Lösungen (ohne Gewähr)

Mechanik 1, Klausur 2 2009

1a) ges.: $\underline{F}_B = \begin{pmatrix} 0 \\ F_{By} \\ F_{Bz} \end{pmatrix}$; $\underline{F}_H = \begin{pmatrix} F_{Hx} \\ F_{Hy} \\ F_{Hz} \end{pmatrix}$

$\underline{R}_1 = \underline{F}_A + \underline{F}_C + \underline{F}_M = \begin{pmatrix} 0 \\ -F \\ -5F \end{pmatrix}$; $\underline{R}_2 = \underline{F}_B + \underline{F}_H = \begin{pmatrix} F_{Hx} \\ F_{By}+F_{Hy} \\ F_{Bz}+F_{Hz} \end{pmatrix}$; $\underline{R}_1 = \underline{R}_2$; $\underline{M}_{01} = \underline{M}_{02}$

$\underline{M}_{01} = \begin{pmatrix} -\frac{a}{2}\cdot 3F \\ \frac{a}{2}\cdot 3F + a\cdot 2F \\ a\cdot F - aF \end{pmatrix} = \begin{pmatrix} -\frac{3}{2}aF \\ \frac{7}{2}aF \\ 0 \end{pmatrix}$; $\underline{M}_{02} = \begin{pmatrix} a\cdot F_{Bz} + \frac{a}{2}F_{Hz} - aF_{Hy} \\ -aF_{Bz} - \frac{a}{2}F_{Hz} + aF_{Hx} \\ aF_{By} + \frac{a}{2}F_{Hy} - \frac{a}{2}F_{Hx} \end{pmatrix}$

$\underline{R}_1 = \underline{R}_2$: $\underline{\underline{F_{Hx} = 0}}$ (1)

$-F = F_{By} + F_{Hy}$ (2)

$-5F = F_{Bz} + F_{Hz}$ (3)

$\underline{M}_{01} = \underline{M}_{02}$: $-\frac{3}{2}F = F_{Bz} + \frac{F_{Hz}}{2} - F_{Hy}$ (4)

$\frac{7}{2}F = F_{Hx} - F_{Bz} - \frac{F_{Hz}}{2}$ (5)

$0 = F_{By} + \frac{F_{Hy}}{2} - \frac{F_{Hx}}{2}$ (6)

$2\cdot$(6) $-$ (2): $\underline{\underline{F = F_{By}}}$ in (2): $\underline{\underline{F_{Hy} = -2F}}$

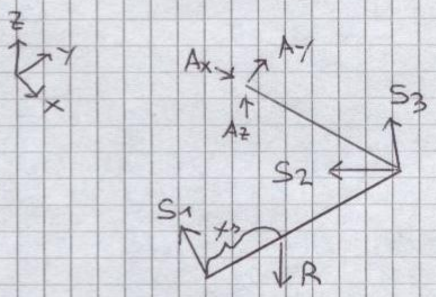
(3)+(5): $-\frac{3}{2}F = \frac{F_{Hz}}{2} \rightarrow \underline{\underline{F_{Hz} = -3F}}$ in (3): $\underline{\underline{F_{Bz} = -2F}}$

$$\underline{\underline{\underline{F}_B = \begin{pmatrix} 0 \\ F \\ -2F \end{pmatrix} \; ; \; \underline{F}_H = \begin{pmatrix} 0 \\ -2F \\ -3F \end{pmatrix}}}$$

b) $P = \underline{v}_0 \cdot \underline{R}_1 + \underline{\omega}\cdot \underline{M}_{01} = \begin{pmatrix} \omega_x \\ \omega_y \\ 0 \end{pmatrix} \begin{pmatrix} -\frac{3}{2}aF \\ \frac{7}{2}aF \\ 0 \end{pmatrix} = \underline{\underline{\frac{7}{2}aF\omega_y - \frac{3}{2}aF\omega_x}}$

Da $\underline{R}_1 = \underline{R}_2$ und $\underline{M}_{01} = \underline{M}_{02}$ (Definition für stat. äquivalente Kräftegruppe) gilt auch $P_1 = P_2$

2)



$x_1 = \frac{1}{3}L$; $R = \frac{q_0 \cdot L}{2} = \frac{2\frac{P}{L}\cdot L}{2} = P$

$\Sigma F_x = 0$: $A_x - \frac{\sqrt{3}}{2}S_1 - \frac{\sqrt{3}}{2\sqrt{2}}S_2 - \frac{1}{\sqrt{2}}S_3 = 0$ (1)

