Exercise 1

a) $v = \lambda f = 0{,}5\,m \cdot 2\,Hz = 1\,\frac{m}{s}$

b) $y(x,t) = A \sin(kx - \omega t)$

with $A = 1\,cm$

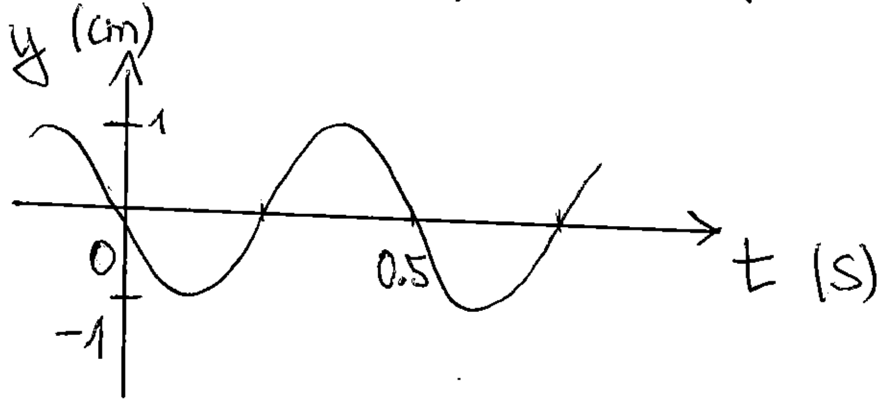
$k = 2\pi/\lambda = \frac{2\pi}{0.5\,m} = 4\pi\,m^{-1}$

$\omega = 2\pi f = 4\pi\,Hz$

c) $x = 1\,m \implies kx = 4\pi$

$\implies y(1m, t) = (1\,cm)\sin(4\pi - 4\pi\,Hz \cdot t)$

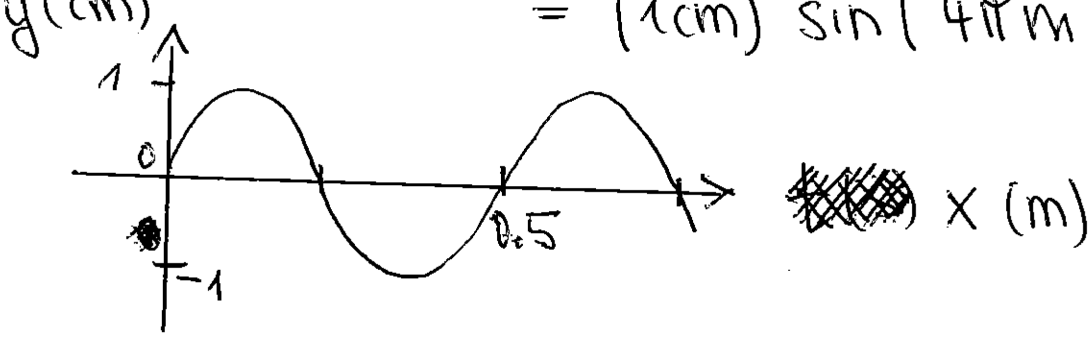
$= (1\,cm)\sin(-4\pi\,Hz \cdot t)$



d) $t = 2.5\,s \implies \omega t = 10\pi$

$\implies y(x, 2.5s) = (1\,cm)\sin(4\pi m^{-1} x - 10\pi)$

$= (1\,cm)\sin(4\pi m^{-1} x)$



e) $$P = \frac{1}{2}\mu A^2 \omega^2 v$$
$$= \frac{1}{2}\, 0.01 \frac{kg}{m}\, (0.01m)^2\, (4\pi\, Hz)^2\, 1 \frac{m}{s}$$
$$= 8\pi^2 \cdot 10^{-6}\, \frac{kg\, m^2}{s^3}$$

f) fundamental mode frequency

$$f_0 = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \frac{1}{1m}\sqrt{\frac{9N}{0.01\, kg/m}} = 30\, Hz$$

$$\Longrightarrow \quad f_1 = 2f_0 = 60\, Hz$$
$$f_2 = 3f_0 = 90\, Hz$$

Exercise 2

a) frequency observed by runner

$$f = \frac{v_s + v}{v_s} f_0 \implies v = \frac{f}{f_0} v_s - v_s$$

$$= \frac{346\,\text{Hz}}{340\,\text{Hz}} 340\,\frac{\text{m}}{\text{s}} - 340\,\frac{\text{m}}{\text{s}}$$

$$= 6\,\text{m/s}$$

b) power is distributed on spheres $\implies I \propto 1/d^2$ ← distance

$$\frac{I(3s)}{I(0s)} = \frac{1/d_{3s}^2}{1/d_{0s}^2} = \left(\frac{d_{0s}}{d_{3s}}\right)^2 = 100 \mathrel{\hat{=}} +20\,\text{dB}$$

d_{0s}: 20m

d_{3s}: $(20 - 3 \cdot 6)\,\text{m} = 2\,\text{m}$

c) we examine path length difference Δ with

$\Delta =$ dist (speaker B, observer) − dist (speaker A, observer)

at $d = 0$: $\Delta = 4\,\text{m} \implies$ constructive interference

at $d = 20\,\text{m}$: $\Delta = \sqrt{(20\,\text{m})^2 + (4\,\text{m})^2} - 20\,\text{m}$
$= \sqrt{416}\,\text{m} - 20\,\text{m} < 1\,\text{m}$

