5
- Theory Recap
- Intro Exercise 6
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#### The key goal is to:

- → Understand the main difference and implications (i.e., when to use which formula)
- → Know how to derive the formulas based on your understanding, not because you memorized them for the exam

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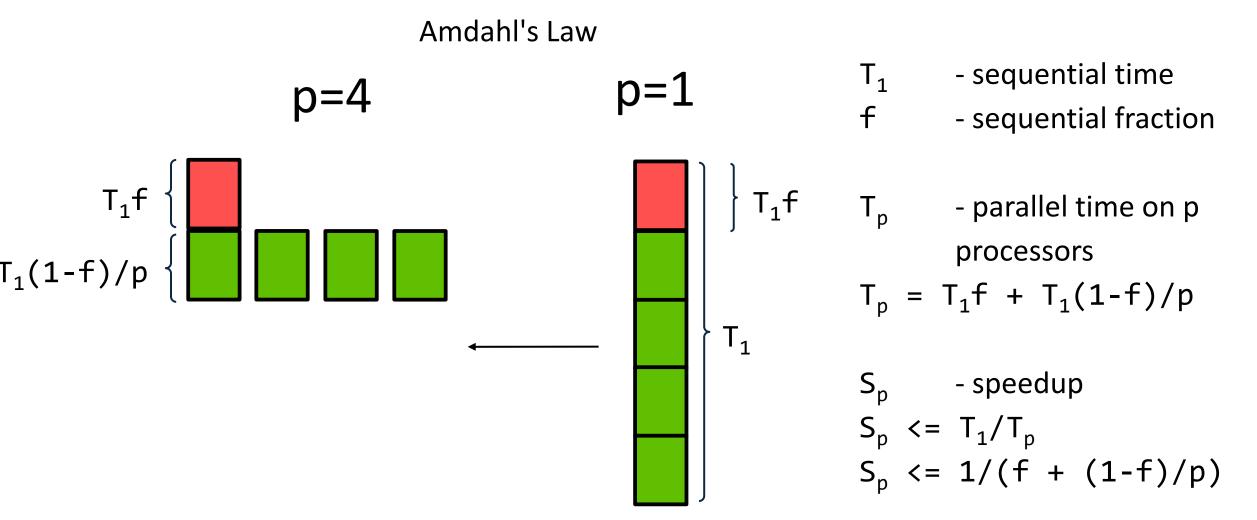
- → Understand the main difference and implications (i.e., when to use which formula)
- → Know how to derive the formulas based on your understanding, not because you memorized them for the exam



#### The key goal is to:

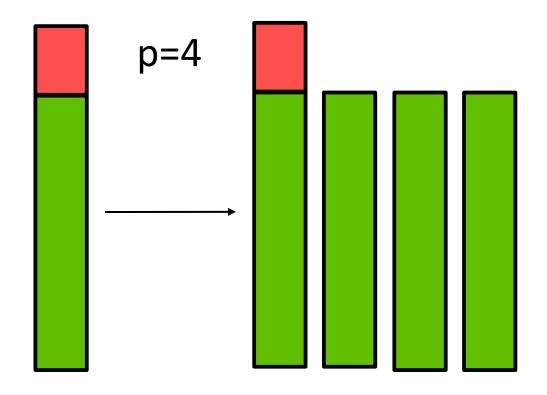
- → Understand the main difference and implications (i.e., when to use which formula)
- → Know how to derive the formulas based on your understanding, not because you memorized them for the exam

### Amdahl's Law Derivation



Less time for the parallel part

#### Gustafson's Law



More work in the same time

T - sequential time of original work

T<sub>1</sub> - sequential time with work\*p

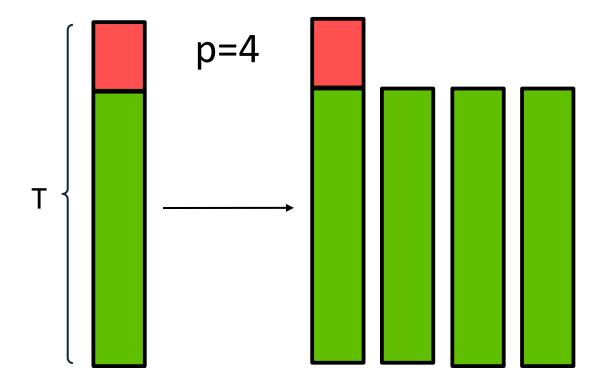
f - sequential fraction

 $T_1 = ?$ 

 $T_p$  - parallel time on p processors  $T_p =$ ?

 $S_p$  - speedup  $S_p = T_1/T_p$  $S_p = ?$ 

#### Gustafson's Law



More work in the same time

T - sequential time of original work

T<sub>1</sub> - sequential time with work\*p

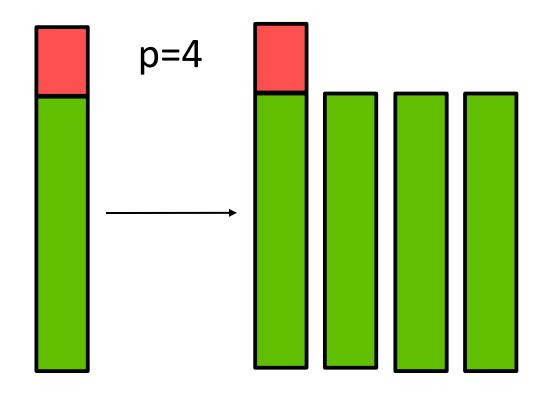
f - sequential fraction

$$T_1 = Tf + T(1-f)p$$

 $T_p$  - parallel time on p processors  $T_p =$ ?

$$S_p$$
 - speedup  
 $S_p = T_1/T_p$   
 $S_p = ?$ 

#### **Gustafson's Law**



More work in the same Time

T - sequential time of original work

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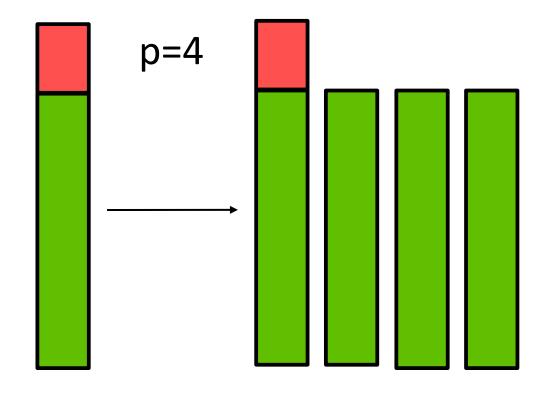
f - sequential fraction

$$T_1 = Tf + T(1-f)p$$

 $T_p$  - parallel time on p processors  $T_p = Tf + T(1-f)p/p = T$ 

$$S_p$$
 - speedup  
 $S_p = T_1/T_p$   
 $S_p = ?$ 

#### **Gustafson's Law**



More work in the same time

T - sequential time of original work

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$$T_1 = Tf + T(1-f)p$$

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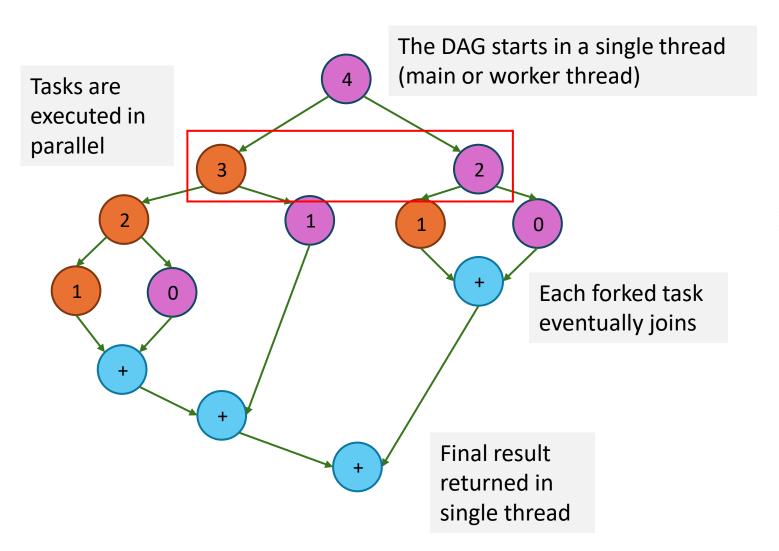
$$S_p$$
 - speedup  
 $S_p = T_1/T_p$   
 $S_p = f + (1-f)p$ 

# fib(4) task graph

```
public class Fibonacci {
   public static long fib(int n) {
      if (n < 2) {
          return n;
      }
      spawn task for fib(n-1);
      spawn task for fib(n-2);
      wait for tasks to complete
      return addition of task results
   }
}</pre>
```

#### essential

# fib(4) task graph FJ



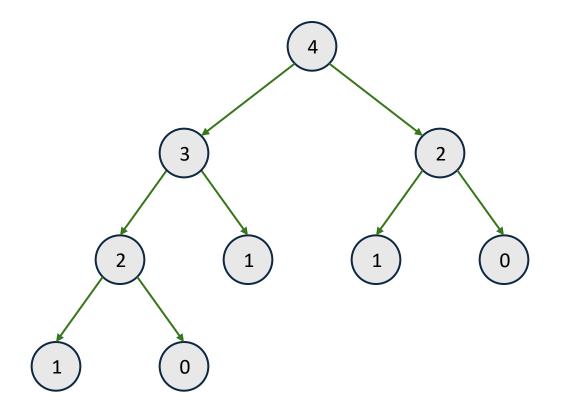
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public class Fibonacci {
   public static long fib(int n) {
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      wait for tasks to complete
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   }
}</pre>
```

#### What is a task?

#### What is an edge?

spawn, same procedure, wait

# fib(4) simplified task graph



Simpler at the expense of not modelling joins and inter-process dependencies

```
public class Fibonacci {
   public static long fib(int n) {
      if (n < 2) {
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```

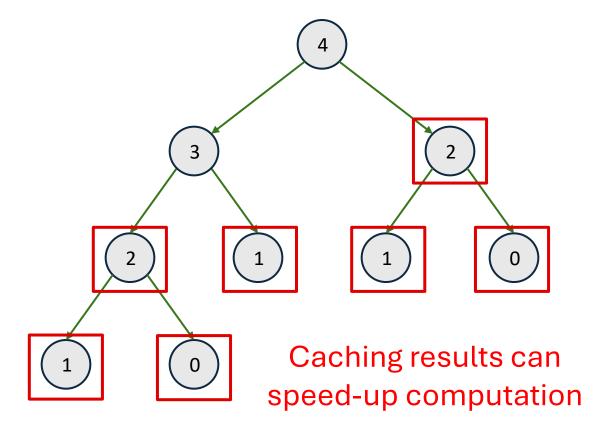
#### What is a task?