$\Sigma F_y = 0$: $A_y - \frac{1}{\sqrt{2}}S_2 + \frac{1}{\sqrt{2}}S_3 = 0$ (2)

$\Sigma F_z = 0$: $A_z + \frac{1}{2}S_1 + \frac{1}{2\sqrt{2}}S_2 - P = 0$ (3)

$\underline{S}_1 = S_1 \begin{pmatrix} -\frac{\sqrt{3}}{2}L \\ 0 \\ \frac{1}{2}L \end{pmatrix} \frac{1}{L}$; $\underline{S}_2 = S_2 \begin{pmatrix} -\frac{\sqrt{3}}{2}L \\ -L \\ \frac{1}{2}L \end{pmatrix} \frac{1}{\sqrt{2}L} = S_2 \begin{pmatrix} -\frac{\sqrt{3}}{2\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{pmatrix}$; $M_{Bx} = 0 = \frac{2}{3}LP - \frac{1}{2}LS_1$ (4)

$\underline{S}_3 = S_3 \begin{pmatrix} -\frac{\sqrt{3}}{2}L \\ \frac{\sqrt{3}}{2}L \\ 0 \end{pmatrix} \frac{2}{\sqrt{6}L} = S_3 \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$

$M_{By} = 0 = \frac{\sqrt{3}}{2}L \cdot A_z \rightarrow \underline{\underline{A_z = 0}}$ (5)

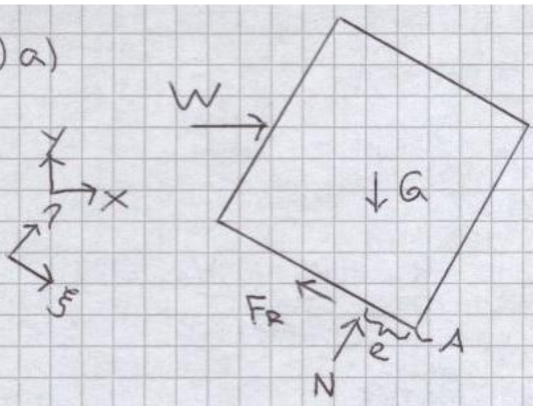
$M_{Bz} = 0 = -\frac{\sqrt{3}}{2}LA_y - \frac{\sqrt{3}}{2}L\cdot S_1$ (6)

(4): $\underline{\underline{S_1 = \frac{4}{3}P}}$; (6): $\underline{\underline{A_y = -S_1 = -\frac{4}{3}P}}$; (1) $\underline{A_x} = \frac{2\sqrt{3}}{3}P + \frac{\sqrt{3}}{3}P + 2P = \underline{\underline{(\sqrt{3}+2)P}}$

(3): $\frac{2}{3}P - P = -\frac{1}{2\sqrt{2}}S_2 \rightarrow \underline{\underline{S_2 = \frac{2\sqrt{2}}{3}P}}$; (2) $-\frac{4}{3}P - \frac{2}{3}P = -\frac{1}{\sqrt{2}}S_3 \rightarrow \underline{\underline{S_3 = 2\sqrt{2}P}}$

$$\underline{\underline{\underline{S}_1 = \begin{pmatrix} -\frac{2\sqrt{3}}{3} \\ 0 \\ \frac{2}{3} \end{pmatrix} P}} \; ; \; \underline{\underline{\underline{S}_2 = \begin{pmatrix} -\frac{\sqrt{3}}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix} P \; ; \; \underline{S}_3 = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} P \; ; \; \underline{A} = \begin{pmatrix} \sqrt{3}+2 \\ -\frac{4}{3} \\ 0 \end{pmatrix} P}}$$

3) a)



$$\sum F_\xi = 0 : \quad \underline{\underline{W \cos\alpha + G \sin\alpha = F_R}}$$

$$\sum F_\eta = 0 : \quad \underline{\underline{N = G \cos\alpha - W \sin\alpha}}$$

b) Standfestigkeit: $0 \leq e \leq D$

$$M_{A_z} = 0 = \frac{D}{2} G \cos\alpha - \frac{h}{2} G \sin\alpha - \frac{h}{2} W \cos\alpha - D \cdot W \sin\alpha - N \cdot e$$

mit $e = 0$: $\frac{G}{2}\left(D \cos\alpha - h \sin\alpha\right) = W\left(\frac{h}{2} \cos\alpha + D \sin\alpha\right)$

$$\rightarrow \underline{\underline{W \leq \frac{G\left(D \cos\alpha - h \sin\alpha\right)}{2\left(\frac{h}{2} \cos\alpha + D \sin\alpha\right)}}}$$