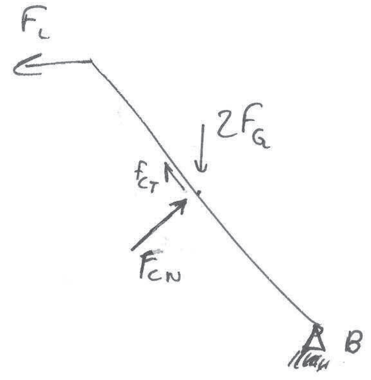
1 a)



B: $2\ell \frac{1}{2}\sqrt{3}\, F_L + 2F_G \frac{\ell}{2} = F_{CN}\,\ell$

$\underline{\underline{\sqrt{3}\, F_L + F_G = F_{CN}}}$ (1)

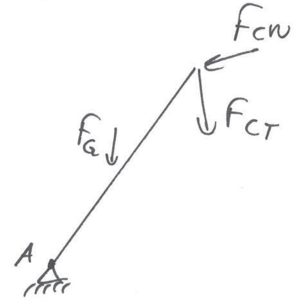
Ruhe: $F_{CN} > 0$

$\sqrt{3}\, F_L > -F_G$

$\underline{\underline{F_L > -\frac{1}{3}\sqrt{3}\, F_G}}$

1 b)

A: $F_G \frac{\ell}{2} \frac{1}{2} + F_{CT} \frac{\ell}{2}\sqrt{3} - F_{CN} \frac{\ell}{2} = 0$



$F_{CT} = \frac{\sqrt{3}}{3} F_{CN} - \frac{\sqrt{3}}{6} F_G = \underline{F_L + \frac{\sqrt{3}}{6} F_G}$

Haften: $|F_{CT}| < \mu_0 F_{CN} = \frac{\sqrt{3}}{4} F_{CN}$

$-\frac{\sqrt{3}}{4} F_{CN} < F_{CT} < \frac{\sqrt{3}}{4} F_{CN}$

$$\underbrace{-\frac{3}{4} F_L - \frac{\sqrt{3}}{4} F_G < F_L + \frac{\sqrt{3}}{6} F_G}_{1} \qquad \underbrace{F_L + \frac{\sqrt{3}}{6} F_G < \frac{3}{4} F_L + \frac{\sqrt{3}}{4} F_G}_{2}$$

1: $\frac{7}{4} F_L > -\frac{5\sqrt{3}}{12} F_G$

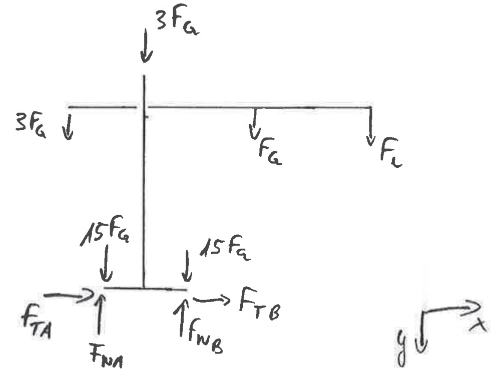
2: $\frac{1}{4} F_L < \frac{\sqrt{3}}{12} F_G$

$$\underline{\underline{-\frac{5\sqrt{3}}{12} F_G < F_L < \frac{\sqrt{3}}{3} F_G}}$$

Aufgabe 2

a) Ja, das System ist statisch unbestimmt, da die Tangentialkräfte nicht eindeutig bestimmt werden können.

b)



R_y: $3F_G + F_G + 3F_G + 30F_G + F_L = F_{NA} + F_{NB}$

$$\underline{37F_G + F_L - F_{NB} = F_{NA}}$$

R_x: $F_{TA} + F_{TB} = 0$

C: $\frac{l}{6}(F_{NB} - F_{NA}) + l\,3F_G - \frac{3}{2}l\,F_G - 3l\,F_L = 0$

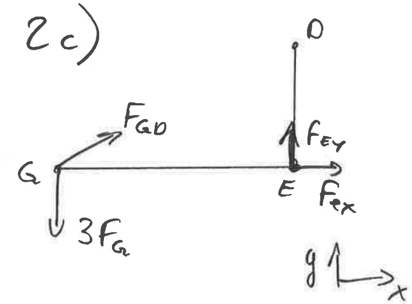
$$\underline{F_{NB} = F_{NA} - 9F_G + 18F_L}$$