Call to Fibonacci

#### What is an edge?

spawn
(no dependency within same procedure)

# fib(4) simplified task graph



Simpler at the expense of not modelling joins and inter-process dependencies

```
public class Fibonacci {
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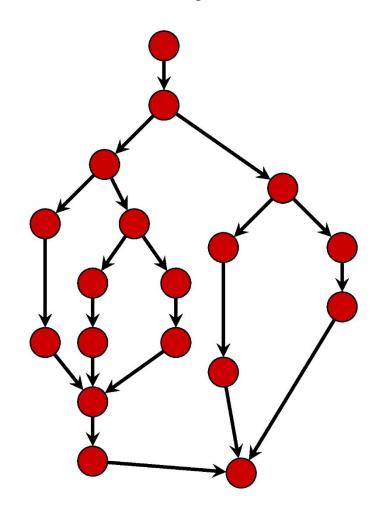
#### What is a task?

Call to Fibonacci

#### What is an edge?

spawn (no dependency within same procedure)

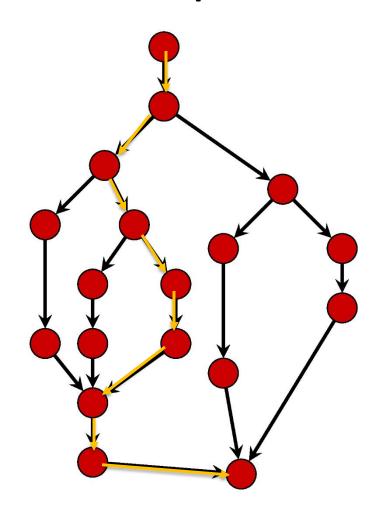
## Task Graphs



Critical path: path from start to end that takes the longest (for some metric)

Example: #nodes

## Task Graphs

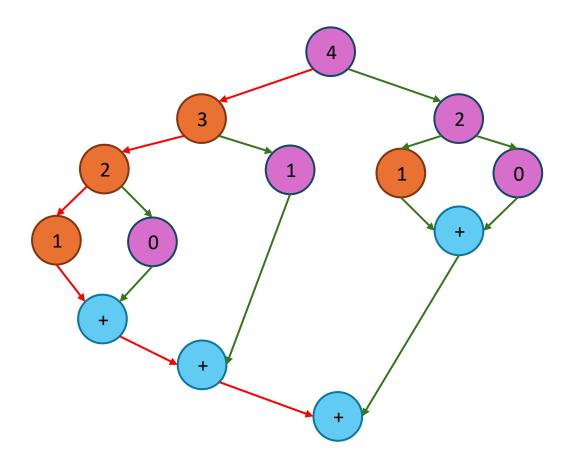


Critical path: path from start to end that takes the longest (for some metric)

Example: #nodes

#### essential

## fib(4) task graph FJ



critical path length is 7 tasks

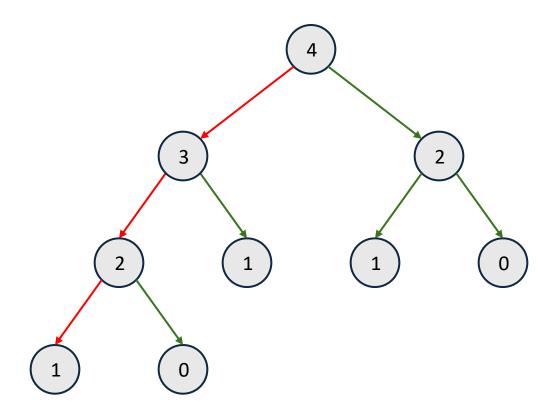
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public class Fibonacci {
   public static long fib(int n) {
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      spawn task for fib(n-1);
      spawn task for fib(n-2);
      wait for tasks to complete
      return addition of task results
   }
}</pre>
```

#### What is a task?

#### What is an edge?

spawn, same procedure, wait

# fib(4) simplified task graph



critical path length is 4 tasks

```
public class Fibonacci {
   public static long fib(int n) {
      if (n < 2) {
         return n;
      }
      spawn task for fib(n-1);
      spawn task for fib(n-2);
      wait for tasks to complete
      return addition of task results
   }
}</pre>
```

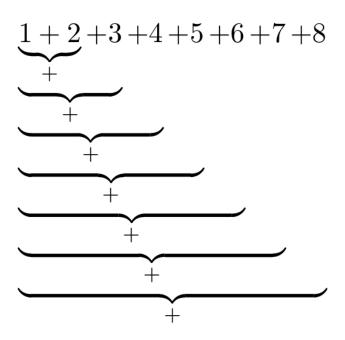
#### What is a task?

Call to Fibonacci

#### What is an edge?

spawn
(no dependency within same procedure)

### Adding eight numbers:



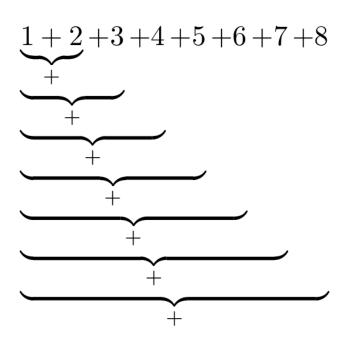
Task: Call to add()

Cut-off: 1

Task: Call to add()

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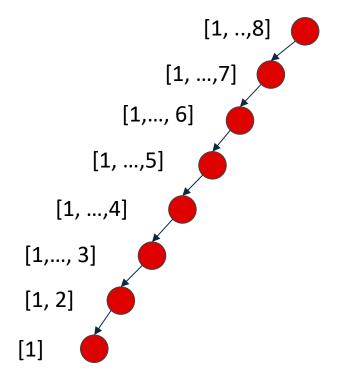
### Adding eight numbers:



Task: Call to add()

Cut-off: 1

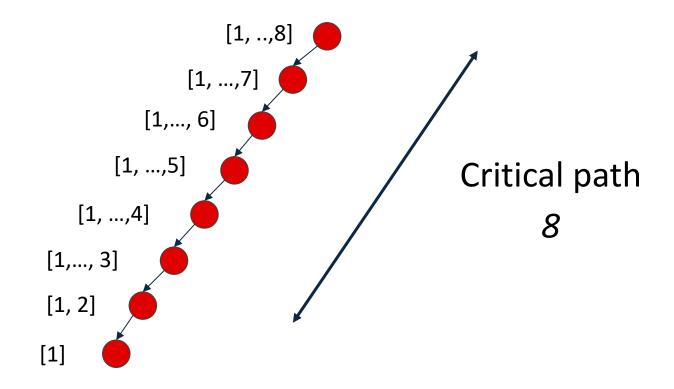
### Adding eight numbers:



Task: Call to add()

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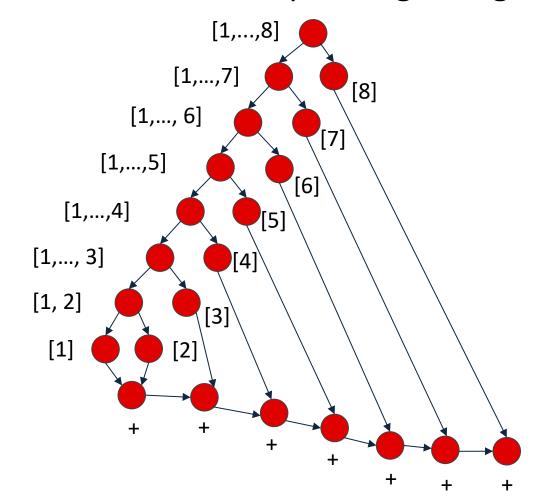


## Task Graph FJ

### Adding eight numbers:

#### Task: fork, join, continuation

Cut-off: 1

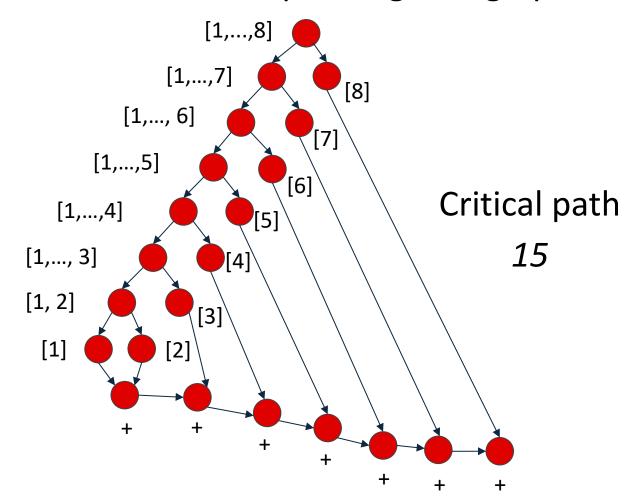


## Task Graph FJ

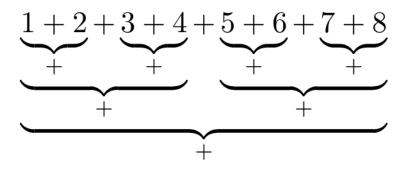
### Adding eight numbers:

#### Task: fork, join, continuation

Cut-off: 1



### Adding eight numbers:



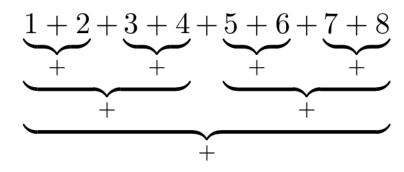
Task: Call to add()

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Task: Call to add()

Cut-off: 1

Adding eight numbers:

