A Joint Coordinate System for the Clinical Description of Three-Dimensional Motions: Application to the Knee

Materials and Mechanics in Medicine HS 2019

Paper 4

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About the Paper

- Published 1983 in Journal of Biomechanics

- 3’304 Citations (end of 2019):
  → Foundation for experimental research in sports medicine, knee orthopaedics, rehabilitation, movement science,… etc.

- Novel method for calculating a joint coordinate system that provides a simple geometric description of the three-dimensional rotational and translational motion between two rigid bodies.

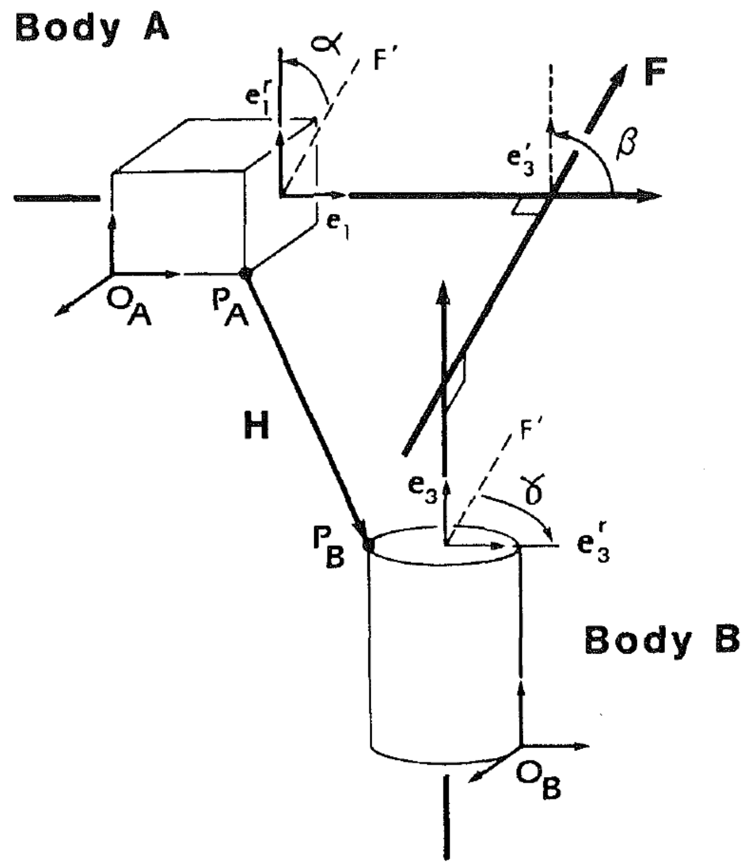
General Idea

Describe relative motion of two rigid bodies as a set of spatial linkages

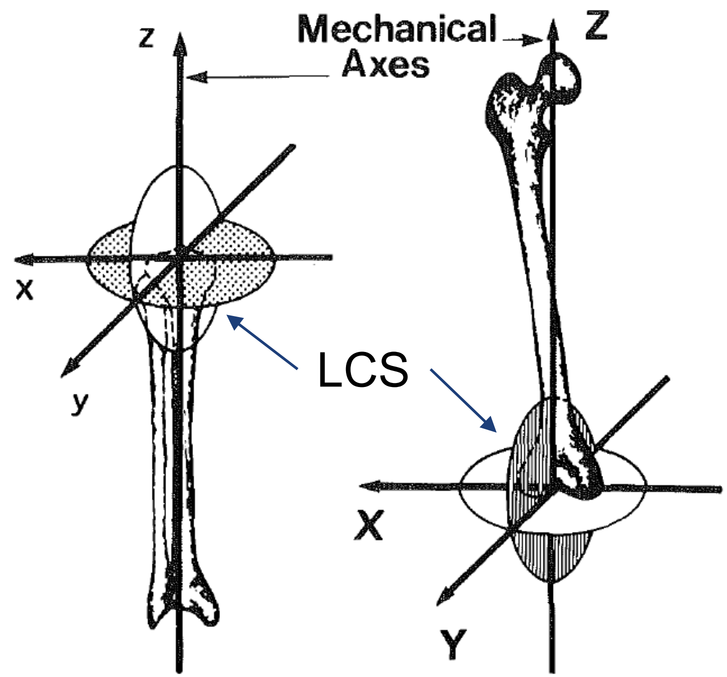
Create a new coordinate system called «joint coordinate system» (JCS) which is aligned with that of the clinical coordinate system.