$\implies$ constructive interference occurs at $\Delta = (4, 3, 2, 1)\,\text{m}$

$\implies$ 4 maxima

Exercise 3

a) step 1: water from 10°C to 0°C

energy that needs to be removed:

$$Q_1 = c_{water} \cdot m \cdot \Delta T = 4.2 \frac{kJ}{kgK} \cdot 1kg \cdot 10K = 42kJ$$

step 2: freezing

$$Q_2 = H \cdot m = 330 \frac{kJ}{kg} \cdot 1kg = 330 kJ$$

step 3: ice from 0°C to −10°C

$$Q_3 = c_{ice} \cdot m \cdot \Delta T = 2.1 \frac{kJ}{kgK} \cdot 1kg \cdot 10K = 21kJ$$

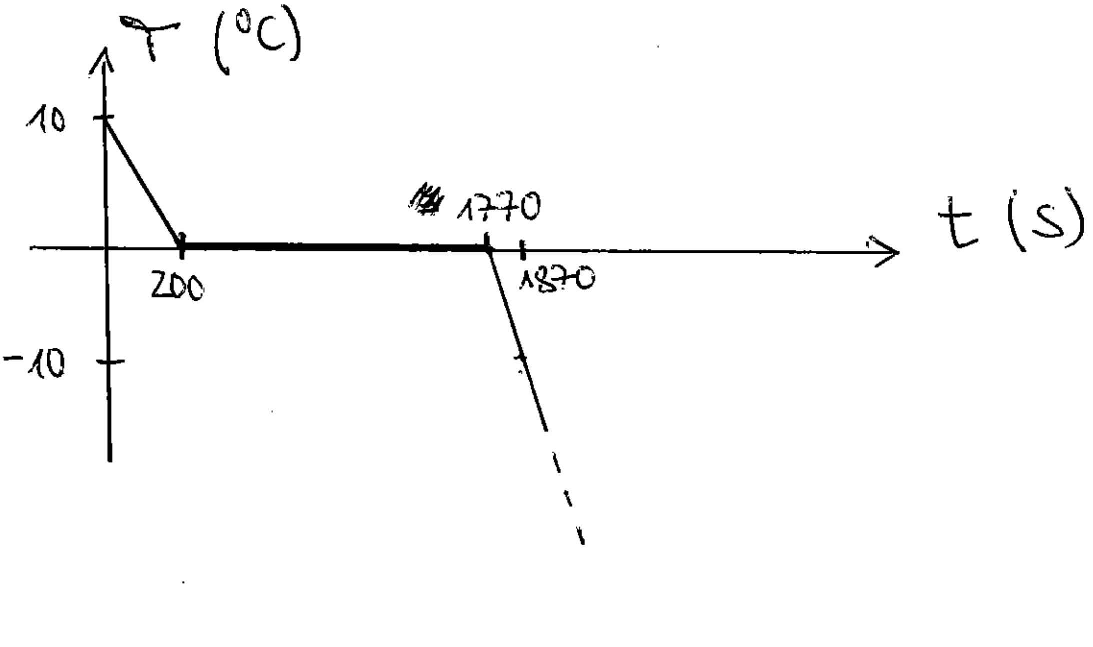
refridgerator removes heat with power:

$$P_{cool} = 4.2 \cdot 50W = 210W$$

time to reach T = −10°C:

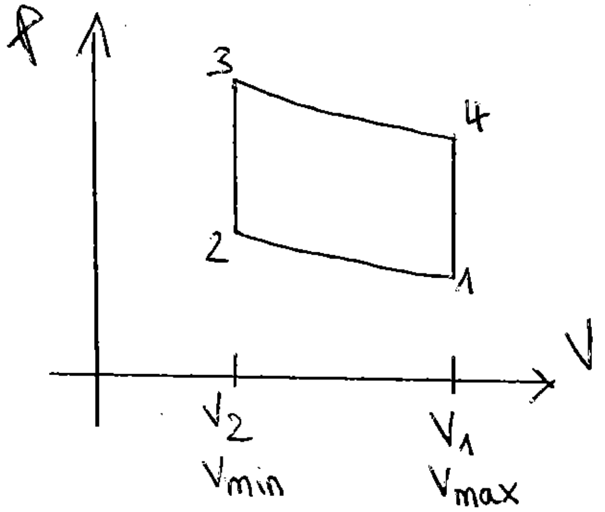
$$t = \frac{Q_1 + Q_2 + Q_3}{P_{cool}} = \frac{393 kJ}{0.21 kW} \cong 1870 s$$

b)