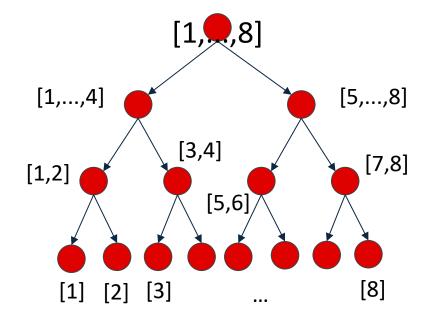


Task: Call to add()

Cut-off: 1

### Adding eight numbers:

$$\underbrace{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8}_{+}$$

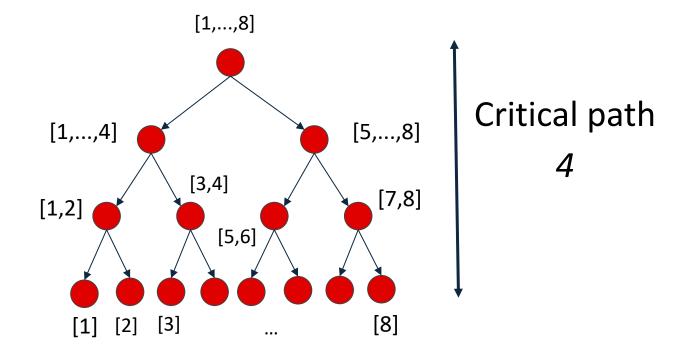


Task: Call to add()

Cut-off: 1

### Adding eight numbers:

$$\underbrace{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8}_{+}$$



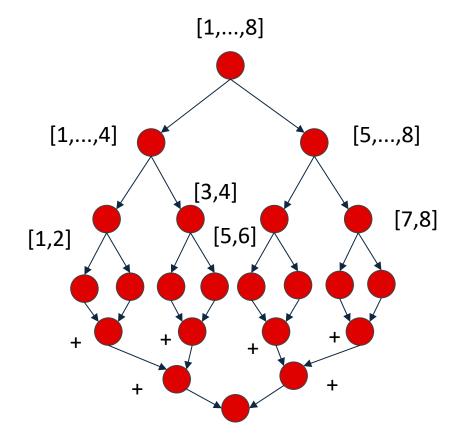
## Task Graph FJ

Task: fork, join, continuation

Cut-off: 1

### Adding eight numbers:

$$\underbrace{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8}_{+}$$



## Task Graph FJ

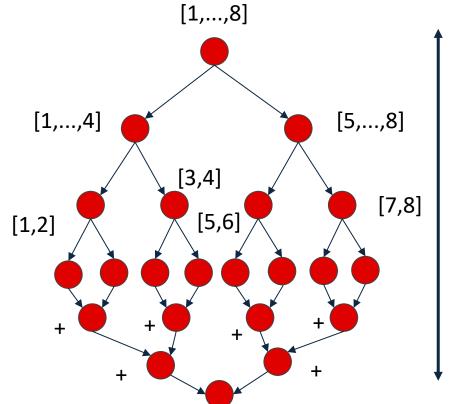
Task: fork, join, continuation

Cut-off: 1

### Adding eight numbers:

$$\underbrace{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8}_{+}$$

### What is the corresponding task graph?



Critical path

## Task Graphs

A wide task graph → higher potential parallelism

A deep task graph → more sequential dependencies

- There is a master solution, feel free to take a look if you had trouble with these theory task
- I'll also upload my code solution

# **Coding Part**

### Task 1: Search And Count

Search an array of integers for a certain feature and count integers that have this feature:

- . Light workload: count number of non-zero values.
- Heavy workload: count how many integers are prime numbers.

We will study single threaded and multi-threaded implementation of the problem.

## Task 1 A: Search And Count - Sequential

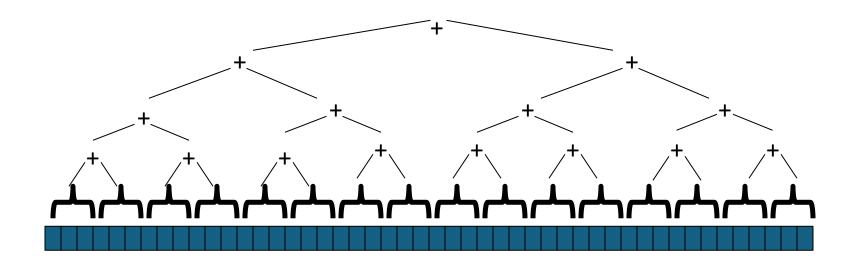
```
private int[] input;
private Workload.Type type;
private SearchAndCountSingle(int[] input, Workload.Type wt) {
  this.input = input;
  this.type = wt;
private int count() {
  int count = 0;
  for (int i = 0; i < input.length; i++) {</pre>
    if (Workload.doWork(input[i], type)) count++;
  return count;
```

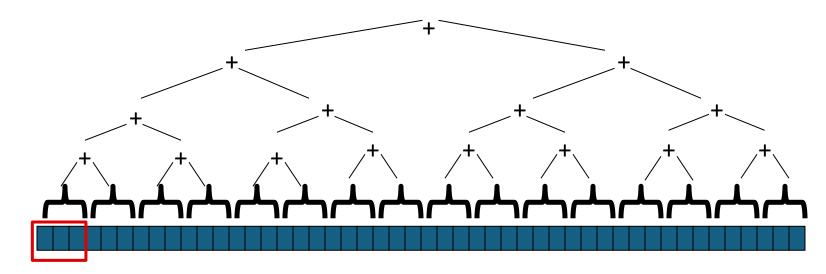
public class SearchAndCountSingle {

Straightforward implementation. Simply iterate through the input array and count how many times given event occurs.

### Basic structure of a divide-and-conquer algorithm:

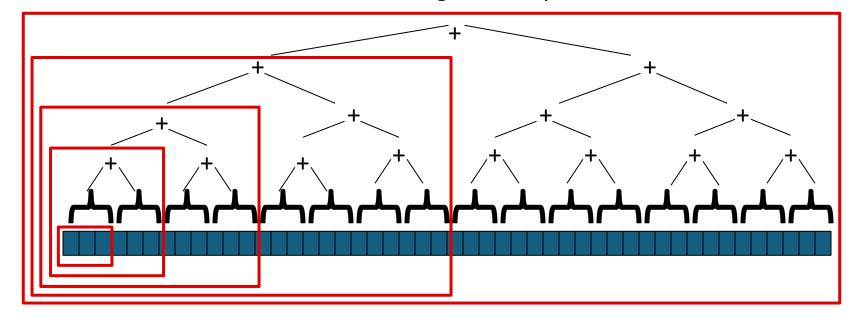
- 1. If problem is small enough, solve it directly
- 2. Otherwise
  - a. Break problem into subproblems
  - b. Solve subproblems recursively
  - c. Assemble solutions of subproblems into overall solution





base case no further split

Tasks at different levels of granularity



What determines a task?

i) input array

ii) start index iii) length/end index

These are fields we want to store in the task

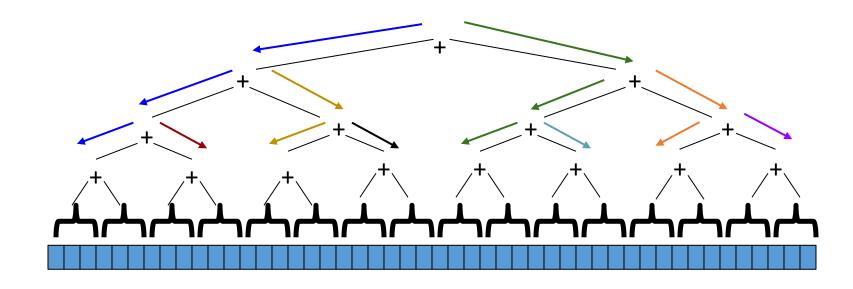
# Feedback: Tasks 1 B-D

thread 1 thread 2 thread 3 thread 4 thread 5

thread 6 thread 7

thread 8

•••



Performance optimization

Same thread is reused instead

of creating a new one

thread 2

thread 1

thread 3

thread 4

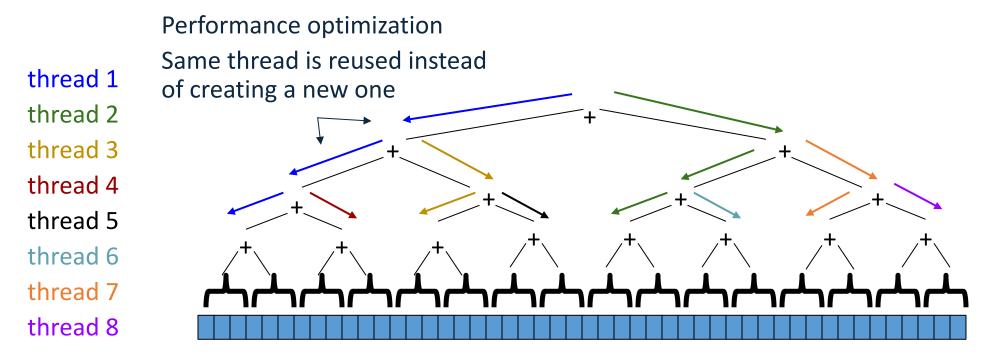
thread 5

thread 6

thread 7

thread 8

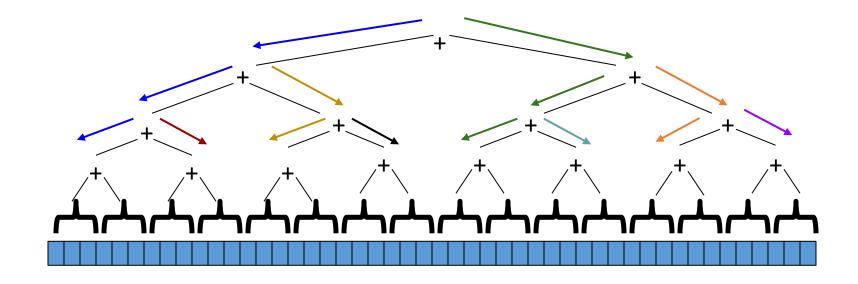
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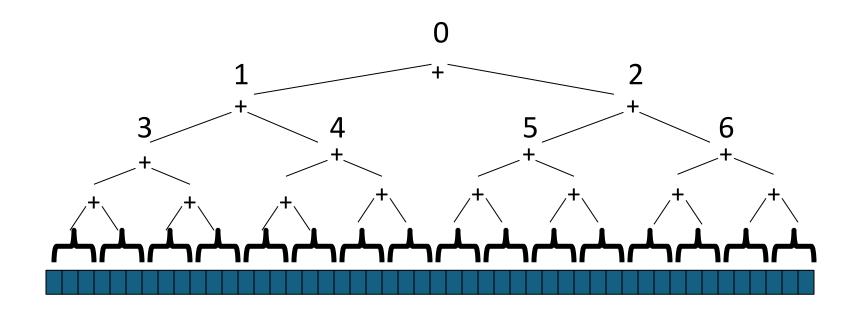
Task B:

Extend your implementation such that it creates only a fixed number of threads. Make sure that your solution is properly synchronized when checking whether to create a new thread

How to achieve this?



