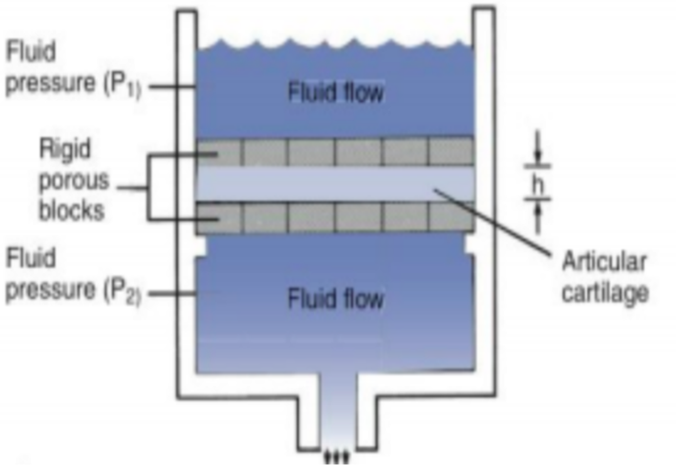


The permeability data below have been obtained for a hydrogel material to be used as a scaffold material for cartilage tissue repair. The data were obtained using the experimental set up shown at the right. The original thickness (shown here as height *"h"*) of the specimen was 5 mm.

The experiment was performed to determine the permeability as a function of its relative compression. One main material descriptor of interest is the so-called "intrinsic permeability" (also called the "absolute permeability" or "specific permeability"). The intrinsic permeability of a material corresponds to an undeformed state with 100% saturation by a single-phase fluid. In biological tissues, for most practical questions, this can be assumed to be water-like fluid (a Newtonian fluid with viscosity of 0.0006913 Pa·s at 37C).

Displacement (mm)	**Permeability (SI units)**
-0.1	2.3×10^{-14}
-0.2	1.96×10^{-14}
-0.3	1.62×10^{-14}
-0.4	1.48×10^{-14}

Task and Questions:

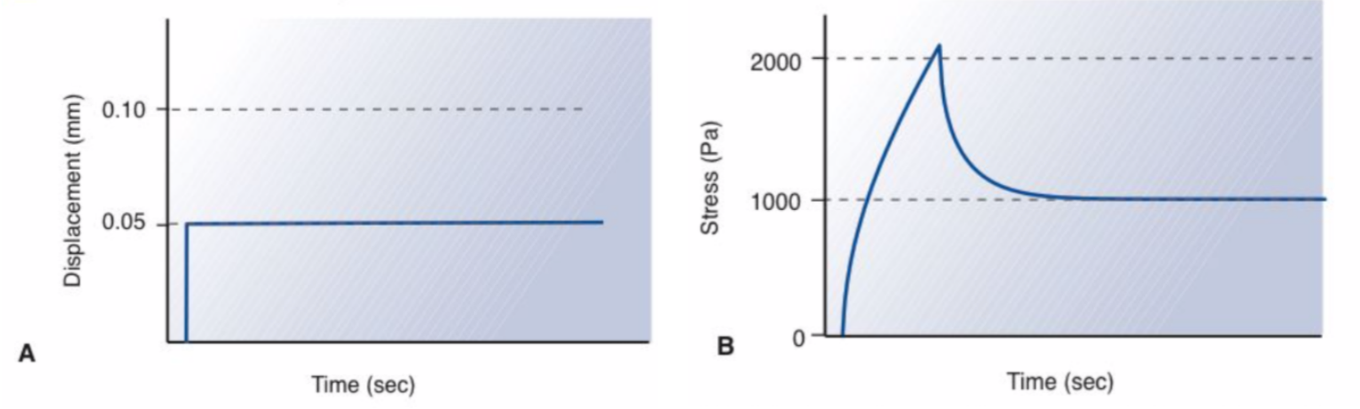
a) From Darcy's Law (below), what are the SI units of permeability?
b) Plot the permeability data as a function of material strain. Assume that an exponential function well fits to the data. Estimate the "intrinsic permeability" of the hydrogel.
c) Considering Darcy's Law and the nature of the hydrogel´s characteristic permeability:
    a. why might strain-dependent permeability benefit the load support mechanisms of a hydrated tissue?
    b. considering both biological and mechanical perspectives, what is the consequence for the hydrogel when it is saturated with fluid of reduced viscosity, for instance in an inflammatory environment? Is this an advantage or a disadvantage? Why?