Bedingung für Ruhe: $F_{NA} \geq 0$:

$$\underline{F_{NA} \geq 23F_G - \frac{17}{2}F_L \geq 0}$$

$$\hookrightarrow \underline{\underline{F_L \leq \frac{46}{17}F_G}}$$

2c)



$\overline{GD} = \sqrt{l^2 + \frac{l^2}{4}} = \frac{1}{2}l\sqrt{5}$

R_x: $F_{EX} + \frac{2}{\sqrt{5}}F_{GD} = 0$ (1)

R_y: $F_{EY} - 3F_G + F_{GD}\frac{1}{\sqrt{5}} = 0$ (2)

G: $3l\,F_{EY} = 0$ (3)

$$\underline{F_{EY} = 0}$$

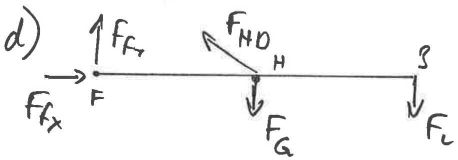
(3) + (2):

$$\underline{\underline{F_{GD} = 3\sqrt{5}\,F_G}}$$

(1): $\underline{\underline{F_{EX}}} = -\frac{2}{\sqrt{5}}F_{GD} = \underline{\underline{-6F_G}}$

---

d)



$\overline{S_{HD}} = \sqrt{\frac{l^2}{4} + l^2\frac{9}{4}} = \frac{l}{2}\sqrt{10}$

R_x: $\underline{\underline{F_{FX}}} = F_{HD}\frac{3}{\sqrt{10}} = \underline{\underline{3F_G + 6F_L}}$

R_y: $F_{FY} + F_{HD}\frac{1}{\sqrt{10}} - F_L = 0$, $\underline{\underline{F_y = -F_L}}$

F: $\frac{3}{2}l\,F_G + 3l\,F_L - F_{HD}\frac{3}{\sqrt{10}}\frac{l}{2} = 0$

$$\underline{\underline{F_{HD} = \sqrt{10}\,(F_G + 2F_L)}}$$

Aufgabe 3

a) Ja, das System ist statisch unbestimmt. Stabkräfte können nicht eindeutig bestimmt werden. Bzw. ein Stab zuviel.

b) Virtuelle Bewegung:

$$\tilde{\hat{v}}_C = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \omega l$$

Virtuelle Leistung:

$$0 = \tilde{P} = \hat{v}_C \cdot \vec{F}_C = \begin{pmatrix} 0 \\ -\omega l \end{pmatrix} \cdot \begin{pmatrix} 0 \\ F_{CG} - F_C \end{pmatrix}$$

$$\rightarrow \underline{\underline{F_{CG} = F_C}} \quad \text{Zugkraft}$$

c) Virtuelle Geschwindigkeiten:

$$\tilde{\hat{v}}_G = \begin{pmatrix} -1 \\ -9 \end{pmatrix} \omega l, \quad \tilde{\hat{v}}_D = \begin{pmatrix} 0 \\ -8 \end{pmatrix} \omega l, \quad \tilde{\hat{v}}_E = \begin{pmatrix} 0 \\ -7 \end{pmatrix} \omega l, \quad \tilde{\hat{v}}_F = \begin{pmatrix} 0 \\ -6 \end{pmatrix} \omega l$$

$$\ldots \tilde{\hat{v}}_P = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \omega l$$

Virtuelle Leistung:

$$0 = \tilde{P} = \tilde{\vec{v}}_G \cdot \vec{F}_G + \sum \tilde{\vec{v}}_i \cdot \vec{F}_i = \begin{pmatrix} -1 \\ -9 \end{pmatrix} \begin{pmatrix} -\frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} \end{pmatrix} F_{AG}\, \omega l \; +$$

$$F_C\, \omega l \,(9+8+7+6+5+4+3+2+1)$$

$$= 5\sqrt{2}\, F_{AG} + 45 F_C$$

$$\hookrightarrow \underline{\underline{F_{AG} = -\frac{9}{2}\sqrt{2}\, F_C}} \quad \text{Druckstab}$$

3d)