Option 1:
Shared counter with synchronized/atomic access

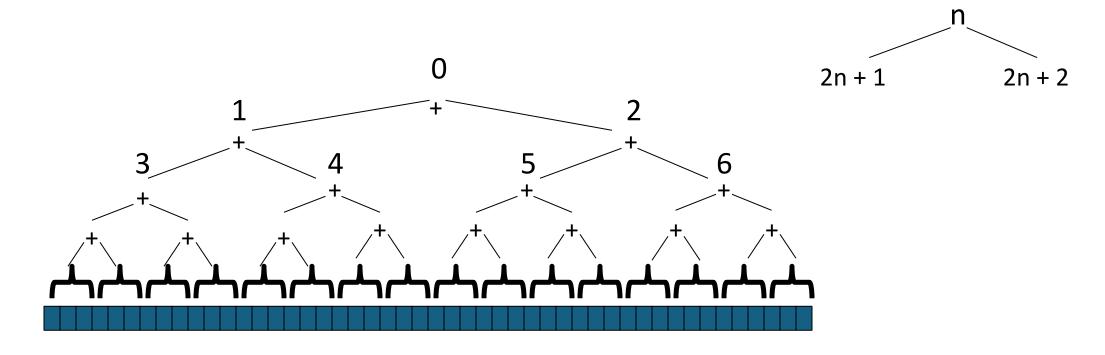


### Option 1:

Shared counter with synchronized/atomic access

#### Option 2:

Assign unique sequential id to each task. Spawn threads for first N tasks.

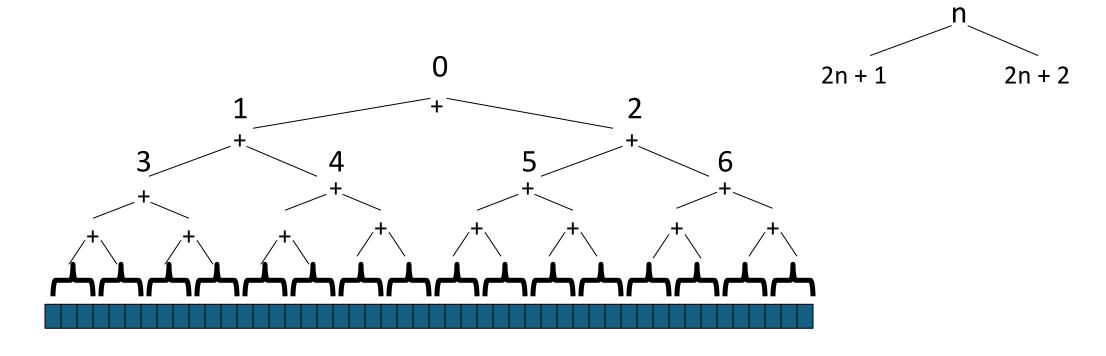


### Option 1:

Shared counter with synchronized/atomic access

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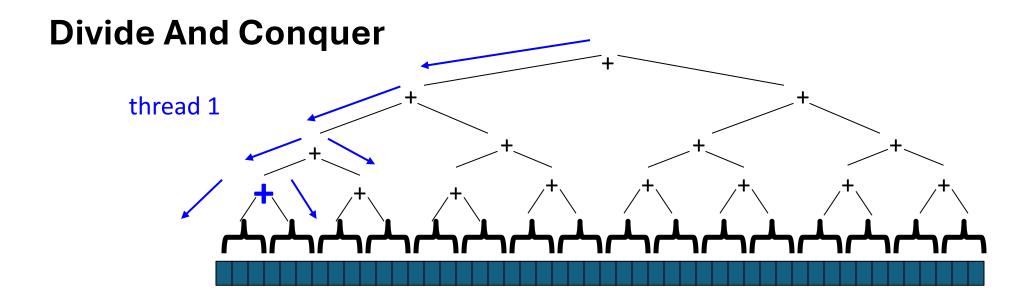
- + no synchronization required
- imbalanced amount of work

### **Divide And Conquer**

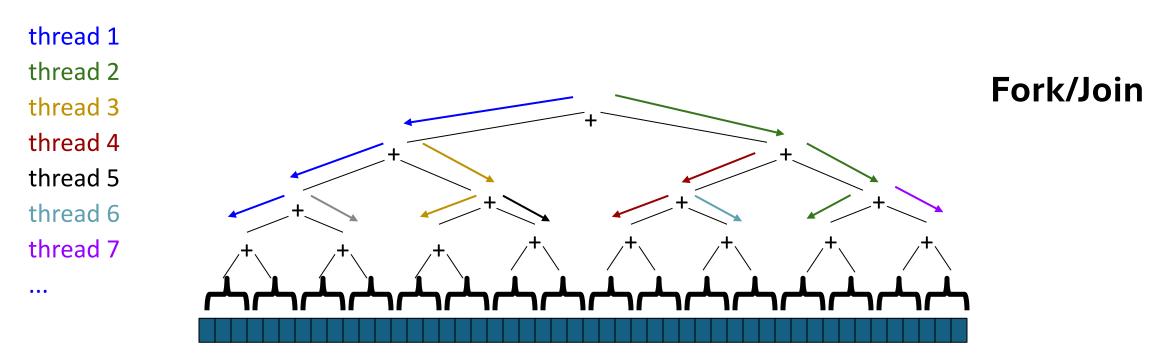
Fundamental design pattern based on recursively breaking down a problem into smaller problems that can be combined to give a solution to the original problem

#### Fork/Join

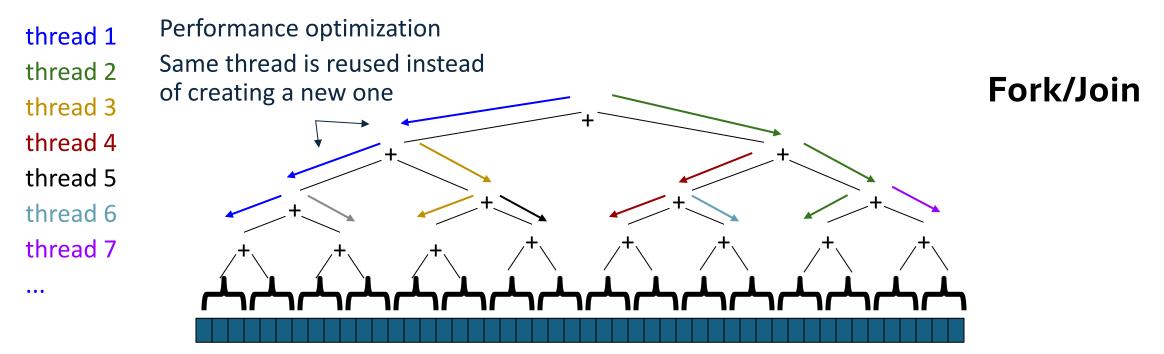
A framework that supports Divide and Conquer style parallelism



recursively breaking down a problem into smaller problems problems are solved sequentially



a framework that supports Divide and Conquer style parallelism problems are solved in parallel



a framework that supports Divide and Conquer style parallelism problems are solved in parallel

## Search And Count - Task Parallel

#### Define the task structure:

```
public class SearchAndCountMultiple extends RecursiveTask<Integer> {
    private int[] input;
    private int start;
    private int length;
    private int cutOff;
    private Workload.Type workloadType;
}
```

```
protected Integer compute() {
```

Recall the template for divide and conquer task parallelism

}

```
protected Integer compute() {
 if (// work is small) {
   // do the work directly
                                           Recall the template for
 else {
   // split work into pieces
                                          divide and conquer
                                           task parallelism
   // invoke the pieces and
      wait for the results
   // combine the results
```

for

task

```
protected Integer compute() {
 if (// work is small) {
   // do the work directly
                                          Recall the template
 else {
   // split work into pieces
                                          divide and conquer
                                           parallelism
                                          Let's fill in the template for
   // invoke the pieces and
      wait for the results
                                           the search and count task
   // combine the results
```

```
protected Integer compute() {
                                     protected Integer compute() {
  if (// work is small) {
                                       if (// work is small) {
   // do the work directly
                                         // do the work directly
  else {
                                       else {
   // split work into pieces
                                        // split work into pieces
   // invoke the pieces and
                                         // invoke the pieces and
      wait for the results
                                            wait for the results
   // combine the results
                                         // combine the results
```

```
public class SearchAndCountMultiple
  extends RecursiveTask<Integer> {
      private int[] input;
      private int start;
      private int length;
      private int cutOff;
      private Workload.Type type;
}
```

```
protected Integer compute() {
                                      protected Integer compute() {
  if (// work is small)
                                        if (length <= cutOff) {</pre>
   // do the work directly
                                         // do the work directly
  else {
                                        else {
   // split work into pieces
                                         // split work into pieces
   // invoke the pieces and
                                         // invoke the pieces and
       wait for the results
                                             wait for the results
   // combine the results
                                          // combine the results
```

```
public class SearchAndCountMultiple
  extends RecursiveTask<Integer> {
      private int[] input;
      private int start;
      private int length;
      private int cutOff;
      private Workload.Type type;
}
```