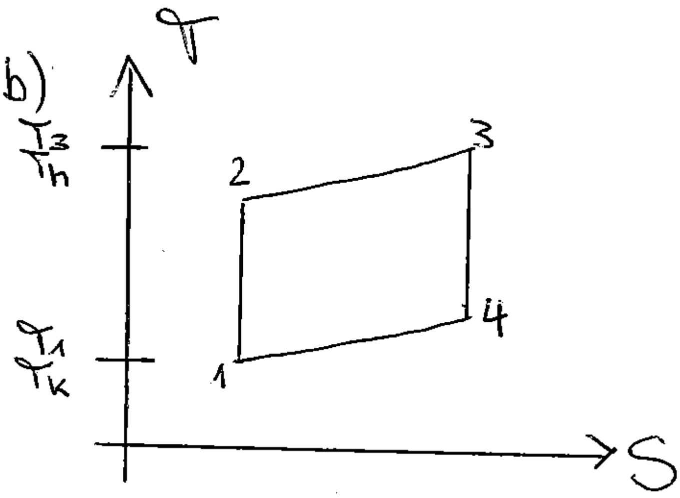


Exercise 4

a)



b)



see also next page!

c) * no work done during isochoric steps $(dV=0)$

$$\Rightarrow W_{2\to3} = W_{4\to1} = 0$$

* no heat transfer during adiabatic steps

$$\Rightarrow W_{1\to2} \overset{\text{1st law}}{=} U_2 - U_1 = \tfrac{3}{2} nR\,(T_2 - T_1)$$

$$W_{3\to4} = U_4 - U_3 = \tfrac{3}{2} nR\,(T_4 - T_3)$$

$$\Rightarrow \Sigma W = \frac{3}{2} nR \left(T_2 - T_1 + T_4 - T_3\right)$$

d) heat is taken from hot reservoir in step $2\to3$

$$\Rightarrow Q_h \overset{\text{1st law}}{=} U_3 - U_2 = \frac{3}{2} nR\,(T_3 - T_2)$$

e) $$\eta = \frac{-\Sigma W}{Q_h} = \frac{-T_2 + T_1 - T_4 + T_3}{T_3 - T_2} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1}{T_2}\,\frac{T_4/T_1 - 1}{T_3/T_2 - 1}$$

$$\frac{T_4}{T_1} \overset{\substack{\text{ideal}\\\text{gas}}}{=} \frac{p_4 V_4}{p_1 V_1} \overset{V_1 = V_4}{=} \frac{p_4 V_4^{\gamma}}{p_1 V_1^{\gamma}} \overset{\substack{\text{adiaba-}\\\text{ticity}}}{=} \frac{p_3 V_3^{\gamma}}{p_2 V_2^{\gamma}} \overset{V_2 = V_3}{=} \frac{p_3 V_3}{p_2 V_2} = \frac{T_3}{T_2}$$

with $\gamma = \frac{c_p}{c_v}$

$$\Longrightarrow \eta = 1 - \frac{T_1}{T_2} \underbrace{\frac{T_4/T_1 - 1}{T_3/T_2 - 1}}_{1} = 1 - \frac{T_1}{T_2} \overset{\text{adiabaticity}}{=} 1 - \frac{1}{\left(\frac{V_1}{V_2}\right)^{\gamma-1}}$$

equations for adiabatic processes:

$$pV^{\gamma} = \text{const}$$
$$TV^{\gamma-1} = \text{const}$$

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b) <u>entropy per cycle</u>:

it is not clearly specified, if the entropy of the mashine or the entropy of the universe is meant here

We therefore regard this question as a bonus question, which gives 1 additional point (the maximum number of point however stays at 16 for the full exercise)

correct answers are: intended answer

* $\Delta S = 0$ for engine (cycle prozess, S is state variable)
* $\Delta S > 0$ for universe (due to isochoric steps)

entropy change of reservoirs:

$$\Delta S_{cycle} = \Delta S_{1\to 2} + \Delta S_{2\to 3} + \Delta S_{3\to 4} + \Delta S_{4\to 1}$$

$$\Delta S_{1\to 2} = \Delta S_{3\to 4} = 0 \quad \text{(adiabatic)}$$

$$\Delta S_{2\to 3} = -\frac{C_V(T_3 - T_2)}{T_3}$$

$$\Delta S_{4\to 1} = -\frac{C_V(T_1 - T_4)}{T_1}$$

Exercise 5

a) duration in reference frame of observer on earth

$$t = \frac{s}{v} = \frac{16.7\ ly}{0.954\ c} = \frac{16.7\ c \cdot y}{0.954\ c} \cong 17.5\ y$$

year

b) distance to star in ref. frame of obs. in spaceship

$$s' = s/\gamma = s\sqrt{1-(v/c)^2}$$
$$= 16.7\ ly \sqrt{1-(\sqrt{0.91})^2}$$
$$= 16.7\ ly \cdot 0.3$$
$$\cong 5.0\ ly$$

c) duration in ref. frame of obs. in spaceship

$$t' = \frac{s'}{v} = \frac{5.0\ ly}{0.954 c} = 5.25\ y$$

or via

$$t' = t/\gamma = t\sqrt{1-(v/c)^2} = 5.25\ y$$