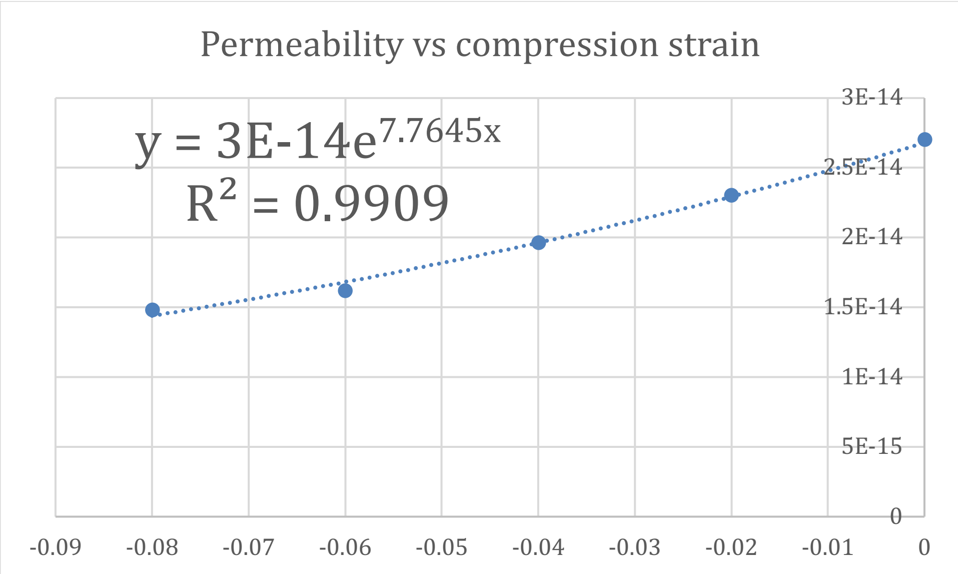
Solutions Exercise Cartilage and Bone

Question1 :

a) From Darcy's Law (below), what are the SI units of permeability?
Answer: m^4 / N·s

b) Plot the permeability data as a function of material strain. Assume that an exponential function well fits to the data. Estimate the "intrinsic permeability" of the hydrogel.



Answer: an exponential fit predicts the intrinsic permeability to lie around 2.7 x 10^{-14} m^4 / N·s

c) Considering Darcy's Law and the nature of the hydrogel´s characteristic permeability:

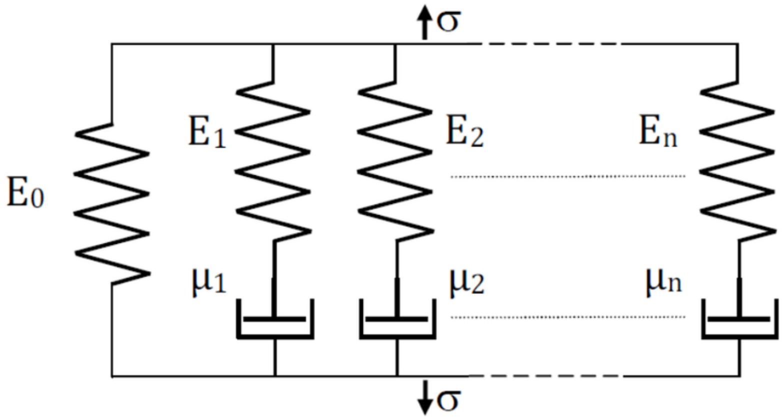
   a. why might strain-dependent permeability benefit the load support mechanisms of a hydrated tissue?
   Answer: as the tissue is increasingly loaded in compression, the corresponding decreased permeability of the tissue results in an effectively stiffer tissue that is more resistant to further deformation. This non-linearity of the the tissue stiffness has several benefits: high fluid exchange under low loads (increased tissue nutrition) but large potential capacity for bearing high mechanical loads.

   b. considering both biological and mechanical perspectives, what is the consequence for the hydrogel when it is saturated with fluid of reduced viscosity, for instance in an inflammatory environment? Is this an advantage or a disadvantage? Why?
   Answer: lower viscosity fluids will diminish the mechanical pressure gradient for a given tissue displacement (tissue compression can be considered as a volume change equivalent to the exudated fluid). Thus inflammatory fluid will equate to more fluid exudation for a given external compressive load applied to the surface. This has biological benefits in terms of increased nutrient transport. It may however compromise the ability of the tissue to bear mechanical loads, and can contribute to mechanical breakdown of the tissue.

Question 2: Stress-relaxation test

a) Assume that a sample of the hydrogel from problem 1 is being tested (with a thickness of 5 mm and a cross-sectional area of 100 mm^2). What is the amount of applied strain? Applied force?
Answer: 0.05 mm displacement on an initial thickness of 5mm corresponds to 1% nominal strain. A stress of 2000 Pa (2 kPA, or 0.002 MPa) on an area of 100 mm^2 corresponds to a force of 0.2 N (a quite small force, at the sensitivity limits of most standard testing machines).

b) The so-called "instantaneous elastic response" and "long-term elastic response" are often of interest in assessing the ability of the material to resist dynamic and static mechanical loads, respectively. Propose a combination of springs and dashpots that would allow one to model and describe these behaviors.



Any form of the generalized Maxwell model (E0 in parallel combination with any spring and daspot in series) would be an acceptable model.

a. From this spring dashpot model, which element(s) is/are responsible for the short-term provision of mechanical resistance (stress at in response to an imposed step displacement)? How can this response be described in terms of these elements physical properties (e.g. spring constants)?
Instantaneous: the springs in parallel: $E_0 + E_1 + E_2 + \ldots. + E_n$

b. Which element(s) is/are responsible for the long-term provision of mechanical resistance (stress at equilibrium)? How can this response be described in terms of these elements physical properties (e.g. spring constants)?
Equilibrium: E_0 (the dashpots will eventually dissipate all stress stored in all other springs, so they have no contribution to the equilibrium modulus)

c. Calculate the value of the "equilibrium modulus" defined as below. How does this modulus compare to healthy articular cartilage? Healthy cortical bone?

$$E_{\infty} = \frac{\sigma_{\infty}}{\varepsilon} = \frac{0.001MPa}{0.01} = 0.1 \text{ MPa}$$

This is two orders (10^2) of magnitude lower than healthy articular cartilage (10 MPa), three orders of magnitude (10^3) lower than ear cartilage (100MPa) and 5 orders of magnitude (10^5) than cortical bone (17´000 MPa).

3) The material test curves below come from bone sampled in different orientations, and pulled in tension in the direction of the arrows indicated. Explain the physiological basis of this mechanical response.

Answer: bone in the shaft of a long bone is so-called lamellar bone. Lamellar bone, which has a regular parallel alignment of collagen into sheets ("lamellae") is mechanically strong (in tension) along the axis of the collagens (the collagens component of the matrix carries tensile loads). The bone is equally strong in compression, nearly independent of collagen orientation (the mineral matrix bears the compression load). As the bone samples deviate away from the collagen axis, the samples are progressively weaker in tension.