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protected Integer compute() {
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  if (// work is small)
                                        if (length <= cutOff) {</pre>
   // do the work directly
                                          // do the work directly
                                        else {
  else {
                                         // split work into pieces
   // split work into pieces
   // invoke the pieces and
                                         // invoke the pieces and
       wait for the results
                                             wait for the results
   // combine the results
                                          // combine the results
```

```
public class SearchAndCountMultipleSSential
   extends RecursiveTask<Integer> {
          private int[] input;
          private int start;
          private int length;
          private int cutOff;
          private Workload.Type type;
```

```
protected Integer compute() {
   if (// work is small)

   // do the work directly

else {
   // split work into pieces
```

```
protected Integer compute() {
    if (length <= cutOff) {
        int count = 0;
        for (int i = start; i < start + length; i++) {
            if (Workload.doWork(input[i], type)) count++;
        }
        return count;
    else {
        // split work into pieces</pre>
```

public class SearchAndCountMultipleSSential

extends RecursiveTask<Integer> {

private int start; private int length;

Same as sequential

implementation

private int[] input;

```
// invoke the pieces and
     wait for the results

// combine the results
}
```

```
// invoke the pieces and
    wait for the results

// combine the results
}
```

```
protected Integer compute() {
  if (// work is small)
   // do the work directly
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                                                     private int[] input;
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protected Integer compute() {
                                                     private Workload.Type type;
  if (length <= cutOff) {</pre>
    int count = 0;
    for (int i = start; i < start + length; i++) {</pre>
      if (Workload.doWork(input[i], type)) count++;
    return count;
  else {
    // split work into pieces
    // invoke the pieces and
       wait for the results
```

// combine the results

```
if (// work is small)
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else {
 // split work into pieces
 // invoke the pieces and
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 // combine the results
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protected Integer compute() {

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                                                    private int length;
                                                    private int cutOff;
protected Integer compute() {
                                                    private Workload.Type type;
  if (length <= cutOff) {</pre>
    int count = 0;
    for (int i = start; i < start + length; i++) {</pre>
      if (Workload.doWork(input[i], type)) count++;
    return count;
  else {
    int half = (length) / 2;
    SearchAndCountMultiple sc1 =
      new SearchAndCountMultiple(input, start, half, cutOff, type);
    SearchAndCountMultiple sc2 =
      new SearchAndCountMultiple(input, start + half, length - half, cutOff, type);
   // invoke the pieces and
       wait for the results
    // combine the results
```

```
protected Integer compute() {
  if (// work is small)
   // do the work directly
  else {
   // split work into pieces
   // invoke the pieces and
       wait for the results
    // combine the results
```

```
public class SearchAndCountMultipleSSential
                                             extends RecursiveTask<Integer> {
                                                    private int[] input;
                                                    private int start;
                                                    private int length;
                                                    private int cutOff;
protected Integer compute() {
                                                    private Workload.Type type;
  if (length <= cutOff) {</pre>
    int count = 0;
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      if (Workload.doWork(input[i], type)) count++;
    return count;
  else {
    int half = (length) / 2;
    SearchAndCountMultiple sc1 =
      new SearchAndCountMultiple(input, start, half, cutOff, type);
    SearchAndCountMultiple sc2 =
     new SearchAndCountMultiple(input, start + half, length - half, cutOff, type);
    // invoke the pieces and
       wait for the results
    // combine the results
```

```
protected Integer compute() {
  if (// work is small)
   // do the work directly
  else {
   // split work into pieces
   // invoke the pieces and
       wait for the results
    // combine the results
```

```
extends RecursiveTask<Integer> {
                                                    private int[] input;
                                                    private int start;
                                                    private int length;
                                                    private int cutOff;
protected Integer compute() {
                                                    private Workload.Type type;
  if (length <= cutOff) {</pre>
    int count = 0;
    for (int i = start; i < start + length; i++) {</pre>
      if (Workload.doWork(input[i], type)) count++;
    return count;
  else {
    int half = (length) / 2;
    SearchAndCountMultiple sc1 =
      new SearchAndCountMultiple(input, start, half, cutOff, type);
    SearchAndCountMultiple sc2 =
      new SearchAndCountMultiple(input, start + half, length - half, cutOff, type);
    sc1.fork();
    sc2.fork();
    int count1 = sc1.join();
    int count2 = sc2.join();
    // combine the results
```

```
protected Integer compute() {
  if (// work is small)
   // do the work directly
  else {
   // split work into pieces
   // invoke the pieces and
       wait for the results
    // combine the results
```

```
extends RecursiveTask<Integer> {
                                                    private int[] input;
                                                    private int start;
                                                    private int length;
                                                    private int cutOff;
protected Integer compute() {
                                                    private Workload.Type type;
  if (length <= cutOff) {</pre>
    int count = 0;
    for (int i = start; i < start + length; i++) {</pre>
      if (Workload.doWork(input[i], type)) count++;
    return count;
  else {
    int half = (length) / 2;
    SearchAndCountMultiple sc1 =
      new SearchAndCountMultiple(input, start, half, cutOff, type);
    SearchAndCountMultiple sc2 =
      new SearchAndCountMultiple(input, start + half, length - half, cutOff, type);
    sc1.fork();
    sc2.fork();
    int count1 = sc1.join();
    int count2 = sc2.join();
    // combine the results
```

```
protected Integer compute() {
  if (// work is small)
   // do the work directly
  else {
   // split work into pieces
   // invoke the pieces and
       wait for the results
   // combine the results
```

```
extends RecursiveTask<Integer> {
                                                     private int[] input;
                                                     private int start;
                                                     private int length;
                                                     private int cutOff;
protected Integer compute() {
                                                     private Workload.Type type;
  if (length <= cutOff) {</pre>
    int count = 0;
    for (int i = start; i < start + length; i++) {</pre>
      if (Workload.doWork(input[i], type)) count++;
    return count;
  else {
    int half = (length) / 2;
    SearchAndCountMultiple sc1 =
      new SearchAndCountMultiple(input, start, half, cutOff, type);
    SearchAndCountMultiple sc2 =
     new SearchAndCountMultiple(input, start + half, length - half, cutOff, type);
    sc1.fork();
    sc2.fork();
    int count1 = sc1.join();
    int count2 = sc2.join();
    return count1 + count2;
```

essentials public class SearchAndCountMultiple

extends RecursiveTask<Integer> {

### Search And Count

```
protected Integer compute() {
  if (// work is small)
   // do the work directly
  else {
   // split work into pieces
   // invoke the pieces and
       wait for the results
    // combine the results
```

```
private int[] input;
                                                    private int start;
                                                    private int length;
                                                    private int cutOff;
protected Integer compute() {
                                                    private Workload.Type type;
  if (length <= cutOff) {</pre>
    int count = 0;
    for (int i = start; i < start + length; i++) {</pre>
      if (Workload.doWork(input[i], type)) count++;
    return count;
  else {
    int half = (length) / 2;
    SearchAndCountMultiple sc1 =
      new SearchAndCountMultiple(input, start, half, cutOff, type);
    SearchAndCountMultiple sc2 =
      new SearchAndCountMultiple(input, start + half, length - half, cutOff, type);
    sc1.fork();
    sc2.fork();
    int count1 = sc1.join();
    int count2 = sc2.join();
    return count1 + count2;
```

### ExecutorService

#### TPS01-J. Do not execute interdependent tasks in a bounded thread pool

Created by Dhruv Mohindra, last modified by Carol J. Lallier on Jun 22, 2015

Bounded thread pools allow the programmer to specify an upper limit on the number of threads that can concurrently execute in a thread pool. Programs must not use threads from a bounded thread pool to execute tasks that depend on the completion of other tasks in the pool.

A form of deadlock called *thread-starvation deadlock* arises when all the threads executing in the pool are blocked on tasks that are waiting on an internal queue for an available thread in which to execute. Thread-starvation deadlock occurs when currently executing tasks submit other tasks to a thread pool and wait for them to complete and the thread pool lacks the capacity to accommodate all the tasks at once.

This problem can be confusing because the program can function correctly when fewer threads are needed. The issue can be mitigated, in some cases, by choosing a larger pool size. However, determining a suitable size may be difficult or even impossible.

Similarly, threads in a thread pool may fail to be recycled when two executing tasks each require the other to complete before they can terminate. A blocking operation within a subtask can also lead to unbounded queue growth [Goetz 2006].

**Fork/Join**: recommended for Divide and Conquer tasks as they have strong task interdependency

ExecutorService: for handling many independent requests where tasks are standalone

• See code

### Plan für heute

- Organisation
- Nachbesprechung Exercise 5
- Theory Recap
- Intro Exercise 6
- Exam Questions
- Kahoot

# Summary

 $T_1$ : Time on one processor

 $T_P$ : Time on P processors

 $T_{\infty}$ : Time on "infinite" processors  $(\lim_{P\to\infty} T_P)$ 

 $S_P = \frac{T_1}{T_P}$ : Speedup generated using P processors

• Amdahls Law assumes a fixed workload in variable time it states:

$$S_P = \frac{T_1}{T_P} \le \frac{1}{f + \frac{1-f}{P}}$$

• Gustafsons Law assumes a fixed time window, but measures the speedup in workload terms:

$$S_P \le f + P(1-f)$$

# Lock Object

Shared object that satisfies the following interface

```
public interface Lock{
    public void lock();  // entering CS
    public void unlock();  // leaving CS
}
```

#### providing the following semantics

new Lock make a new lock, initially "not held"

acquire blocks (only) if this lock is already currently "held"
Once "not held", makes lock "held" [all at once!]

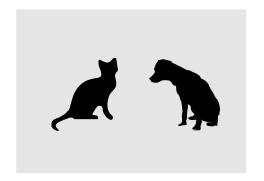
release makes this lock "not held"
If >= 1 threads are blocked on it, exactly 1 will acquire it

18

# Required Properties of Mutual Exclusion

### Safety Property

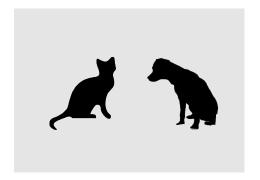
At most one process executes the critical section code



# Required Properties of Mutual Exclusion

### Safety Property

At most one process executes the critical section code



#### Liveness

 Minimally: acquire\_mutex must terminate in finite time when no process executes in the critical section



# Almost-correct pseudocode

```
class BankAccount {
  private int balance = 0;
                                           One lock for
  private Lock lk = new Lock();
                                           each account
  void withdraw(int amount) {
    lk.lock(); // may block
     int b = getBalance();
     if(amount > b)
       throw new WithdrawTooLargeException();
     setBalance(b - amount);
     lk.unlock();
  // deposit would also acquire/release lk
```

# Almost-correct pseudocode

```
class BankAccount {
  private int balance = 0;
                                           One lock for
  private Lock lk = new Lock();
                                           each account
  void withdraw(int amount) {
    lk.lock(); // may block
     int b = getBalance();
     if(amount > b)
       throw new WithdrawTooLargeException();
     setBalance(b - amount);
     lk.unlock();
  // deposit would also acquire/release lk
```

Lock won't be released if exception is thrown!

# Solution: Use try/finally block!