Darcy's law:

$$q = \frac{-k\,A}{\mu}\frac{dp}{dL}$$

where q = volume exudate rate (m^3/s), k is the intrinsic permeability, A= area of exudation surface (m^3), l = length of the specimen (m), p is the applied pressure (Pa), and μ is the viscosity of the fluid (Pa·s).

2. Given the material testing data curves,



a) Assume that a sample of the hydrogel from problem 1 is being tested (with a thickness of 5 mm and a cross-sectional area of 100 mm^2). What is the amount of applied strain? Applied force?

b) The so-called ”instantaneous elastic response” and “long-term elastic response” are often of interest in assessing the ability of the material to resist dynamic and static mechanical loads, respectively. Propose a combination of springs and dashpots that would allow one to model and describe these behaviors.

   a. From this spring dashpot model, which element(s) is/are responsible for the short-term provision of mechanical resistance (stress at in response to an imposed step displacement)? How can this response be described in terms of these elements physical properties (e.g. spring constants)?

   b. Which element(s) is/are responsible for the long-term provision of mechanical resistance (stress at equilibrium)? How can this response be described in terms of these elements physical properties (e.g. spring constants)?

c) Calculate the value of the “equilibrium modulus” defined as below. How does this modulus compare to healthy articular cartilage? Healthy ear cartilage? Healthy cortical bone?

$$E_\infty = \frac{\sigma_\infty}{\varepsilon} = \frac{stress\ at\ equilibrium}{applied\ strain}$$

3) The material test curves below come from bone sampled in different orientations, and pulled in tension in the direction of the arrows indicated. Explain the physiological basis of this mechanical response.

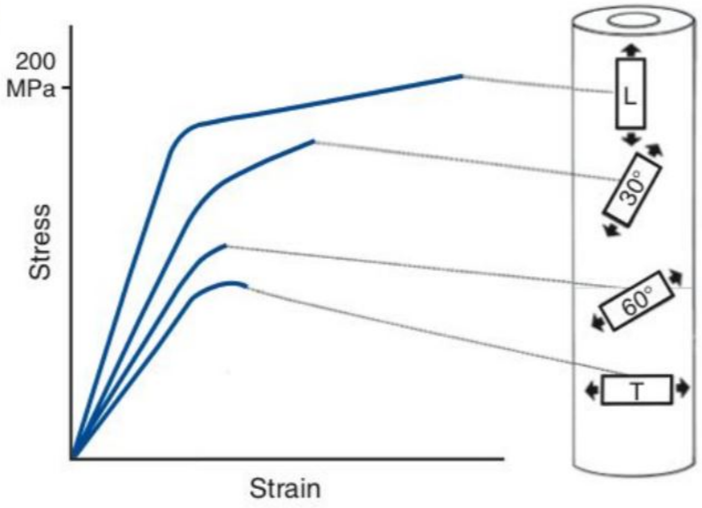


FIG. 2-16

Anisotropic behavior of cortical bone specimens from a human femoral shaft tested in tension (pulled) in four directions: longitudinal (*L*), tilted 30° with respect to the neutral axis of the bone, tilted 60°, and transverse (*T*). The modulus of toughness is clearly also anisotropic in bone and is greatest when tension is applied longitudinally. *Data from Frankel, V.H., Burstein, A.H. (1970). Orthopaedic Biomechanics. Philadelphia: Lea & Febiger.*