17.10.2018 | TM HS 2018 - Klausur I

CG

Musterlösung: Aufgabe 1

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a) 2 starre Körper ✓ ~~AR~~ ① AR 1

b) $\underline{v}_C = \underline{\omega} \times \underline{AC} = \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} \ell/2 \\ \ell\sqrt{3}/2 \\ 0 \end{pmatrix} = \frac{\omega \ell}{2} \begin{pmatrix} \sqrt{3} \\ -1 \\ 0 \end{pmatrix}$

$\underline{v}_B = \begin{pmatrix} v_x \\ 0 \\ 0 \end{pmatrix}$

$\underline{v}_B \cdot \underline{CB} = \underline{v}_C \cdot \underline{CB}$

$\begin{pmatrix} v_x \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -\sqrt{3} \\ 0 \end{pmatrix} \cancel{\frac{\ell}{2}} = \frac{\omega \ell}{2} \begin{pmatrix} \sqrt{3} \\ -1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -\sqrt{3} \\ 0 \end{pmatrix} \cancel{\frac{\ell}{2}}$

$3 v_x = \frac{\omega \ell}{2} \left(3\sqrt{3} + \sqrt{3} \right)$

$v_x = \frac{2}{3} \sqrt{3}\, \omega \ell$

$\rightarrow \underline{v}_B = \frac{2}{3} \sqrt{3}\, \omega \ell \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ ✓ ✓

~~AR~~ ① AR 2 : x - Komponente

~~AR~~ ① AR 3 : y & z - Komponente

ODER Betrag & Richtung

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a)

$\underline{v}_D = \tilde{v}\frac{\sqrt{10}}{10}\begin{bmatrix}1\\0\\-3\end{bmatrix}$

$\underline{DE} = l\begin{pmatrix}1\\-1\\1\end{pmatrix}$

$\underline{v}_D \cdot \underline{DE} = \underline{v}_E \cdot \underline{DE}$

$\tilde{v}\frac{\sqrt{10}}{10}\begin{pmatrix}1\\0\\-3\end{pmatrix}\cdot l\begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}0\\v\\-5v\end{pmatrix}\cdot l\begin{pmatrix}2\\-1\\1\end{pmatrix}$

$\tilde{v}\frac{\sqrt{10}}{10}(1-3) = -v-5v$

$\tilde{v}\frac{\sqrt{10}}{10}(-2) = -6v$

$\tilde{v} = 3\frac{10}{\sqrt{10}}v \longrightarrow \underline{v}_D = 3v\begin{bmatrix}1\\0\\-3\end{bmatrix} \xrightarrow{\|\underline{v}_D\|} v_D = 3v\sqrt{10}$ ✓ $①^{AR}_4$

$①^{AR}_1$ $①^{AR}_2$ $①^{AR}_3$

b)

$\underline{v}_D = \underline{v}_E + \underline{w}\times\underline{r}_{ED}$ ✓ $①^{AR}_5$ für Ansatz

$\underline{w}\times\underline{r}_{ED} = \underline{v}_D - \underline{v}_E$

$\begin{pmatrix}w_x\\0\\w_z\end{pmatrix}\times l\begin{pmatrix}-1\\1\\-1\end{pmatrix} = \begin{bmatrix}3v\\0\\-9v\end{bmatrix} - \begin{bmatrix}0\\v\\-5v\end{bmatrix} = v\begin{bmatrix}3\\-1\\-4\end{bmatrix}$

$\begin{bmatrix}-w_z l\\-w_z l + w_x l\\w_x l\end{bmatrix} = v\begin{bmatrix}3\\-1\\-4\end{bmatrix} \begin{matrix}\longrightarrow w_z = -3\frac{v}{l}\\ \longrightarrow w_x = -\cancel{\frac{v}{l}} + w_z\\ \longrightarrow w_x = -4\frac{v}{l}\end{matrix} \longrightarrow \underline{w} = -\frac{v}{l}\begin{pmatrix}4\\0\\3\end{pmatrix} \xrightarrow{\|\cdot\|} w = 5\frac{v}{l}$ ✓ $①^{AR}_9$