Virtuelle Geschwindigkeiten:

$$\tilde{\hat{v}}_J = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \hat{\omega} l, \quad \tilde{\hat{v}}_C = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \omega l, \quad \tilde{\hat{v}}_D = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \omega l, \quad \tilde{\hat{v}}_E = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \omega l$$

$$\tilde{\hat{v}}_F = \begin{pmatrix} 0 \\ -6 \end{pmatrix} \omega l, \quad \tilde{\hat{v}}_L = \begin{pmatrix} 0 \\ -5 \end{pmatrix} \omega l, \quad \tilde{\hat{v}}_M = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \omega l, \quad \tilde{\hat{v}}_N = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$$

$$\tilde{\hat{v}}_O = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \omega l, \quad \tilde{\hat{v}}_P = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \omega l$$

Virtuelle Leistung:

$$0 = \tilde{P} = \tilde{\hat{v}}_J \cdot F_J + \sum v_i F_i + \tilde{\hat{v}}_F \cdot F_F$$

$$= \begin{pmatrix} -1 \\ 3 \end{pmatrix} \hat{\omega} l \cdot \begin{pmatrix} \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} \end{pmatrix} F_J + \begin{pmatrix} 0 \\ -6 \end{pmatrix} \omega l \begin{pmatrix} -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} \end{pmatrix} F_F + F_C\, \omega l\,(+15)$$

$$= -2\sqrt{2}\, F_J - 3\sqrt{2}\, F_F + 15 F_C$$

$$= -5\sqrt{2}\, F_{JF} + 15 F_C$$

$$\hookrightarrow \underline{\underline{F_{JF} = \frac{3}{2}\sqrt{2}\, F_C}} \quad \text{Zugkraft}$$

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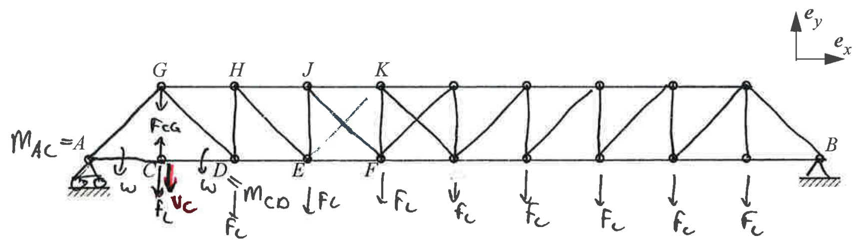
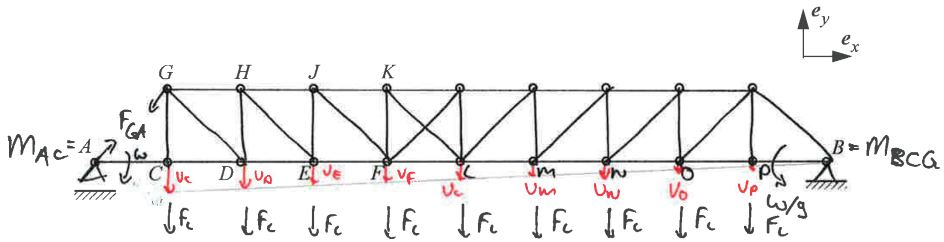
Aufgabe 3 *(Skizzenblatt)*

b)



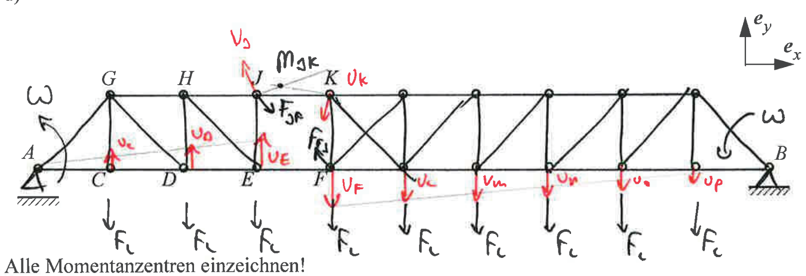
Alle Momentanzentren einzeichnen!

c)



Alle Momentanzentren einzeichnen!

d)