```
Lock lk = new ReentrantLock();
public static long criticalWork() {
  lk.lock();
  try {
    //do some work
    return result;
  } finally {
    lk.release();
```

Always gets executed (even after exception or return)

### Possible mistakes

Incorrect: Use different locks for withdraw and deposit

- Mutual exclusion works only when using same lock
- balance field is the shared resource being protected

Poor performance: Use same lock for every bank account

No simultaneous operations on different accounts

Incorrect: Forget to release a lock (blocks other threads forever!)

Previous slide is wrong because of the exception possibility!

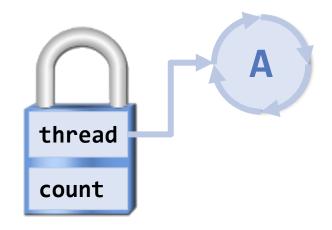
```
if(amount > b) {
   lk.unlock(); // hard to remember!
   throw new WithdrawTooLargeException();
}
```

### Re-entrant lock

A re-entrant lock (a.k.a. recursive lock)

"remembers"

- the thread (if any) that currently holds it
- a count



When the lock goes from *not-held* to *held*, the count is set to 0 If (code running in) the current holder calls **lock** (acquire):

- it does not block
- it increments the count

#### On unlock (release):

- if the count is > 0, the count is decremented
- if the count is 0, the lock becomes not-held

### Re-entrant locks work

This simple code works fine provided 1k is a reentrant lock

Okay to call setBalance directly

 Okay to call withdraw (won't block forever)

```
int setBalance(int x) {
 lk.lock();
  balance = x;
  lk.unlock();
void withdraw(int amount) {
  lk.lock();
  setBalance(b - amount);
  lk.unlock();
```

# Waiting for lock

### lock()

Acquires the lock and blocks indefinitely until it is available

### tryLock()

- Attempts to acquire the lock without blocking
- Returns true if successful, false if the lock is already held
- Supports timeouts

### Race condition

A **Race Condition** occurs in concurrent programming when the correctness of the system depends on the specific interleaving or ordering of operations executed by multiple threads or processes.

Typically, problem is some *intermediate state* that "messes up" a concurrent thread that "sees" that state

Note: This lecture makes a big distinction between *data races* and *bad interleavings*, both instances of race-condition bugs

 Confusion often results from not distinguishing these or using the ambiguous "race condition" to mean only one

### The distinction

**Data Race** [aka Low Level Race Condition, low semantic level] Erroneous program behavior caused by insufficiently synchronized accesses of a shared resource by multiple threads, e.g. Simultaneous read/write or write/write of the same memory location

(for mortals) always an error, due to compiler & HW

**Bad Interleaving** [aka *High Level Race Condition, high semantic level*] Erroneous program behavior caused by an unfavorable execution order of a multithreaded algorithm that makes use of otherwise well synchronized resources.

"Bad" depends on your specification

# On low- and high-level data races

Shared data **balance**, access protected by synchronized

#### Forgot **synchronized** in **withdraw**:

- withdraw accesses balance only under lock (via setBalance / getBalance)
- No concurrent read / write or write / write accesses of balance
  - → No low-level data race
- Two withdraw operations can be interleaved – if this is a problem depends or } the specification of our bank account
- We might get unwanted interleavings (intermediate states that should not be observed / violating invariants)
  - → We can still have a high-level data race

```
public synchronized void setBalance(int x) {
public synchronized int getBalance() { .. }
public synchronized void withdraw(int amount)
    b = getBalance()
    setBalance(b - amount);
public synchronized void deposit(int amount) {
    b = getBalance()
    setBalance(b + amount);
```

# Why Locks?

See code examples

# Why Locks?

- example models a bank where multiple threads transfer money between accounts
- Problem:
  - Locking Order Issue: Each transfer call synchronizes first on this and then on target
  - If two threads try to transfer in opposite directions at the same time, they will deadlock
  - e.g. if Thread-1 locks accountA and waits for accountB, while Thread-2 locks accountB and waits for accountA, both will be stuck indefinitely

# Why Locks?

- To prevent deadlocks, it ensures that locks are always acquired in a consistent order
- Prevent deadlock by introducing order with System.identityHashCode() to determine which account should be locked first (or any kind of unique ID)

# Parallel Patterns

## The prefix-sum problem

```
Given int[] input,
produce int[] output where:
output[i] = input[0] + input[1] + ... + input[i]
```

## Sequential prefix-sum

```
int[] prefix_sum(int[] input){
   int[] output = new int[input.length];
   output[0] = input[0];

for(int i = 1; i < input.length; i++)
      output[i] = output[i-1] + input[i];

return output;
}</pre>
```

#### Does not seem parallelizable

- Work: O(n), Span: O(n)
- This algorithm is sequential, but a different algorithm has: Work O(n), Span O(log n)

# Parallel prefix-sum

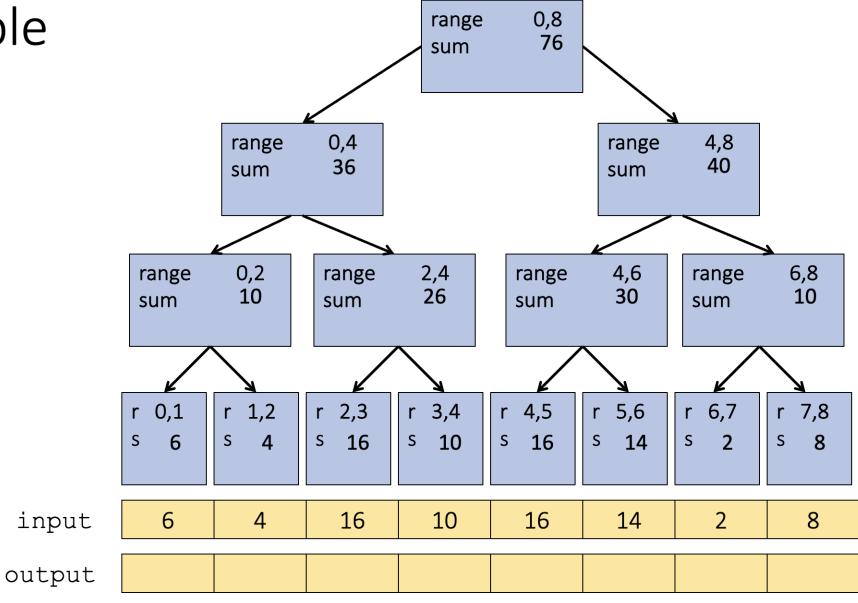
The parallel-prefix algorithm does two passes

- Each pass has O(n) work and O(log n) span
- So in total there is O(n) work and O(log n) span
- So like with array summing, parallelism is n/log n

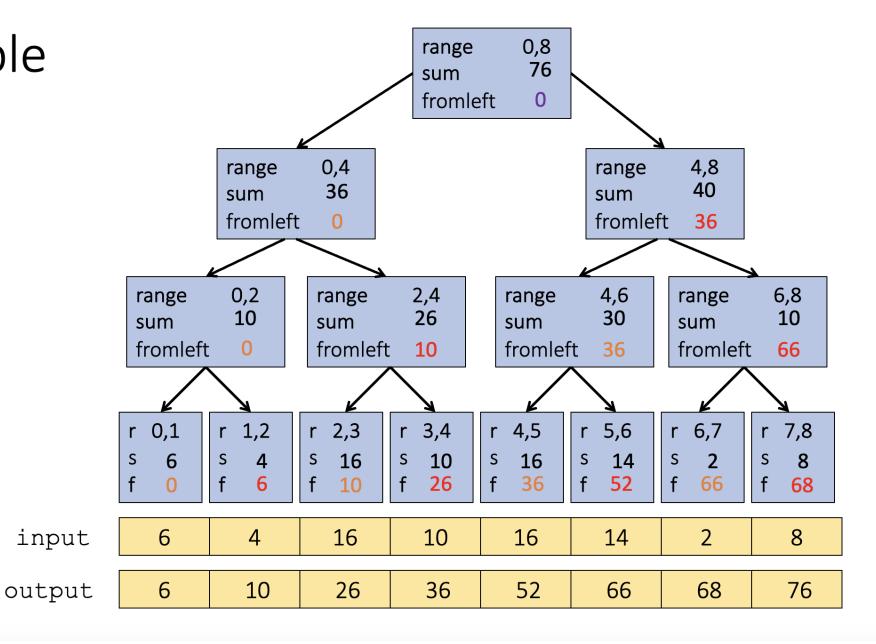
First pass builds a tree bottom-up: the "up" pass

Second pass traverses the tree top-down: the "down" pass

# Example



# Example



### Parallel Patterns

- We are now quite familiar with how to parallize algorithms
- There are a few recurring patterns that are important to know

Map, Reduction, Stencil, Scan, Pack

### Reduction

- A reduction is an operation that produces a single answer from a collection (array etc) via an associative operator.
- Needs to be associative. Otherwise divide-and-conquer won't work

Example: array sum

# Map

- Operates on each element of the input data indenpendently (each array element)
- Output is the same size → no size reduction
- Doesn't have to be the same operation on each element

Example: add two arrays

### Stencil

- Like map but can take more than one element as input
- Generalization of map and thus also no size reduction

### Example:

Image → apply averaging filter on each pixel Update a value based on its neighbors

Never do it in-place because you would then take values that are already output values.

### Scan

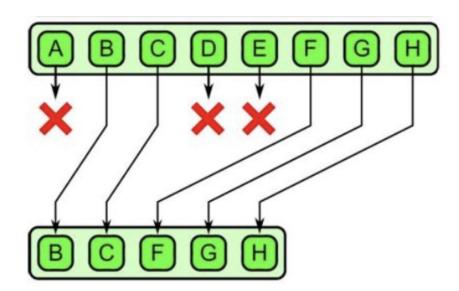
- Collection of data X 

  return collection of data Y
- Y(i) = functionOf( Y(i 1) & X(i) )
- Seems sequential because of dependencies
- Can parallelize if function is associative → O(log(n)) span

Example: parallel prefix sum

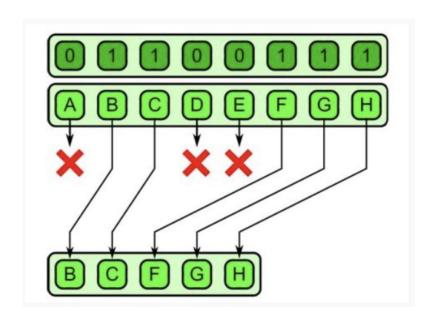
## Pack

Collection of data X → return collection of data X if fulfill condition



### Pack

- First compute bit vector
- Then find index in result array (prefix sum on bit vector)



### Plan für heute

- Organisation
- Nachbesprechung Exercise 5
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- Exam Questions
- Kahoot

# Assignment 6

#### Task Parallelism:

- . Merge Sort
- . Longest Sequence

# Merge sort algorithm

In this exercise you will implement the merge sort algorithm using task parallelism.

The merge sort algorithm partitions the array into smaller arrays, sorts each one separately and then merges the sorted arrays.

- By default, the partitioning of the array continues recursively until the array size is 1 or 2, which then is sorted trivially.
- Try larger cutoff values (e.g partition arrays down to minimum size 4 instead of 2) and see how this affects the algorithm performance.
- · Discuss the asymptotic running time of the algorithm and the obtained speedup.

Given a sequence of numbers:

find the longest sequence of the same consecutive number

Given a sequence of numbers:

find the longest sequence of the same consecutive number

Given a sequence of numbers:

find the longest sequence of the same consecutive number.

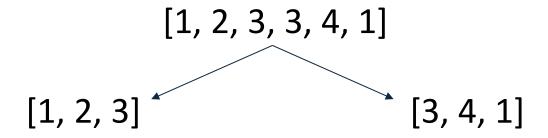
If multiple sequences have the same length, return the first one (the one with lowest starting index)

### Task:

Implement task parallel version that finds the longest sequence of the same consecutive number.

### Challenge:

The input array cannot be divided arbitrarily. For example:

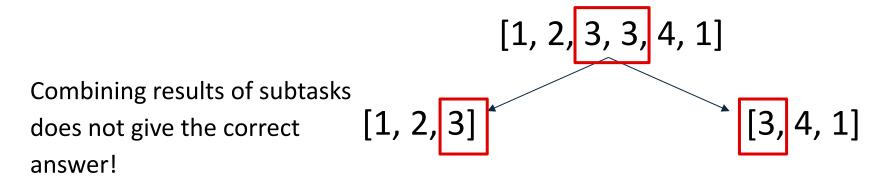


### Task:

Implement task parallel version that finds the longest sequence of the same consecutive number.

### Challenge:

The input array cannot be divided arbitrarily. For example:



## Hint

ExecutorService ex = Executors.newWorkStealingPool();

## Plan für heute

- Organisation
- Nachbesprechung Exercise 5
- Theory Recap
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- Exam Questions
- Kahoot