$①^{AR}_6$ $①^{AR}_7$ $①^{AR}_8$

c) $\underline{w} \neq 0,\ \underline{v}_D \neq 0 \longrightarrow \underline{w}\cdot\underline{v}_D \neq 0 \longrightarrow$ Schraubung

✓ $①^{AR}_{11}$ ✓ $①^{AR}_{10}$

d) $①^{AR}_{12}$ ✓ für Ansatz

$\underline{v}_C = \underline{v}_D + \underline{w}\times\underline{r}_{DC} = 3v\begin{pmatrix}1\\0\\-3\end{pmatrix} + \begin{pmatrix}w_x\\0\\w_z\end{pmatrix}\times\begin{pmatrix}0\\-2l\\0\end{pmatrix} = \begin{pmatrix}3v+2w_z l\\0\\-9v-2w_x l\end{pmatrix}$ ✓

ODER:

$①^{AR}_{13}$

$\underline{v}_C = \underline{v}_E + \underline{w}\times\underline{r}_{EC} = \begin{pmatrix}0\\v\\-5v\end{pmatrix} + \begin{pmatrix}w_x\\0\\w_z\end{pmatrix}\times\begin{pmatrix}-l\\-l\\-l\end{pmatrix} = \begin{pmatrix}w_z\cdot l\\v + l(w_x - w_z)\\-5v - w_x l\end{pmatrix}$ ✓

Kinemate in C: $\begin{Bmatrix}\underline{w}_C\\\underline{w}\end{Bmatrix}$ $①^{KR}_{14}$

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a) $\underline{F}_T = \sqrt{2}\, F\, \underline{e}_{F_T} = \sqrt{2}\, F \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \frac{1}{\sqrt{2}} = F \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ ✓ ①$_2$ ①$_1^{AR}$

$\underline{R} = \underline{F}_T + \underline{F}_V + \underline{F}_G = F \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ -3F \\ 0 \end{pmatrix} + F \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} =$

$= F \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$ ✓ ①$_2^{AR}$ ✓ ①$_3^{AR}$ ✓ ①$_4^{AR}$

b) $\underline{M}_0 = \sum \underline{r}_i \times \underline{F}_i = \underline{r}_T \times \underline{F}_T + \underline{r}_V \times \underline{F}_V + \underbrace{\underline{r}_G \times \underline{F}_G}_{\text{durch } 0 \to \text{ kein Moment bezogen auf } 0} =$ ✓ ①$_5^{AR}$ → Ansatz richtig

$= \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix} \times F \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} e \\ a \\ b \end{pmatrix} \times \begin{pmatrix} 0 \\ -3F \\ 0 \end{pmatrix} =$

$\underline{M}_0 = Fh \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 3F \begin{pmatrix} b \\ 0 \\ -e \end{pmatrix} = F \begin{pmatrix} 3b \\ h \\ -3e \end{pmatrix}$ ✓ ①$_6^{AR}$ ✓ ①$_7^{AR}$ ✓ ①$_8^{AR}$

c) $\underline{M}_T = \sum \underline{r}_{Ti} \times \underline{F}_i = \underline{M}_0 + \underline{r}_{T0} \times \underline{R} = \underline{M}_0 + \underline{R} \times \underline{r}_{0T} =$ ✓ ✓ ✓ ①$_9^{AR}$ → Ansatz

$= F \begin{pmatrix} 3b \\ h \\ -3e \end{pmatrix} + F \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix} = F \begin{pmatrix} 3b \\ h \\ -3e \end{pmatrix} + Fh \begin{pmatrix} -3 \\ -h \\ 0 \end{pmatrix} =$

$= F \begin{pmatrix} 3(b-h) \\ 0 \\ -3e \end{pmatrix} = 3F \begin{pmatrix} b-h \\ 0 \\ -e \end{pmatrix}$ ✓ ①$_{10}^{AR}$ ✓ ①$_{11}^{AR}$ ✓ ①$_{12}^{AR}$

Dyname in T : $\begin{Bmatrix} \underline{M}_T \\ \underline{R} \end{Bmatrix}$ ✓ ①$_{13}^{AR}$