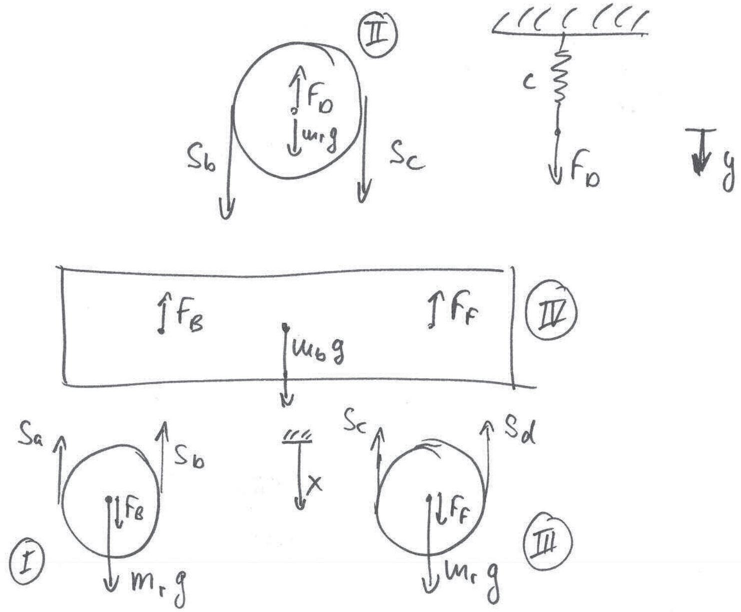
Alle Momentanzentren einzeichnen!

 Reserveskizzen auf der Rückseite

Aufgabe 4

a) Freiheitsgrad $= 1$

b)



c) Ja, der Balken ist zweifach statisch unbestimmt gelagert.

4.d)

I $\begin{cases} m_r \ddot{x} = m_r g + F_B - S_a - S_b & (1) \\ I_C \ddot{\varphi}_1 = r S_a - r S_b & (2) \end{cases}$

II $\begin{cases} 0 = m_r g - F_D + S_b + S_c & (3) \\ I_C \ddot{\varphi}_2 = r S_b - r S_c & (4) \end{cases}$

III $\begin{cases} m_r \ddot{x} = m_r g + F_F - S_c - S_d & (5) \\ I_C \ddot{\varphi}_3 = r S_c - r S_d & (6) \end{cases}$

IV $\quad m_b \ddot{x} = m_b g - F_B - F_F \qquad (7)$

Kraftgesetz der Feder:

$$\underline{\underline{F_d = c y}} \qquad (8)$$

4) e) kinematische Relationen

$$r \dot{\varphi}_1 = \dot{x} \quad \rightarrow \quad \underline{\underline{\dot{\varphi}_1 = \frac{\dot{x}}{r}}}$$

$$r \dot{\varphi}_2 = \dot{x} + r \dot{\varphi}_1 \quad \rightarrow \quad \underline{\underline{\dot{\varphi}_2 = \frac{2\dot{x}}{r}}}$$

$$r \dot{\varphi}_3 = 2\dot{x} + \dot{x} \quad \rightarrow \quad \underline{\underline{\dot{\varphi}_3 = 3 \frac{\dot{x}}{r}}}$$

$$\underline{\underline{\dot{y}}} = \dot{x} + r \dot{\varphi}_3 = \underline{\underline{4 \dot{x}}}$$

f)

$$S_d = c y = 4 c x$$

$$S_c = S_d + \frac{I_c}{r} \ddot{\varphi}_3 = 4 c x + 3 \frac{I_c}{r^2} \ddot{x}$$

$$S_b = S_c + \frac{I_c}{r} \ddot{\varphi}_2 = 4 c x + 5 \frac{I_c}{r^2} \ddot{x}$$

$$S_a = S_b + \frac{I_c}{r} \ddot{\varphi}_1 = 4 c x + 6 \frac{I_c}{r^2} \ddot{x}$$

$$(1) \rightarrow F_B = S_a + S_b - m_r g + m_r \ddot{x} = 8 c x + \left(m_r + 11 \frac{I_c}{r^2}\right) \ddot{x} - m_r g$$

$$(3) \rightarrow F_D = S_b + S_c + m_r g = 8 c x + 8 \frac{I_c}{r^2} \ddot{x} + m_r g$$

$$(5) \rightarrow F_F = S_c + S_d + m_r \ddot{x} - m_r g = 8 c x + \left(m_r + 3 \frac{I_c}{r^2}\right) \ddot{x} - m_r g$$

$$(7) \Rightarrow \quad m_b \ddot{x} = m_b g - F_B - F_F$$

$$= m_b g - \left(m_r + 11 \frac{I_c}{r^2}\right) \ddot{x} - 8 c x + m_r g - \left(m_r + \frac{3 I_c}{r^2}\right) \ddot{x} - 8 c x + m_r g$$

$$\underline{\underline{\left(m_b + 2 m_r + 14 \frac{I_c}{r^2}\right) \ddot{x} + 16 c x = (m_b + 2 m_r) g}}$$