```
public class Main {
    public static Thread CreateThread(int start) {
      return new Thread(new Runnable() {
        @Override
        public void run() {
          for (int i = start; i < 7; i+=2) {</pre>
               System.out.println("Number " + i);
9
10
      });
11
12
13
    public static void main(String[] args) throws InterruptedException {
      CreateThread(1).start();
15
      CreateThread(2).start();
16
17
18 }
Markieren Sie alle Ausgaben, welche durch
                                            Mark all the print sequences that can be
den Codeausschnitt ausgegeben werden
                                            produced by running the program shown
können.
                                            above.
                     1 Number 1
                     1 Number 1
                                         1 Number 6
                                                              1 Number 2
2 Number 2
                     2 Number 6
                                         2 Number 5
                                                              2 Number 4
3 Number 3
                     3 Number 3
                                         3 Number 4
                                                              3 Number 6
4 Number 4
                     4 Number 4
                                         4 Number 3
                                                              4 Number 1
5 Number 5
                     5 Number 5
                                         5 Number 2
                                                              5 Number 3
```

6 Number 1

6 Number 5

6 Number 6

6 Number 2

```
public class Main {
    public static Thread CreateThread(int start) {
      return new Thread(new Runnable() {
        @Override
        public void run() {
          for (int i = start; i < 7; i+=2) {</pre>
              System.out.println("Number " + i);
10
      });
11
12
13
    public static void main(String[] args) throws InterruptedException {
      CreateThread(1).start();
15
      CreateThread(2).start();
16
17
18 }
```

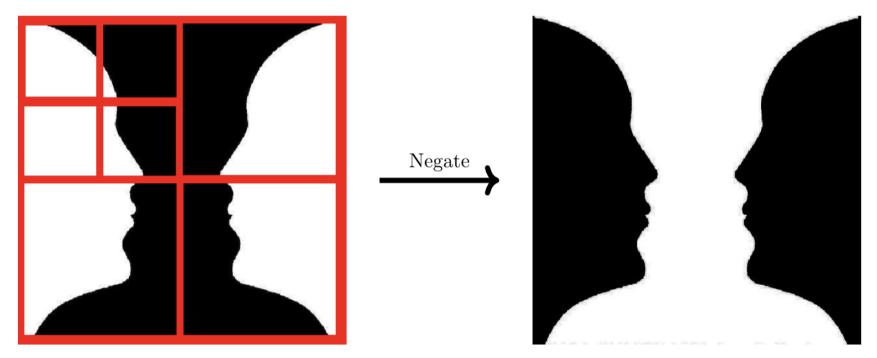
Markieren Sie alle Ausgaben, welche durch den Codeausschnitt ausgegeben werden können. Mark all the print sequences that can be produced by running the program shown above.

nomina.	_	above.	
1 Number 1	1 Number 1	1 Number 6	1 Number 2
2 Number 2	2 Number 6	2 Number 5	2 Number 4
з Number 3	3 Number 3	3 Number 4	з Number 6
4 Number 4	4 Number 4	4 Number 3	4 Number 1
5 Number 5	5 Number 5	5 Number 2	5 Number 3
6 Number 6	6 Number 2	6 Number 1	6 Number 5

#### Fork/Join Framework (16 points)

3. Der folgende Code zielt darauf ab, ein Bild zu negieren, indem es mithilfe des Fork/Join-Frameworks rekursiv in mehrere Unterfenster (vier pro Rekursionsschritt) unterteilt wird. Die Unterfenster können dann parallel negiert werden. Das folgende Beispiel verdeutlicht die Unterteilung des Bildes und die Negierung der einzelnen Unterfenster.

The following code aims to negate an image by recursively subdividing it into multiple subwindows (four per recursion step) using the Fork/Join framework. The subwindows can then be negated in parallel. The example below illustrates the subdivision of the image and negation of the individual subwindows.



Bitte lesen Sie den Code sorgfältig durch und beantworten Sie dann die Fragen zum Code:

Please read the code carefully and then answer the questions regarding the code:

```
public class ImageNegationFJ extends RecursiveAction {
    final static int CUTOFF = 32;
    double[][] image, invertedImage;
    int startx, starty;
    int length;
    public ImageNegationFJ(double[][] image, double[][] invertedImage,
            int startx, int starty, int length) {
        this.image = image;
        this.invertedImage = invertedImage;
        this startx = startx;
        this starty = starty;
        this.length = length;
    @Override
    protected void compute() {
```

```
@Override
protected void compute() {
    if (this.length <= CUTOFF) {</pre>
        for (int offsetX = 0; offsetX < this.length; offsetX++) {</pre>
            for (int offsetY = 0; offsetY < this.length; offsetY++) {</pre>
                this.invertedImage[this.startx + offsetX][this.starty + offsetY] = 1
                        - this.image[this.startx + offsetX][this.starty + offsetY];
     else {
        int halfSize = (this.length) / 2;
        ImageNegationFJ upperLeft = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx, this.starty, halfSize);
        ImageNegationFJ upperRight = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx + halfSize, this.starty,
                halfSize);
        ImageNegationFJ lowerLeft = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx, this.starty + halfSize,
                halfSize);
        ImageNegationFJ lowerRight = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx + halfSize,
                this.starty + halfSize, halfSize);
        upperLeft.fork();
        upperLeft.join();
        upperRight.fork();
        upperRight.join();
        lowerLeft.fork();
        lowerLeft.join();
        lowerRight.compute();
```

```
@Override
protected void compute() {
    if (this.length <= CUTOFF) {</pre>
        for (int offsetX = 0; offsetX < this.length; offsetX++) {</pre>
            for (int offsetY = 0; offsetY < this.length; offsetY++) {</pre>
                this.invertedImage[this.startx + offsetX][this.starty + offsetY] = 1
                        - this.image[this.startx + offsetX][this.starty + offsetY];
    int halfSize = (this.length) / 2;
        ImageNegationFJ upperLeft = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx, this.starty, halfSize);
        ImageNegationFJ upperRight = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx + halfSize, this.starty,
                halfSize);
        ImageNegationFJ lowerLeft = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx, this.starty + halfSize,
                halfSize);
        ImageNegationFJ lowerRight = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx + halfSize,
                this.starty + halfSize, halfSize);
        upperLeft.fork();
       upperLeft.join();
        upperRight.fork();
        upperRight.join();
        lowerLeft.fork();
        lowerLeft.join();
        lowerRight.compute();
```

(a) Welche Annahme trifft der Code bezüglich der Abmessungen des Arrays, das das Eingabebild darstellt?

What assumption does the code make (2) concerning the dimensions of the array representing the input image?

#### **Tobias Steinbrecher** @tsteinbreche · 8 months ago · edited 8 months ago



The image should be square  $s \times s$  and we should have  $s = d2^k$ , where  $d \le 32$ . This is necessary, because we want length to be divisible by 2 in the case length > 32. If this would not be the case, we would do floor division and leave pixels unprocessed.

+ Add Comment ··· More