This JCS requires a «floating axis» F which is perpendicular to the main axes of the rigid bodies. F acts as a mathematical constraint.

Euler angles (alpha, beta, gamma)
→ Flexion, Abduction, Internal Rotation, etc…



Local vs. global coordinate system



Local coordinate system (LCS) is a refrence system within a larger reference system (GCS)

LCS has own origin and axes (orientation), which are attached to the body in question

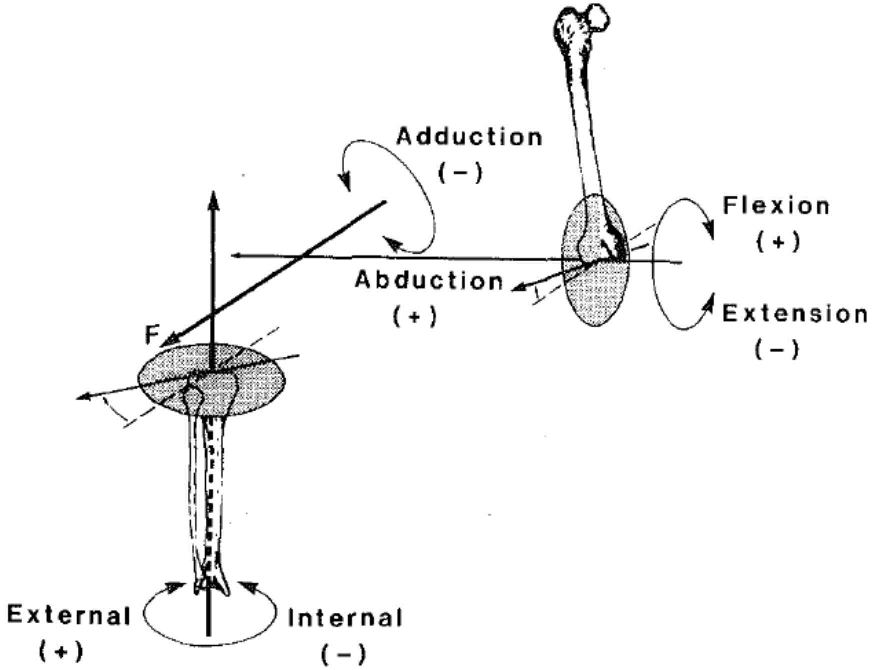
For the Joint Coordinate System, a LCS is defined close to the instantaneous center of rotation for each bone and orientated with respect to certain bony landmarks.

Joint Coordinate Systems

A reference system for joints of the body in relation to the whole body (GCS) and to other body segments (LCS)

Purpose:

- To be able to define the relative position between 2 bodies. I.e. Description of motion
- Use clinical terms of motion and rotation



Standard Euler Angles and Euler Angle of JCS

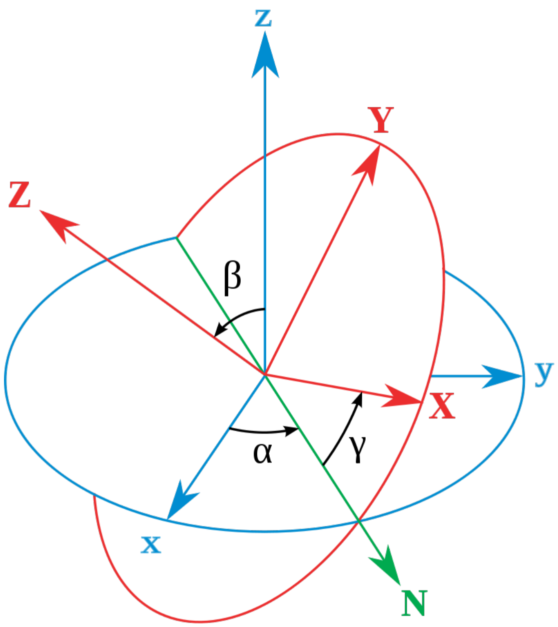
Sequence dependency differs depending on which system is being looked at in order to describe 3-dimensional rotation about axes

Mech III/ Biomech II: Standard Euler Angles in Cart. Sys:

- Dependent upon the order in which rotations occur
- Classified into rotations about 2 or 3 axes

NEW: Euler Angle in a Joint Coordinate System:

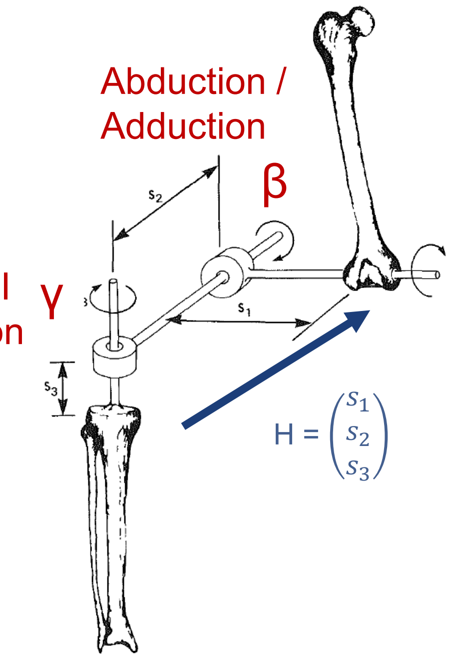
- Independent upon the order in which rotations occur
- All angles are due to rotations about all 3 axes, one of which is the floating axis



Proper Euler angles

Coordinate Transformations

- Rotation + Translation = Transformation
- All details of the clinical rotation parameters Table 2 (page 138 or 3)
- *Eq. 16* describes the transformation matrix which is usually determined experimentally and is necessary to calculate the clinical rotations and translations knowing the elements $B_{i,j}$



$$\mathbf{R} = [R]\mathbf{r} + \mathbf{H} \tag{8a}$$

where **H** is the vector which locates the tibial origin with respect to the femoral coordinate system and $[R]$ is a (3×3) rotation matrix. Equation (8a) can be rewritten as

$$[U] = \begin{bmatrix} 1 & 0 & \cos\beta \\ 0 & \cos\alpha & \sin\alpha\sin\beta \\ 0 & -\sin\alpha & \cos\alpha\sin\beta \end{bmatrix} \tag{10}$$

$$\mathbf{R} = [B]\mathbf{r} \tag{16a}$$

where

$$[B] = \left[\begin{array}{c|c} 1 & 0 \quad 0 \quad 0 \\ \hline (S_1 + S_3\cos\beta) & \\ (S_2\cos\alpha + S_3\sin\alpha\sin\beta) & [R] \\ (-S_2\sin\alpha + S_3\cos\alpha\sin\beta) & \end{array}\right] \tag{16b}$$

Coordinate Transformations – Short Example

Drawer test for ACL/PCL rupture in right knee:

- Translation in S_2
- S_1 & S_3 remain neutral
- angles neutral: $\beta = \pm {}^{\pi}/_{2}, \gamma = {}^{\pi}/_{2}$
- Extension $\alpha = 0$

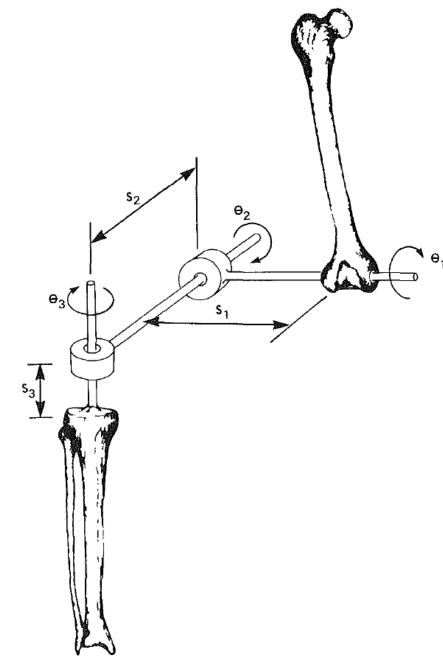
Coordinate Transformations – Short Example

$$\boldsymbol{R} = [B]\boldsymbol{r} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ S_1 + S_3\cos\beta & 1 & 0 & \cos\beta \\ S_2\cos\alpha + S_3\sin\alpha\cos\beta & 0 & \cos\alpha & \sin\alpha\sin\beta \\ -S_2\sin\alpha + S_3\cos\alpha\sin\beta & 0 & -\sin\alpha & \cos\alpha\sin\beta \end{pmatrix} \cdot \begin{pmatrix} 1 \\ r_x \\ r_y \\ r_z \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ S_2 & 0 & 1 & 0 \\ -S_3 & 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ r_1 = 0 \\ r_2 = 0 \\ r_3 = 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ S_2 \\ -S_3 \end{pmatrix}$$

Drawer test for ACL/PCL rupture in right knee:

- Translation in S_2
- S_1 & S_3 remain neutral
- angles neutral: $\beta = {}^{\pi}/_{2}, \gamma = 0$
- Extension $\alpha = 0$
- $\boldsymbol{r}$ → deviation from origin (lever)



Summary

Describe the motion of the knee joint
Purpose:

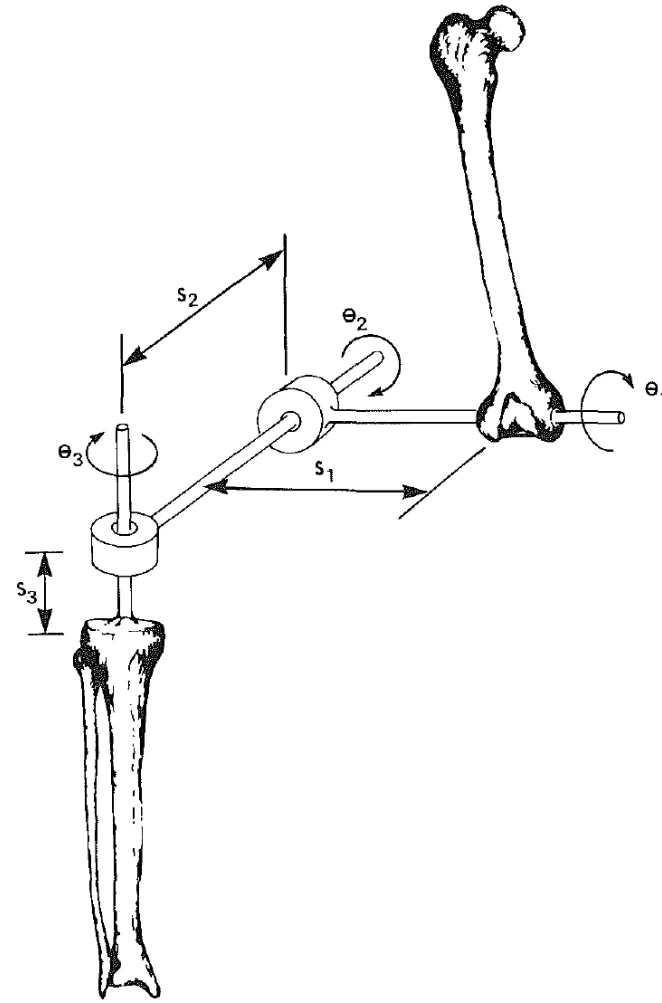
- ensure that all three rotations have functional meaning for the knee

How is it different than cardan/Euler rotations?

- NOT an orthogonal system
- Two segment-fixed axes and a FLOATING axis

Essentially, we must define the anatomical axes of interest from bony markers, the clinical axes of rotation, and the origin of the joint coordinate system for a complete analysis of motion

→ One of many ways to quantify motion of two rigid bodies



Appendix

$$[R]^T = \begin{bmatrix} \cos\gamma\sin\beta & -\cos\alpha\sin\gamma - \cos\gamma\sin\alpha\cos\beta & \sin\alpha\sin\gamma - \cos\gamma\cos\alpha\cos\beta \\ \sin\gamma\sin\beta & \cos\alpha\cos\gamma - \sin\gamma\sin\alpha\cos\beta & -\cos\gamma\sin\alpha - \cos\alpha\cos\beta\sin\gamma \\ \cos\beta & \sin\beta\sin\alpha & \cos\alpha\sin\beta \end{bmatrix}$$

Table 2 Clinical rotations and translations

Clinical rotations

- $\alpha = \text{Flexion } (+ve)$ — $\sin\alpha = -\mathbf{e}_2 \bullet \mathbf{K}$
- $\beta = \begin{cases} \pi/2 + \text{Adduction, right knee} \\ \pi/2 - \text{Adduction, left knee} \end{cases}$ — $\cos\beta = \mathbf{I} \bullet \mathbf{k}$
- $\gamma = \text{External rotation } (+ve)$ — $\sin\gamma = \begin{cases} -\mathbf{e}_2 \bullet \mathbf{i} \text{ right knee} \\ \mathbf{e}_2 \bullet \mathbf{i} \text{ left knee} \end{cases}$

Clinical translations

- q_1 $(+ve)$ lateral tibial displacement
- q_2 $(+ve)$ anterior tibial drawer displacement
- q_3 $(+ve)$ joint distraction