```
@Override
protected void compute() {
    if (this.length <= CUTOFF) {</pre>
        for (int offsetX = 0; offsetX < this.length; offsetX++) {</pre>
            for (int offsetY = 0; offsetY < this.length; offsetY++) {</pre>
                this.invertedImage[this.startx + offsetX][this.starty + offsetY] = 1
                         - this.image[this.startx + offsetX][this.starty + offsetY];
    } else {
        int halfSize = (this.length) / 2;
        ImageNegationFJ upperLeft = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx, this.starty, halfSize);
        ImageNegationFJ upperRight = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx + halfSize, this.starty,
                halfSize);
        ImageNegationFJ lowerLeft = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx, this.starty + halfSize,
                halfSize);
        ImageNegationFJ lowerRight = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx + halfSize,
                this.starty + halfSize, halfSize);
        upperLeft.fork();
        upperLeft.join();
        upperRight.fork();
        upperRight.join();
        lowerLeft.fork();
        lowerLeft.join();
        lowerRight.compute();
```

(b) Parallelisiert der Code die beabsichtigte Aufgabe korrekt oder gibt es weitere Optimie-2024-07-30T13:53:26.664575+00:00 rungsmognenkeiten: Wenn ja, welche Optimierung würden Sie vorschlagen und warum? Does the code correctly parallelize the (4) intended task or is there further optimization that could be done? If so, which optimization would you propose and why?

Not good:

```
upperLeft.fork();
upperLeft.join();
upperRight.fork();
upperRight.join();
lowerLeft.fork();
lowerLeft.join();
lowerRight.compute();
```

**Tobias Steinbrecher** @tsteinbreche · 8 months ago · edited 7 months ago



No, the parallelization is incorrect, as we have subsequent fork() and join() calls, which means that we wait for the corresponding subproblem to be finished, before calling fork() on the next one. To fix this, we should do the following:

```
upperLeft.fork();
upperRight.fork();
lowerLeft.fork();
lowerRight.compute();
upperLeft.join();
upperRight.join();
lowerLeft.join();
```

```
public class ImageNegationFJ extends RecursiveAction {
    final static int CUTOFF = 32;
    double[][] image, invertedImage;
    int startx, starty;
    int length;
    public ImageNegationFJ(double[][] image, double[][] invertedImage,
            int startx, int starty, int length) {
        this.image = image;
        this.invertedImage = invertedImage;
        this.startx = startx;
        this.starty = starty;
        this.length = length;
    @Override
    protected void compute() {
```

(c) Vervollständigen Sie das folgende Codegerüst, indem Sie die oben implementierte ImageNegationFJ Klasse und die ForkJoinPool Klasse verwenden, um die Variable negatedImage mit den negierten Werten zu füllen.

Complete the following code skeleton (4) by using the above implemented ImageNegationFJ class and the ForkJoinPool class to fill the variable negatedImage with the negated values.

```
double[][] image = {{0, 1}, {1, 0}};
int imageSize = image.length;
double[][] negatedImage = new double[imageSize][imageSize];
```

```
double[][] image = {{0,1}, {1,0}};
int imageSize = image.length;
double[][] negatedImage = new double[imageSize][imageSize];
ForkJoinPool fjp = new ForkJoinPool();
ForkJoinTask t = new ImageNegationFJ(image, negatedImage, 0, 0 ,imageSize);
fjp.invoke(t);
```

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- (d) Unter der Annahme, dass die Klasse ImageNegationFJ korrekt parallelisiert ist, wie viele Threads verwendet der ForkJoin-Pool effektiv, um das 2 × 2 negatedImage Array aus Aufgabe 3c) zu füllen?
- Assuming that the ImageNegationFJ (2) class is correctly parallelized, how many threads does the ForkJoinPool effectively use to fill the  $2 \times 2$  negatedImage array from task 3c)?

```
double[][] image = \{\{0,1\}, \{1,0\}\};
```

```
@Override
protected void compute() {
    if (this.length <= CUTOFF) {</pre>
        for (int offsetX = 0; offsetX < this.length; offsetX++) {</pre>
            for (int offsetY = 0; offsetY < this.length; offsetY++) {</pre>
                this.invertedImage[this.startx + offsetX][this.starty + offsetY] = 1
                       - this.image[this.startx + offsetX][this.starty + offsetY];
    } else {
        int halfSize = (this.length) / 2;
        ImageNegationFJ upperLeft = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx, this.starty, halfSize);
        ImageNegationFJ upperRight = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx + halfSize, this.starty,
                halfSize);
        ImageNegationFJ lowerLeft = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx, this.starty + halfSize,
                halfSize);
        ImageNegationFJ lowerRight = new ImageNegationFJ(this.image,
                this invertedImage, this startx + halfSize,
                this.starty + halfSize, halfSize);
       upperLeft.fork();
        upperLeft.join();
        upperRight.fork();
        upperRight.join();
        lowerLeft.fork();
        lowerLeft.join();
        lowerRight.compute();
```

final static int CUTOFF = 32;

- (d) Unter der Annahme, dass die Klasse ImageNegationFJ korrekt parallelisiert ist, wie viele Threads verwendet der ForkJoin-Pool effektiv, um das 2 × 2 negatedImage Array aus Aufgabe 3c) zu füllen?
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double[][] image = \{\{0,1\}, \{1,0\}\};
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@Override
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   if (this.length <= CUTOFF) {</pre>
       for (int offsetX = 0; offsetX < this.length; offsetX++) {</pre>
           for (int offsetY = 0; offsetY < this.length; offsetY++) {</pre>
                this.invertedImage[this.startx + offsetX][this.starty + offsetY] = 1
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    } else {
       int halfSize = (this.length) / 2;
       ImageNegationFJ upperLeft = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx, this.starty, halfSize);
        ImageNegationFJ upperRight = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx + halfSize, this.starty,
                halfSize);
       ImageNegationFJ lowerLeft = new ImageNegationFJ(this.image,
                this.invertedImage, this.startx, this.starty + halfSize,
                halfSize);
       ImageNegationFJ lowerRight = new ImageNegationFJ(this.image,
                this invertedImage, this startx + halfSize,
                this.starty + halfSize, halfSize);
       upperLeft.fork();
       upperLeft.join();
       upperRight.fork();
       upperRight.join();
       lowerLeft.fork();
       lowerLeft.join();
        lowerRight.compute();
```

### final static int CUTOFF = 32;

**Tobias Steinbrecher** @tsteinbreche · 8 months ago · edited 8 months ago

Because of the sequential cutoff, only **one** Thread would be used *effectively*.

(e) Gehen Sie von einem konstanten Overhead von  $16\,\mathrm{MB} = 2^4\,\mathrm{MB}$  pro Thread aus und dass pro Split immer vier neue Threads erstellt werden. Dies bedeutet, dass die Anzahl der Threads nicht durch den ForkJoinPool festgelegt wird, sodass kein Thread wiederverwendet wird und es zu keinem Work Stealing zwischen den Threads kommt. Was ist der niedrigste Wert für CUTOFF, wenn Sie ein Bild der Größe  $4000 \times 4000$  eingeben, bevor Ihnen bei einem RAM der Größe  $10\,\mathrm{GB}$  der Speicher ausgeht? Hinweis:  $1\,\mathrm{GB} = 2^{10}\,\mathrm{MB}$ .

Assume a fixed overhead of  $16 \,\mathrm{MB} = (4) \,2^4 \,\mathrm{MB}$  per thread and that there are always four new threads created per split. This means that the number of threads is not fixed by the ForkJoinPool, so no thread is re-used and there is no work stealing among the threads. What is the lowest value for CUTOFF if you input an image of size  $4000 \times 4000$  before you run out of memory using a RAM of size  $10 \,\mathrm{GB}$ ? Hint:  $1 \,\mathrm{GB} = 2^{10} \,\mathrm{MB}$ .

- (e) Gehen Sie von einem konstanten Overhead von  $16\,\mathrm{MB} = 2^4\,\mathrm{MB}$  pro Thread aus und dass pro Split immer vier neue Threads erstellt werden. Dies bedeutet, dass die Anzahl der Threads nicht durch den ForkJoinPool festgelegt wird, sodass kein Thread wiederverwendet wird und es zu keinem Work Stealing zwischen den Threads kommt. Was ist der niedrigste Wert für CUTOFF, wenn Sie ein Bild der Größe  $4000 \times 4000$  eingeben, bevor Ihnen bei einem RAM der Größe  $10\,\mathrm{GB}$  der Speicher ausgeht? Hinweis:  $1\,\mathrm{GB} = 2^{10}\,\mathrm{MB}$ .
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Number of threads, which we can use:

$$N = rac{10 \cdot 2^{10}}{2^4} = 10 \cdot 2^6 = 10 \cdot 4^3$$

In each recursive call, we will use 4 new threads (under given assumptions). Thereby, we have the constraint (i:= number of divisions)

$$4^i \leq 10 \cdot 4^3 \iff i \leq \log_4(10) + 3 \iff i \leq 4$$

and the smallest possible value is  ${\sf CUTOFF} = 4000/2^4 = {f 250}$  to avoid a fifth division.

### Plan für heute

- Organisation
- Nachbesprechung Exercise 5
- Theory Recap
- Intro Exercise 6
- Exam Questions
- Kahoot



### Feedback

- Falls ihr Feedback möchtet sagt mir bitte Bescheid!
- Schreibt mir eine Mail oder auf Discord

## Danke

• Bis nächste Woche!